

Why Bassreflex is not Suitable for Low-Frequency Musical Sound Reproduction

Robert-H Munnig Schmidt

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email:rob@rmsacoustics.nl

www.rmsacoustics.nl



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1 Introduction

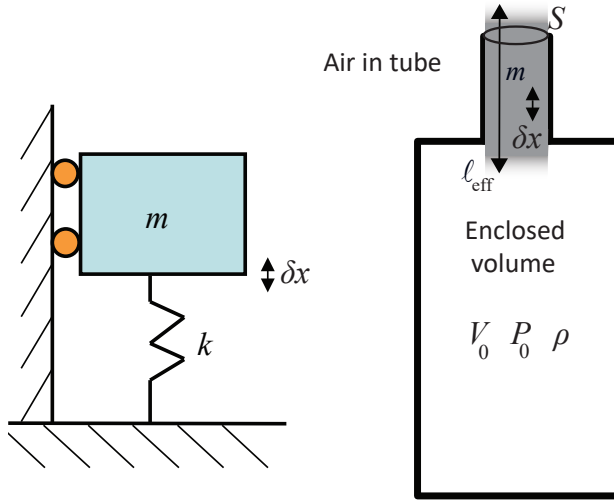
The faithful reproduction of the lowest frequencies by loudspeakers can only be fulfilled by applying large drivers and high power amplifiers, because of the large volume of air that needs to be moved. Generally such large loudspeakers do not get much sympathy from people who strive for home esthetics rather than sound quality due to constraints in space and the dominant techno-style shape of a loudspeaker. In principle small and inconspicuous loudspeakers can be realised by using smaller drivers, however a small radiating surface requires a large diaphragm excursion to compensate the small radiating surface and generate sufficient sound pressure. Unfortunately, a large mechanical excursion will automatically account for an increased distortion level by non-linearity of the actuator and the suspension.

To alleviate this issue, loudspeaker manufacturers have applied both impedance matching devices (horns) and acoustic dynamic phenomena to increase the acoustic output of a loudspeaker system without increasing the diaphragm excursion. This paper deals with the second option, in particular with the bass reflex system, which is still the most widely used example of applying dynamics phenomena to enhance the performance at low frequencies. In a bassreflex system a passive resonating mass is added, which is elastically coupled to the driver diaphragm and partly takes over the low frequency sound generation below a certain frequency while simultaneously reducing the excursion of the main loudspeaker.

The original purpose of this paper was to give students at the university a real life example of how dynamic eigenmodes determine the vibrational properties of mechanical structures. For that reason the dynamic analysis of a bassreflex system is dominant in this paper.

The second (and my personal) goal was, however, to determine the real value of such an approach for high quality loudspeakers. And, unfortunately for the majority of loudspeaker manufacturers, the conclusion is that a bass reflex system is at best a compromise but mostly utterly useless when a transparent reproduction of the low frequency sound of music is strived for.

The paper starts with the Helmholtz Resonator, which is the basis of the bass-reflex principle. Then, after some first thoughts about the applicability of a resonator at low frequencies, a demonstrator is introduced where the principle was shown in the classroom for a bassreflex system with passive diaphragm radiator. This example is analysed for its dynamic properties followed by the same analysis for an air-port resonator. Finally conclusions are drawn from the analysis.



$$\delta V = S \delta x$$

$$\delta P = \lambda P_0 \frac{\delta V}{V_0}$$

$$\delta F = S \delta P = \frac{\lambda P_0 S^2 \delta x}{V_0}$$

$$k = \frac{\delta F}{\delta x} = \frac{\lambda P_0 S^2}{V_0}$$

$$m = \rho S \ell_{\text{eff}}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{\lambda P_0 S^2}{V_0 \rho S \ell_{\text{eff}}}} = \frac{1}{2\pi} \sqrt{\frac{\lambda P_0 S}{V_0 \rho \ell_{\text{eff}}}}$$

$$c = \sqrt{\frac{\lambda P_0}{\rho}} \Rightarrow f_0 = \frac{c}{2\pi} \sqrt{\frac{S}{V_0 \ell_{\text{eff}}}}$$

Figure 1: A Helmholtz resonator consists of an enclosed volume acting as an air-spring with a tube shaped opening, the port. The mass of the air in the port resonates with the stiffness of the air-spring.

2 Helmholtz Resonator

In most cases a so called “Helmholtz Resonator” is used of which the principle is shown in Figure 1. A Helmholtz resonator consists of a combination of an enclosed volume of air (the cabinet) acting as a spring with a moving amount of air (the mass) in a tube, the *bassreflex-port*. A well known example of a standalone Helmholtz resonator is the generation of sound by blowing air over the opening in a bottle. The resonance frequency of a Helmholtz resonator is calculated as follows starting with the stiffness k of the air in the enclosure:

$$k = \lambda S^2 \frac{P_0}{V_0}, \quad (1)$$

with λ being a correction factor for the expansion of air (1.4), S the cross section of the air-port, P_0 the atmospheric pressure and V_0 the volume of the enclosed air. The mass of the air in the port is equal to $\rho_a S_p \ell_{p,\text{eff}}$, where ρ_a equals the density of air and $\ell_{p,\text{eff}}$ equals the effective length of the moving air. The effective length is somewhat larger than the physical length of the port alone as the air immediately near the openings of the port also moves rapidly, decreasing with the distance to the opening. As an empirically found ballpark figure, the effective length equals the length of the port plus approximately 0.73 times the diameter d_p of the port ($\ell_{p,\text{eff}} = \ell_p + 0.73d_p$).

With these values the Helmholtz resonance frequency can be calculated:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k_a}{m_p}} = \frac{1}{2\pi} \sqrt{\frac{\lambda P_0 S_p^2}{V_e \rho_a S_p \ell_{p,\text{eff}}}} = \frac{1}{2\pi} \sqrt{\frac{\lambda P_0 S_p}{V_e \rho_a \ell_{p,\text{eff}}}} \quad (2)$$

With the expression for the speed of sound c_0 the eigenfrequency equals:

$$c_0 = \sqrt{\frac{\lambda P_0}{\rho_a}} \Rightarrow f_0 = \frac{c_0}{2\pi} \sqrt{\frac{S_p}{V_e \ell_{p,\text{eff}}}} \quad (3)$$

When combining a Helmholtz resonator with a loudspeaker in one enclosure, the bassreflex port takes over the sound generation from the loudspeaker at the resonance frequency of the Helmholtz resonator. As a result the diaphragm of the driven loudspeaker hardly moves at that frequency, allowing more electrical power to be supplied to the system, thereby increasing the maximum acoustical output of the system.

3 Bassreflex Systems for Very Low Frequencies

A disadvantage of using a resonator is that it collects energy which is delivered back with some delay after the input signal is terminated, causing coloration of the sound by a delayed resonance at the resonance frequency of the Helmholtz resonator. The delayed resonance can be limited by damping which dissipates the vibration energy into heat. With a standard bassreflex system damping is determined by the loudspeaker-actuator in combination with the amplifier. A second damping factor is the dissipated energy of the port which is difficult to tune as it is influenced by the shape of the port and the amount of damping material used inside the cabinet near the port. This situation almost guarantees differences between each manufactured loudspeaker and to reduce these deviations a special version of the bassreflex principle has been introduced where the moving air is replaced by an additional loudspeaker diaphragm, named a *passive radiator* with well defined dynamic properties. Still, as will be shown in Section 5.2, it is impossible to create a response without any delay unless the benefits of the bassreflex principle are fully sacrificed.

Another and even more important drawback of the bassreflex system is the higher-order ($\approx 4^{\text{th}}$) drop-off below the Helmholtz resonance frequency, which makes it virtually impossible to boost the sound power by filter corrections below this frequency. This higher-order drop-off is best understood when realising that at very low frequencies the bassreflex port is just an acoustic short-circuit between the front and the back side of the loudspeaker, which cancels the total sound pressure, just like with a loudspeaker without an enclosure.

When trying to achieve a response until 20Hz with an acceptable dynamic response without too much delay, one would need to bring the Helmholtz resonance also at 20 Hz and this can only be achieved with extremely large enclosures. The alternative to use a very thin pipe will not work as then the velocity in the pipe becomes too large and turbulence will increase the flow resistance. In fact one can already conclude from mere qualitative reasoning that a bassreflex system does not solve anything for real high end subwoofers. For these reasons the principle is not used for the subwoofers of RMS Acoustics and Mechatronics.

In the following section the dynamic analysis of bass reflex systems is presented, further underlining these conclusive statements on the low-frequency limitations of the principle.

4 Bassreflex Dynamics

As part of the university classroom lectures on dynamics of motion systems, I have often used a demonstrator with two coupled loudspeakers working according to the bassreflex principle. The charm of the system is the easy observability of the dynamic effects and the mental connection to real life systems as most of the students have loudspeakers where the bassreflex principle is used. The demonstrator is based on the same enclosure design that will be presented in the paper on “Sensorless Velocity Feedback Subwoofer”, which also was developed for as a classroom demonstrator, using two large loudspeakers and motional feedback.

The main difference of the approach in this section when compared with the well known Thiele-Small analysis and many other related methods is found in the more mechanical oriented approach. Regular analysis translates the lumped mechanical elements like the rigid body the spring and the damper into their electronic equivalents like inductor, capacitor and resistor. Depending on the method used, the actuator is replaced by a gyrator or transformer and the analysis is further done as if it was an electronic circuit. The usefulness of this electronic equivalent method is proven over the years with many easily applicable computer programs of which the free version of Scan-Speak which can be downloaded at their website is a good example.

While this electronic equivalent method has its advantage in the possibility to use dedicated software from the electronic domain, it is not capable of utilising the knowledge on dynamics which has been gained over the years in the mechanical domain with for instance vibration modal analysis which can model effectively break-up phenomena and decoupling of compliant bodies. The propagation of sound in any medium is a physical phenomenon with a clear relation to the mechanical domain” which is a strong argument to remain in the mechanical domain when searching improvements in reproducing music by loudspeakers. In this respect this section can be seen is a starting point for learning the mechanical dynamics approach on sound reproduction.

5 Bassreflex with Diaphragm Resonator

The enclosure as shown in Figure 2 was originally designed to be used as an active controlled closed-box system as described in a separate paper on velocity feedback. Due to the two loudspeakers that share the same enclosure it allows to experiment with the passively radiating diaphragm principle by using one of the loudspeakers as the active driven loudspeaker and the other as the passive radiator. The damping of each loudspeaker can be controlled by the impedance between the external connections, either from the amplifier for the driven loudspeaker or by a series resistance for the passive radiator.

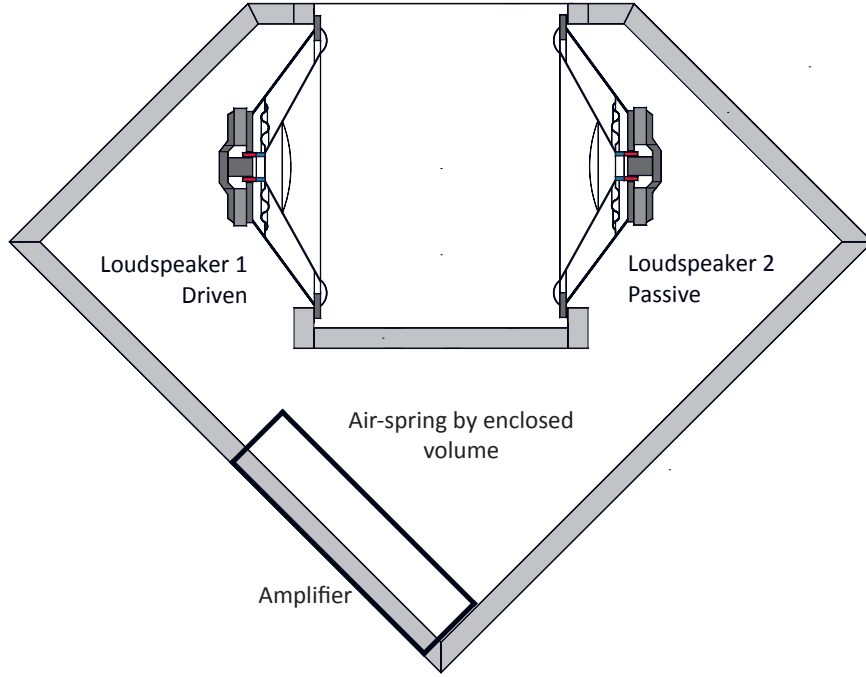


Figure 2: The enclosure of the subwoofer with two loudspeakers. With the bassreflex principle one of the loudspeakers is driven by an amplifier while the other is passively coupled to the driven loudspeaker by the stiffness of the air spring enclosed by the cabinet.

5.1 Frequency Response of Diaphragm Resonator

The bass-reflex principle is directly related to the theory about the dynamic response of two elastically coupled bodies to a force on one of the bodies. This theory¹ shows that the motion amplitude of driven loudspeaker should become zero at the anti-resonance frequency determined by the moving mass of the passive radiator and the stiffness of the coupling spring between the diaphragms, which is determined by the enclosed air.

The stiffness of the air between the two diaphragms can be calculated with the same equation as used with the Helmholtz resonator. With the surface area of the diaphragm $A_d = 0.047 \text{ m}^2$, an air pressure of 10^5 Pa , a volume of the enclosure $V_{\text{en}} = 0.06 \text{ m}^3$ and $\lambda = 1.2$ due to the fibre filling:

$$k_a = \lambda S_d^2 \frac{P_0}{V_e} = 1.2 \cdot 0.047^2 \frac{10^5}{0.06} \approx 4.4 \text{ [N/m]}, \quad (4)$$

For the total stiffness that the passive radiator experiences the stiffness of the suspension needs to be added, which equals the inverse of the compliance ($k_s = 1/0.53 \cdot 10^{-3} = 1.88 \cdot 10^3$ and leads to a total stiffness of:

$$k_{\text{tot}} = (1.89 + 4.4) \cdot 10^3 \approx 6.3 \cdot 10^3 \text{ [N/m]} \quad (5)$$

¹For more detailed background information see the book “The Design of High Performance Mechatronics” as presented on the website <http://rmsacoustics.nl/education.html>

Specs:						
Electrical Data						
Nominal impedance	Zn	4	ohm	Power handling		
Minimum impedance	Zmin	3	ohm	100h RMS noise test (IEC)	--	W
Maximum impedance	Zo	65.7	ohm	Long-term Max System Power (IEC)	--	W
DC resistance	Re	2.6	ohm	Max linear SPL (rms) @ power	--	dB/W
Voice coil inductance	Le	1.6	mH	Short Term Max power	--	W
Capacitor in series with x ohm	Cc	--	uF	Voice Coil and Magnet Parameters		
T-S Parameters				Voice coil diameter	51	mm
Resonance Frequency	fs	19.1	Hz	Voice coil height	32.6	mm
Mechanical Q factor	Qms	9.29		Voice coil layers	4	
Electrical Q factor	Qes	0.38		Height of the gap	8	mm
Total Q factor	Qts	0.37		Linear excursion +/-	13	mm
Ratio fs/Qts	F	--		Max mech. excursion +/-	--	mm
Force factor	Bl	10.3	Tm	Flux density of gap	--	mWb
Mechanical resistance	Rms	1.69	Kg/s	Total useful flux	2.3	mWb
Moving mass	Mms	130.6	g	Diameter of magnet	147	mm
Suspension compliance	Cms	0.53	mm/N	Height of magnet	35	mm
Effective cone diameter	D	24.4	cm	Weight of magnet	2.2	Kg
Effective piston area	Sd	466	cm ²			
Equivalent volume	Vas	159	ltrs			
Sensitivity		91.2	dB			
Ratio BL/ (Re)		6.4				

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Figure 3: Characteristics of the applied loudspeaker, the Peerless XXLS 12.

With the moving mass of 0.13 kg this results in a natural frequency of the passive radiator with this spring equal to:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{6.3 \cdot 10^3}{0.13}} \approx 35 \text{ [Hz]} \quad (6)$$

Measurement of this natural frequency showed a slightly lower frequency of 33 Hz which might indicate that the filling works better in achieving an isothermal compression/expansion ($\lambda < 1.2$) or that the stiffness of the suspension is lower. This small deviation is acceptable within the accuracy of the approximated values and the model is sufficiently correct to take the estimated values for the moving mass and spring stiffness of the air in the enclosure and calculate the response of the two loudspeakers, taking into account all springs and dampers. Figure 4 shows the lumped-element model used to derive the frequency response functions where m_1 equals the mass of the driven loudspeaker and m_2 the mass of the passive radiator. c_1 , c_2 , k_1 and k_2 are the springs and dampers of each element to the enclosure caused by the guiding diaphragm (surround, spider and the electromagnetic damping. In the model the previously found resemblance between radiated power and acceleration is used and both sound-pressure responses are added together for the total sound pressure. This corresponds with the earlier found conclusion that two loudspeakers that generate the same sound pressure by a certain movement of the diaphragm will together generate a sound power that is four times the sound power of one loudspeaker ($P_a \propto p_a^2$, $2p_a \Rightarrow 4P_a$).

Starting with m_1 :

$$m_1 s^2 x_1 = F - c_1 s x_1 - k_1 x_1 + k_{air}(x_2 - x_1) \quad (7)$$

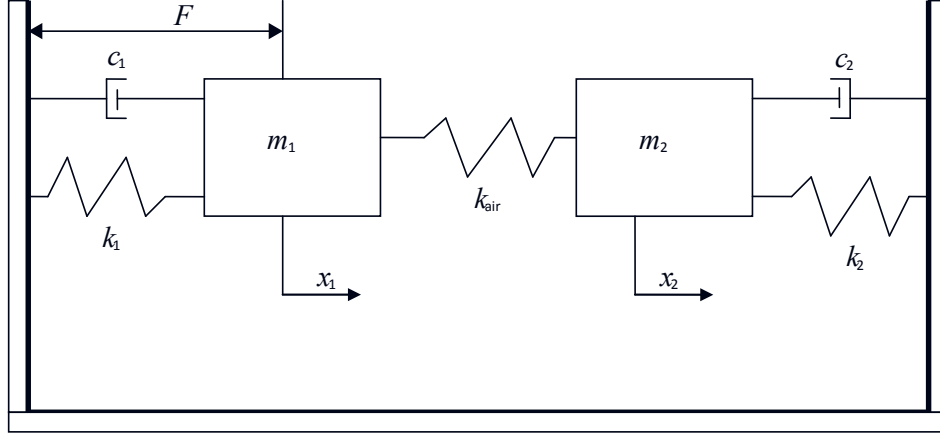


Figure 4: The lumped-element model of the bass reflex system with passive radiator is used to derive the frequency response functions.

From this follows:

$$F = x_1(m_1 s^2 + c_1 s + k_1 + k_{air}) - x_2 k_{air} \quad (8)$$

The motion equation for mass m_2 equals

$$m_2 s^2 x_2 = -c_2 s x_2 - k_2 x_2 + x_1 k_{air} - x_2 k_{air} \quad (9)$$

and the displacement x_2 can be written as function from x_1 :

$$x_2 = x_1 \frac{k_{air}}{m_2 s^2 + c_2 s + k_2 + k_{air}} \quad (10)$$

Filling this in Equation (8) and careful applying some algebra leads to the following equations:

$$\frac{x_1}{F} = \frac{m_2 s^2 + c_2 s + k_2 + k_{air}}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + k_1 k_2 + k_1 k_{air} + k_2 k_{air}} \quad (11)$$

And:

$$\frac{x_2}{F} = \frac{k_{air}}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + k_1 k_2 + k_1 k_{air} + k_2 k_{air}} \quad (12)$$

With:

$$\begin{aligned} a_4 &= m_1 m_2 \\ a_3 &= m_1 c_2 + m_2 c_1 \\ a_2 &= m_1 k_2 + c_1 c_2 + m_2 k_1 + (m_1 + m_2) k_{air} \\ a_1 &= c_1 k_2 + c_2 k_1 + (c_1 + c_2) k_{air} \end{aligned}$$

Replacing s with $j\omega$ and multiplying with $s^2 = -\omega^2$ ultimately leads to the following radial frequency response functions for the sound pressure. Note that this is only a proportionality relation to the sound pressure as only the acceleration is calculated.

To calculate the real sound pressure it must be multiplied with the radiating efficiency of the diaphragm at a certain distance:

$$P_{a,1}(\omega) \propto \frac{m_2 \omega^4 - j \cdot c_2 \omega^3 - (k_2 + k_{\text{air}}) \omega^2}{a_4 \omega^4 - j \cdot a_3 \omega^3 - a_2 \omega^2 + j \cdot a_1 \omega + k_1 k_2 + k_1 k_{\text{air}} + k_2 k_{\text{air}}} \quad (13)$$

and for the passive radiator:

$$P_{a,2}(\omega) \propto \frac{-k_{\text{air}} \omega^2}{a_4 \omega^4 - j \cdot a_3 \omega^3 - a_2 \omega^2 + j \cdot a_1 \omega + k_1 k_2 + k_1 k_{\text{air}} + k_2 k_{\text{air}}} \quad (14)$$

The total sound pressure is then equal to the difference of these equations as being caused by the motion difference between the two diaphragms.

With the help of MATLAB the responses for different levels of damping are calculated using the above equations. By subtracting both responses the sound response is obtained because the difference of movement creates the acoustic pressure/power. The first Bode-plot of Figure 5 shows the effect of the situation when the damping is very low as would be the case when the amplifier has a high output impedance, like a current source. At very low frequencies both masses move in phase until a clear resonance at around 19 Hz. This resonance is caused by the surround diaphragm and spider of both loudspeakers and corresponds with the given resonance frequency characteristics of the used loudspeaker when not mounted in an enclosure. They move both in the same direction so the air volume in enclosure does not change by this movement, causing no additional stiffness.

At a higher frequency the passive radiator will dynamically decouple from the driven loudspeaker because the spring can not supply enough force to accelerate the passive radiator. Eventually this causes a negative peak, the anti-resonance in the response of the driven loudspeaker at the predicted 33 Hz. At the second resonance frequency both masses will move in counter phase. Now each loudspeaker works on half the volume of the enclosure which means that the gas spring of the enclosure is equally divided over each loudspeaker so they both get twice the stiffness of the total enclosed air between both loudspeakers:

$$k = 2 \cdot 4.4 = 8.8 \cdot 10^3 \text{ [N/m]} \quad (15)$$

Adding the stiffness of the diaphragm suspension results in the total stiffness per loudspeaker:

$$k = (1.89 + 8.8) \cdot 10^3 \approx 11 \cdot 10^3 \text{ [N/m]} \quad (16)$$

This results in a resonance frequency of:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 45 \text{ [Hz]} \quad (17)$$

As the diaphragms now move in the opposite direction of each other they will create a sound pressure and as a result the summed response shows a very strong resonance.

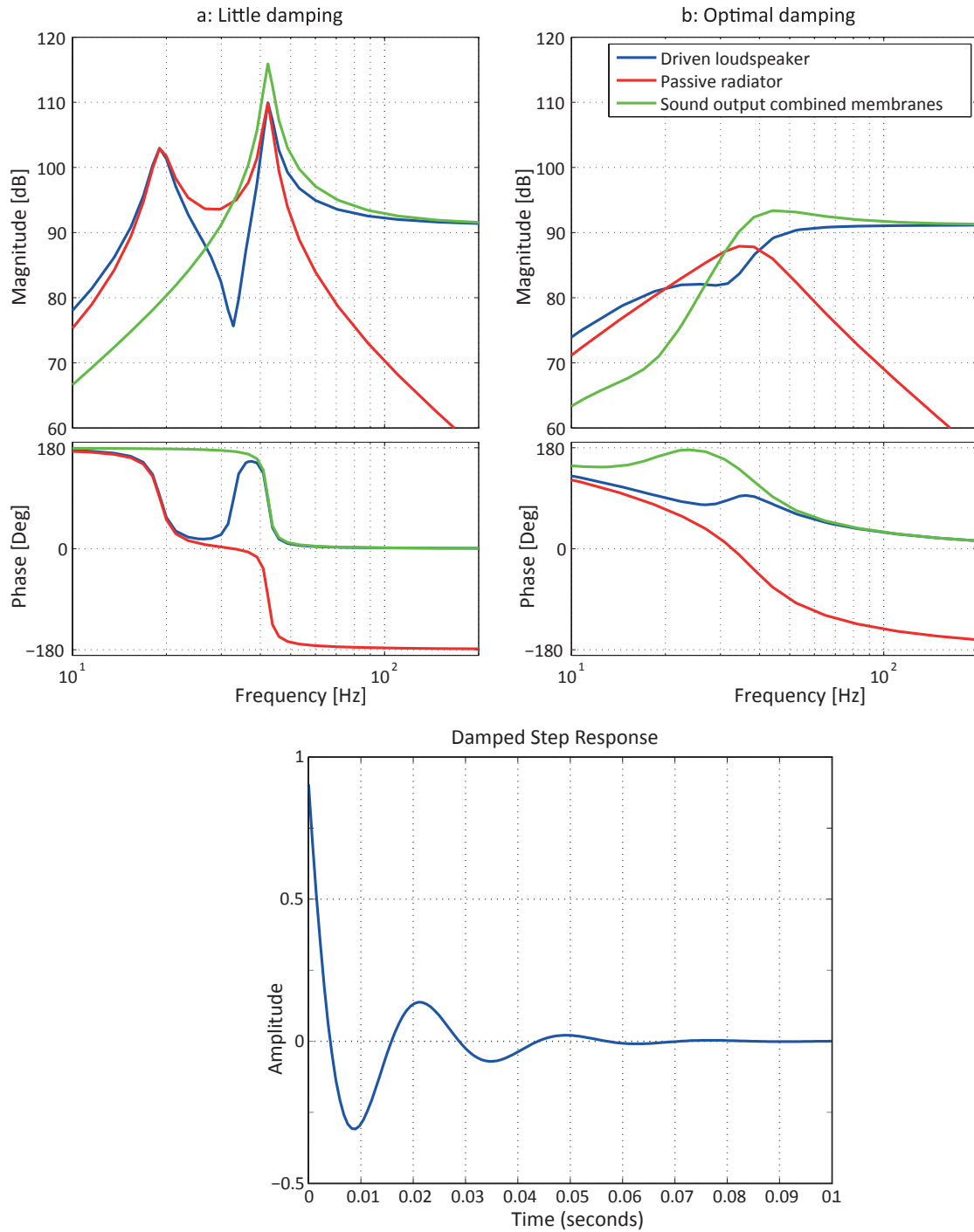


Figure 5: Bode plot of the undamped and damped responses from both the driven diaphragm (blue), the passive radiator (red) and the combined responses (black). Below the first resonance at ≈ 20 Hz both diaphragms move in the same direction and give no sound pressure. The damping matches the situation when the driven loudspeaker is connected to a voltage source amplifier. The damped step response is still quite nervous. This can be improved by also damping the passive radiator but then the beneficial effect of the resonator in the low frequency response is also reduced. Note the fourth order 24 dB/octave slope below the maximum value at 40 Hz in the combined response

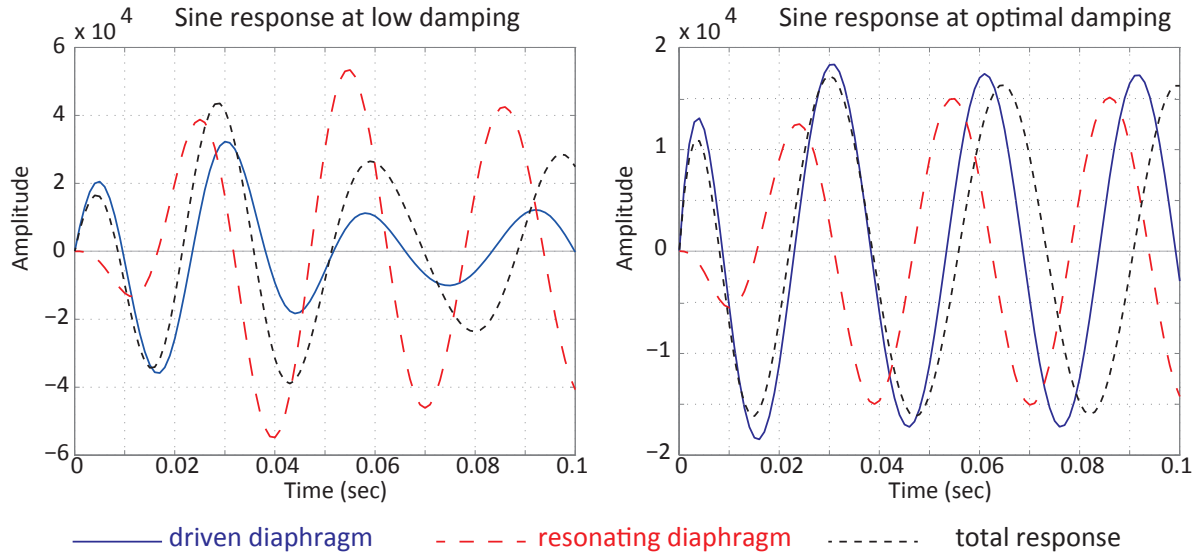


Figure 6: The response of the driven and the resonator diaphragm on a starting sinusoidal signal with a frequency equal to the Helmholtz frequency, shows clearly that first the driven diaphragm will create the sound pressure while after a few periods the resonator takes over. This is most clearly seen with low damping but then also the total response shows overshoot. When both diaphragms have some amount of damping the system can be made to act without overshoot, however, in that case the total response becomes almost equal to the response of the driven diaphragm.

In order to reduce this resonance peak, damping is applied on the driven loudspeaker by using a voltage source amplifier. The effect is shown in the second Bode-plot of Figure 5 and also in the stepresponse. The added damping clearly reduces the high peak in the frequency response but the sound contribution of the second passive diaphragm is also reduced. Still the summed output shows an acceptable resonance with less than 3 dB increase in magnitude at 40 Hz and a -3 dB bandwidth @ 30 Hz. The stepresponse is still not very well damped with almost two full periods of ≈ 2.5 ms (≈ 40 Hz) which would clearly cause an over emphasis of 40Hz at low frequency transients, creating a “boombox” sound. More damping could be applied at the passive radiator to reduce the resonance but this also reduces the beneficial effect of the reduction of the loudspeaker excursion as demonstrated in Figure 5, when comparing a: and b:. Furthermore it is quite expensive to use a full loudspeaker to only contribute some damping at this very limited frequency area. For this reason normally the electromagnetic actuator is omitted with the passive radiator and only the mass is tuned while the surround is made from damping rubber to reduce the resonance to an acceptable level.

5.2 Time Domain Response of Sine Signal

The stepresponse of Figure 5 showed a clear periodic reaction with an undamped resonator. Musical signals are, however, never like a step function but rather like a discontinuous series of sine functions and it is interesting to see the behaviour of

both diaphragms on such signals.

Figure 6 shows the calculated time-domain response of a bassreflex system with a passive resonator for two situations, where in both cases a sinusoidal signal with a frequency, equal to the Helmholtz resonance frequency of the passive resonator diaphragm, is started at $t = 0$. In the first situation both the driven diaphragm and the resonating diaphragm have a moderate level of damping and it clearly confirms that the dip in the frequency response of the driven diaphragm from Figure 5 only occurs after some time, because the resonator needs to build up its energy. Furthermore, at a higher level of damping of both diaphragms, as shown in the right graph of Figure 6, the contribution of the resonating diaphragm to the total sound pressure is almost gone. This also corresponds to the frequency response curves from Figure 5.

Two important conclusions can be drawn from these graphs.

- The often assumed benefit that a bassreflex system could allow the use of a smaller driven loudspeaker than with a closed box enclosure for the same maximum low frequency sound pressure is only true for continuous signals and a low damping resonating diaphragm. With varying and sudden bass, like with a base drum, this benefit is non-existing.
- A low level of damping will always create overshoot in the response but a higher level of damping will reduce the benefit of the bassreflex principle. For this reason small subwoofers for computers and cheap home-movie surround systems are always equipped with undamped resonators, resulting in an exaggerated boom bass, which is sometimes nice when watching a war movie but more often very tiring, while cause a headache.

Like most things in real life, there is no such thing as a free lunch. Mostly benefits on one aspect are counteracted by drawbacks on other aspects.

5.3 Modal analysis of Diaphragm Resonator.

The analytical expression of the frequency response becomes quickly quite complicated when describing higher order dynamic structures with several lumped bodies, springs and dampers. For that reason a dynamic system is often analysed by means of its vibration eigenmodes. This is allowed when the system dynamics are essentially linear as then the total dynamic behaviour can be modelled as the superposition of the behaviour of the system in its separate eigenmodes. The theory of eigenmodes is based on the property that a non-rigid dynamic system, described as a series of bodies connected by springs and dampers, shows several characteristic resonance frequencies. Excitation at these frequencies will cause a synchronous periodic movement of all bodies of the system. The characteristic periodic movement is called an “eigenmode” where the German and Dutch word “eigen” means “own”, reflecting the fact that it is a characteristic system property. The corresponding

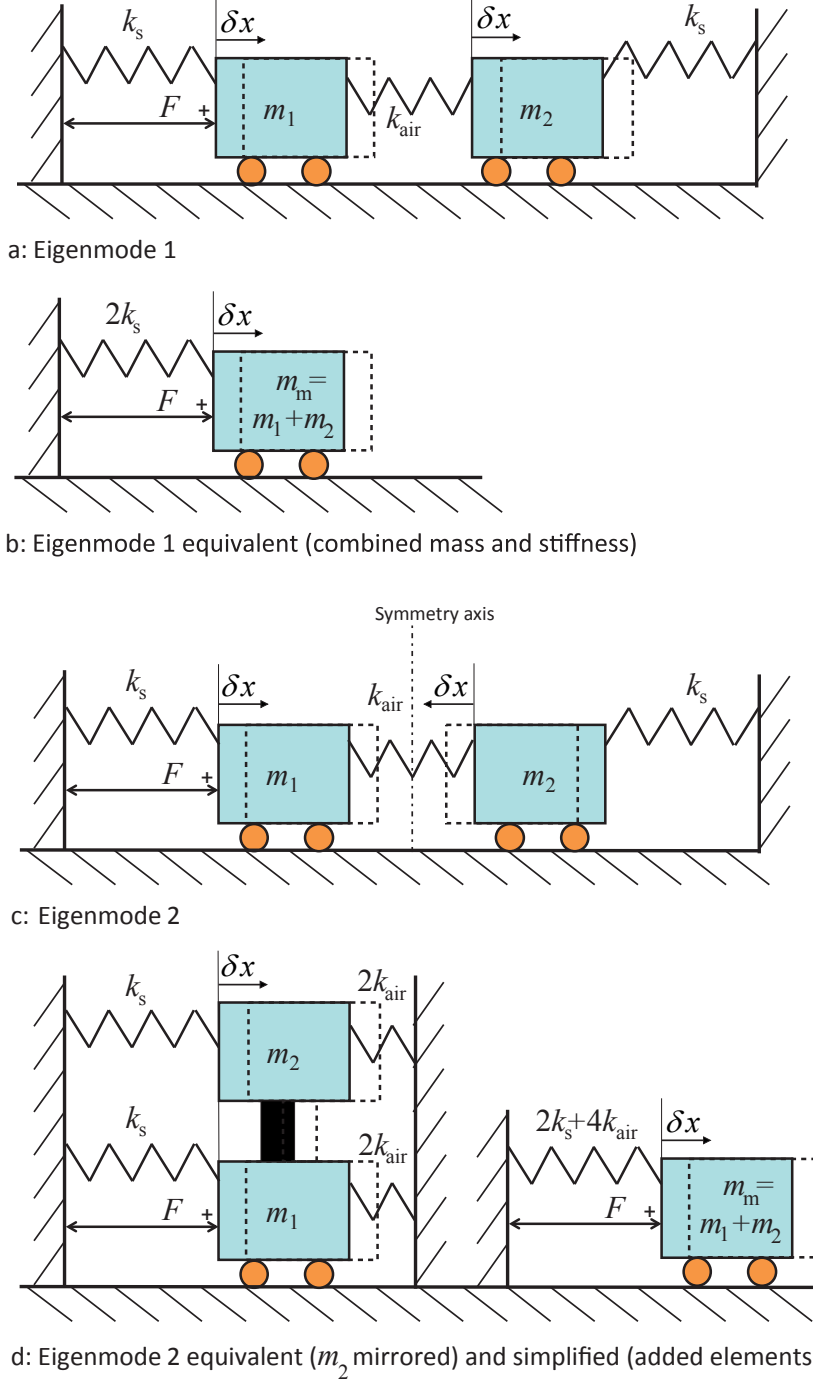


Figure 7: Splitting of the fourth-order dynamic system in two second-order mass-spring systems according to the eigenmodes of the system. The first eigenmode is the rigid-body mode where both diaphragms m_1 and m_2 move in the same direction as if they were one body with modal mass m_m . The mass and suspension stiffness of both diaphragms can then be added to determine the dynamic response of the first eigenmode. The second eigenmode is a bit more complicated to comprehend. It is the mode where both masses move opposite to each other with the same amplitude as if driven by a mechanism. The symmetry allows a mirroring of the second body with its related springs and like with the first eigenmode the modal mass $m_m = m_1 + m_2$. Special attention is needed for the connecting air spring which is a factor four larger in the equivalent simple mass-spring system.

resonance frequency is called the eigenfrequency of that mode, while the movement amplitude as function of the bodies is called the “mode-shape” described by the shape function, a vector notation with terms for each body, where the sign of the value represents the phase at that point relative to the reference body.

As an example the undamped response of Figure 5 shows two clearly distinguishable eigenfrequencies, one at 20 Hz and one at 45 Hz. The eigenmode that corresponds to 20 Hz has a mode shape that is uniform and equal for both loudspeaker diaphragms (Shape function $\begin{bmatrix} 1 & 1 \end{bmatrix}^T$). The second eigenmode at 45 Hz has a mode shape where both diaphragms move opposite to each other with an equal amplitude (Shape function $\begin{bmatrix} 1 & -1 \end{bmatrix}^T$).

From this example one could conclude that the amount of eigenmodes is equal to the square root of the order of the system, which is correct. In principle the exact model of a real system should consist of an infinite amount of springs, dampers and bodies with a corresponding large amount of eigenmodes. In a loudspeaker these are most visible at the higher frequencies where diaphragm-breakup, edge diffraction and other dynamic phenomena represent each their own eigenmodes. In practice the infinite amount of eigenmodes can be reduced to a smaller set by neglecting eigenmodes with a very high eigenfrequency, outside the frequency range of interest. With the example of the passive radiator bassreflex system this set can be restricted to the two mentioned eigenmodes, because this analysis focuses on low frequency sound reproduction.

The reason why this modal analysis is introduced here with this symplified system is its usefulness to explain the anti-resonance of the driven loudspeaker as not being a resonance at all. For that reason it also does not correspond to an eigenmode.

Figure 7 shows the mode-shapes that belong to the two eigenmodes of this fourth-order system while re-arranging the lumped-elements such that their modal behaviour can be directly determined. One should be aware that this simplification is only valid for this specific symmetric situation with equal mass and stiffness values. In the next section it will be shown that an asymmetric system like the bassreflex system with air-port needs some additional adaptations to enable the analysis.

The first eigenmode is the low frequency mode where both diaphragms move in the same direction with equal amplitude and phase, supported by the suspension rubber and spider. With this mode the connecting air-spring is not deforming and its influence can thus be neglected from the analysis. This first eigenmode will have an eigenfrequency which is equal to the eigenfrequency of the unmounted loudspeaker, because the combined masses of the two diaphragms work together with a total, so called “modal mass” $m_m = m_1 + m_2$ on the combined stiffness of the two suspensions:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m_1 + m_2}} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \approx 18 \quad [\text{Hz}] \quad (18)$$

when $m = m_1 = m_2$ and $k = k_1 = k_2$.

This corresponds with the first resonance peak in Figure 8.

The second eigenmode is, as mentioned, related to the opposite movement of the

two bodies, elastically coupled with the connecting air-spring and an eigenfrequency as calculated in Equation (17). The movement amplitude is defined by the masses of the two bodies where a larger mass shows a lower amplitude to correlate the acceleration with the equal force in the connecting spring ($F = ma$).

To imagine both eigenmodes is not extremely difficult. The first eigenmode is the most easy to imagine with the two loudspeakers moving in the same direction, even when only one of them is driven. Like a car with a caravan.

To imagine the second eigenmode one should think of two balls connected with a spring hanging in outer space. An astronaut grabs these balls, stretches the spring and lets the balls loose. Now it is easy to see that the balls first approach each other until the spring is compressed, then they separate again, etc. When the balls have a different mass the smaller ball will move more quickly. In extremis, like with a car as the first body connected via its suspension to the earth as second body, a bumping car will hardly move the earth due to the immense mass difference.

In the complete system the combination of both modes determines the total behaviour and as a result a force in the positive x -direction will also result in a movement of the second body in the positive x -direction but less than with the first eigenmode only. For the first body the response of the second eigenmode is positively added to the first eigenmode while the second eigenmode is subtracted from the first eigenmode for the second body.

It is interesting to see how the elements can be rearranged in ones imagination for the analytical understanding in this case where both moving diaphragms have the same mass. Especially the impact of the air-spring will prove to be significant. In this specific situation with two equal masses the magnitude of the movement of both masses is equal. As a consequence the middle of the air-spring does not move. One might call it a “node” where only force is transferred and for that reason this middle point could be connected to the stationary world like a wall without effect on the eigenmode from a dynamic point of view. This means that each body works on half the spring with double the stiffness as the full spring.

The presence of a imaginary wall in the middle allows to imagine the second body mirrored to the other side of the wall, while it also could be directly connected to the first body. This is allowed for the analysis as the movements are equal for this eigenmode. As a last step all mass and stiffness values can be added like with the first eigenmode. This means that also for this second eigenmode the modal mass m_m becomes equal to $m_1 + m_2$ and the air-spring stiffness appears with a factor four times k_{air} in the equivalent simplified mass-spring system. For the example this stiffness equals $\approx 1.8 \cdot 10^4$ N/m

The combined frequency-response transfer function can be derived from the two responses as shown in Figure 7.

The frequency responses start at low frequencies on a different level because of the difference in stiffness of both modes due to the air-spring. At 18 Hz the common

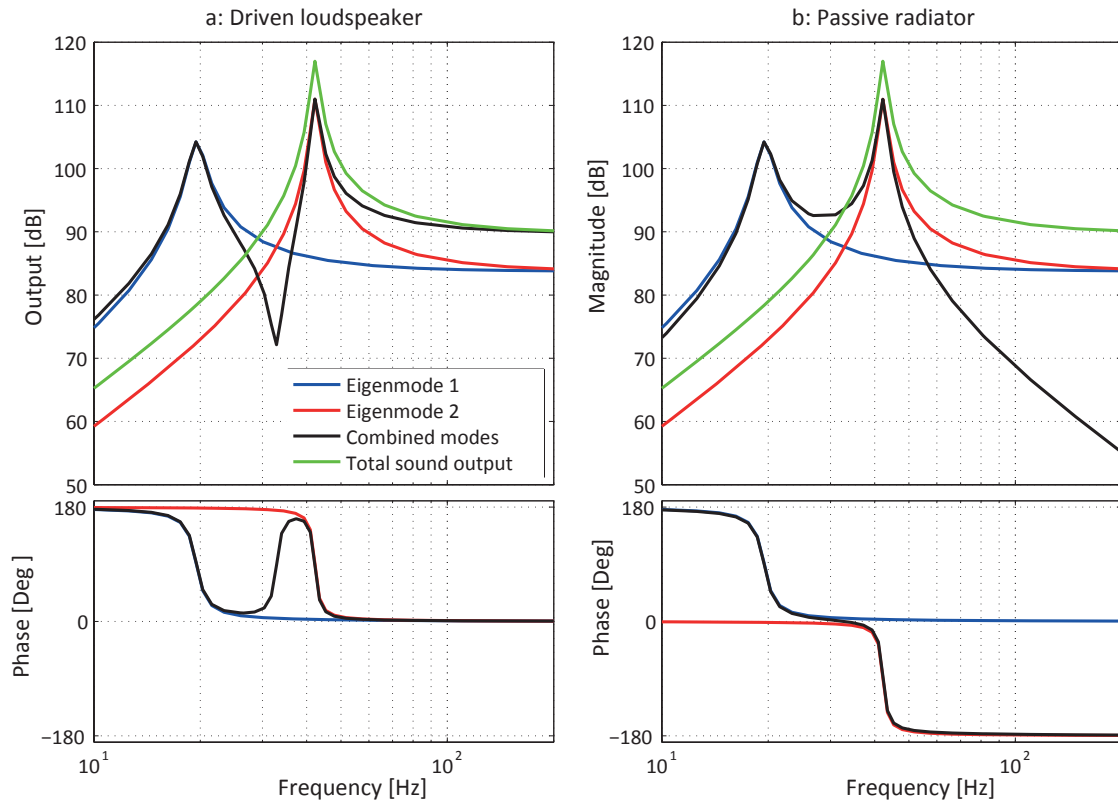


Figure 8: Bode plots of the diaphragms as combined response from the two undamped eigenmodes. The difference in phase of the driven loudspeaker m_1 and the passive radiator m_2 in the second eigenmode causes an “anti-resonance” at approximately 33 Hz because at that frequency the contributions of both eigenmodes are equal but with an opposite sign. With the passive radiator the phases are equal hence the values simply add to the double value (+6dB). Note the combined sound response which is simply double (+6dB) the response of the second eigenmode for each loudspeaker.

eigenfrequency of the first eigenmode is visible in the response of both diaphragms. At ≈ 35 Hz the magnitude of both modes is equal and to determine the combined movement it is important to look at the phase of both modes. For the driven loudspeaker the first eigenmode has almost 0° phase at 35 Hz while the second eigenmode has $+180^\circ$ phase. This means both contributions to the movement of the first diaphragm will cancel each other out and cause the anti-resonance which appears to be no resonance at all but just the combination of two equal opposite movements.

For the passive radiator the situation is different because here the second eigenmode moves in the opposite direction of the driven loudspeaker. An opposite movement means 180° phase and as a consequence the first and second eigenmode have the same 180° phase at 35 Hz for the passive radiator. As a consequence the movements add to a factor two (+6 dB). At higher frequencies first the eigenfrequency of the second eigenmode at ≈ 45 Hz shows its characteristic resonance peak. Above 45 Hz the driven loudspeaker follows a flat response corresponding by a constant acceleration, like with a closed box loudspeaker. The passive radiator however shows a -2 slope at increasing frequencies which is caused by the fact that both eigenmodes approach

the same mass-determined response corresponding with $1/(m_1 + m_2)$ with a phase difference of 180° which means that they cancel each other out more at higher frequencies.

The effect of damping is equal as shown with the analytical calculations. It should be noted that damping of the driven loudspeaker acts on both eigenmodes but with different levels. The quality factor $Q = \sqrt{km}/c$ is higher with the high stiffness of the second eigenmode which means that more damping is needed to suppress this second eigenmode. This is not always sufficiently possible leading to “boom-bass”. A loudspeaker must be designed according to the application by tuning the electromagnetic properties of the actuator to the mass and the enclosure otherwise the result will not be acceptable.

Further it is good to be aware that the sound is only produced by the second eigenmode which matches the green line in the figure which is a factor 2 (+6 dB) above the movement of each diaphragm apart for this second eigenmode only. This factor 2 is due to the fact that the calculation is made for both eigenmodes separately, when driven with a unit force. When combined this force would be divided by two over the two eigenmodes, thereby cancelling the factor 2 in reality.

From this observation one can conclude that it is better to only drive the system in its second eigenmode with sufficient damping. This requires that the first eigenmode is not excited and that is only possible when the passive radiator is also driven. When doing so the system becomes identical to a closed box with two drivers, which do deliver +6dB when compared to one driver because of the twice supplied electrical power.

6 Bassreflex with Air-Port Resonator

A passive radiator is always more expensive than an air-port made by means of a plastic tube. This low cost level is the main reason that the latter is mostly applied even though it is more difficult to achieve a well defined low frequency behaviour. Figure 9,a: shows a schematic drawing of the principle where the passive radiator is replaced by a volume of air contained within a tube that is open both to the outside and to the inside of the enclosure. The air-volume in the bassreflex-port will act as the second passively radiating body and determine a resonating eigenfrequency with the spring of the enclosure and the mass of the driven loudspeaker diaphragm. Unfortunately the modal analysis of this system is less simple as with the previously described symmetrical system. This is caused by the difference in diameter of the bassreflex port and the loudspeaker diaphragm and the different mass values. Still it will be shown that the same “anti-resonance” effect occurs on the driven loudspeaker as with the passive radiator. This corresponds approximately with the eigenfrequency of a Helmholtz resonator determined by the air-mass in the port and the enclosed volume of air as explained in Figure 1. It is also important to note that the same dynamic limitations that were described in Section 5.2 are valid

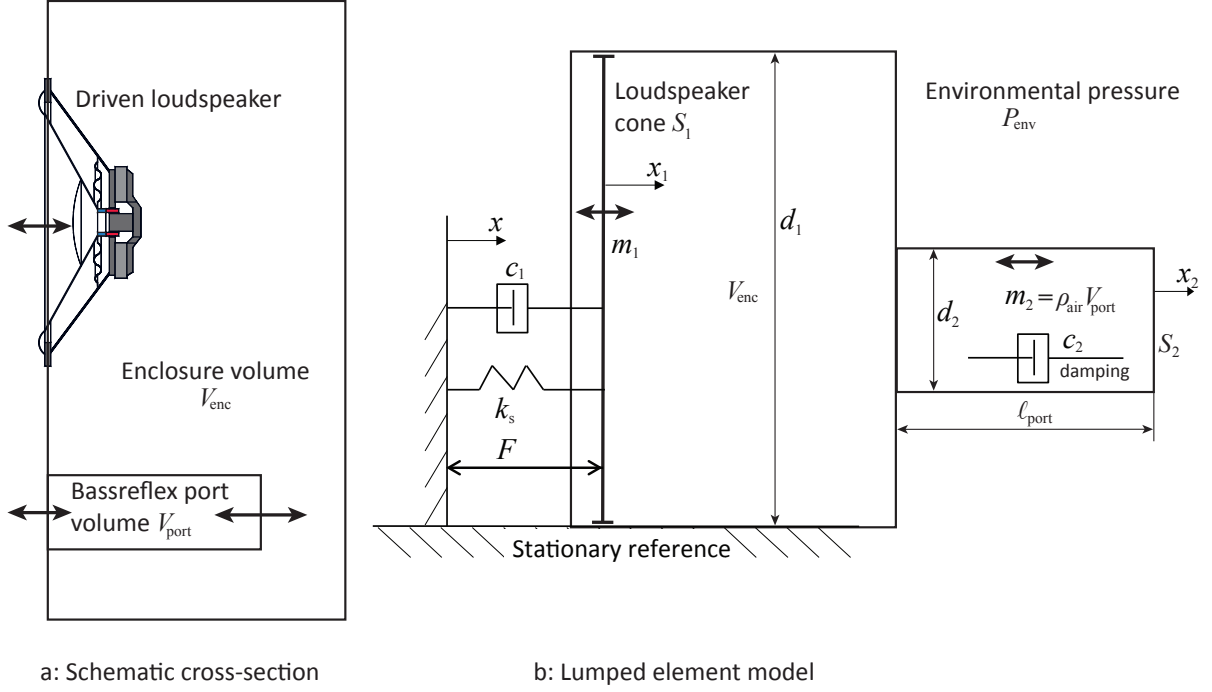


Figure 9: A normal bassreflex loudspeaker enclosure (a:) applies a volume of air as passive radiator. This volume of air is enclosed by a pipe or port that is open at both sides, connecting the enclosure volume to the environment. The mass of the volume of air inside the air-port or bassreflex-port port will resonate with the stiffness of the air spring by the enclosure volume causing a reduction of the excursion amplitude of the driven loudspeaker diaphragm at that frequency. The lumped element equivalent scheme (b:) is used to derive the frequency transfer functions.

for bassreflex systems with an air-port resonator, because, as will be shown, the dynamics are essentially equal.

6.1 Frequency Response of Air-Port Resonator

The frequency response for both moving elements is determined with the help of the lumped-element model of Figure 9,b:. New terms in this model are the radiating surfaces $S_1 = 0.25\pi d_1^2$ for the driven loudspeaker and $S_2 = 0.25\pi d_2^2$ for the opening of the bassreflex port, where d stands for the diameter. The mass of the passive radiator is determined by the volume of the port $V_{\text{port}} = S_2 \ell_{\text{eff}}$ and the density of air ($\rho \approx 1.2 \text{ kg/m}^3$). As mentioned with the Helmholtz resonator, the effective length ℓ_{eff} equals the length of the port plus approximately 0.73 times the diameter of the port ($\ell_{\text{eff}} = \ell + 0.73d_2$). The factor λ for the adiabatic compression/expansion of the air in the enclosure can be accounted with as a factor reducing the volume of air in the enclosure V_e .

Starting with the driven loudspeaker with mass m_1 :

$$m_1 s^2 x_1 = F - \frac{x_1 S_1}{V_e} P_0 S_1 - x_1 k_s + \frac{x_2 S_2}{V_e} P_0 S_1 - c_1 s x_1 \quad (19)$$

where k_s equals the stiffness of the surround suspension of the loudspeaker and P_0 equals the air pressure of the environment ($\approx 10^5$ Pa). Both fractions in the above equation represent the relative volume change by a movement of the respective elements which, after multiplication with the average environmental pressure and the radiating surface S_1 of the loudspeaker, gives the force on that surface due to the movement of each element.

Written as force in terms of x_1 and x_2 this equation is written as:

$$F = x_1 \left(m_1 s^2 + c_1 s + \frac{S_1^2 P_0}{V_e} + k_s \right) - x_2 \frac{S_1 S_2 P_0}{V_e} \quad (20)$$

Doing the same steps as with the loudspeaker the motion equation for the passive radiating air-mass m_2 can be derived, resulting in the following relation between x_1 and x_2 :

$$x_2 \left(m_2 s^2 + c_2 s + \frac{S_2^2 P_0}{V_e} \right) = x_1 \frac{S_1 S_2 P_0}{V_e} \quad (21)$$

To simplify further calculations three stiffness terms are defined:

$$k_1 = \frac{S_1^2 P_0}{V_e} + k_s, \quad k_2 = \frac{S_2^2 P_0}{V_e}, \quad \text{and} \quad k_3 = \frac{S_1 S_2 P_0}{V_e} \quad (22)$$

With these terms Equation (20) is simplified into:

$$F = x_1 (m_1 s^2 + c_1 s + k_1) - x_2 k_3 \quad (23)$$

and with Equation (21) the displacement x_2 can be written as function from x_1 :

$$x_2 = x_1 \frac{k_3}{m_2 s^2 + c_2 s + k_2} \quad (24)$$

Filling this in Equation (23) and careful applying some algebra leads to the following transfer functions from force to motion:

$$\frac{x_1}{F} = \frac{m_2 s^2 + c_2 s + k_2}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + k_1 k_2 - k_3^2} \quad (25)$$

and:

$$\frac{x_2}{F} = \frac{k_3}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + k_1 k_2 - k_3^2} \quad (26)$$

with:

$$a_4 = m_1 m_2$$

$$a_3 = m_1 c_2 + m_2 c_1$$

$$a_2 = m_1 k_2 + c_1 c_2 + m_2 k_1$$

$$a_1 = c_1 k_2 + c_2 k_1$$

Replacing s with $j\omega$ and multiplying the numerator with $s^2 = -\omega^2$ to get the acceleration response, ultimately leads to the following proportional radial frequency response functions for the soundpressure of the loudspeaker diaphragm:

$$P_{a,1}(\omega) \propto \frac{m_2\omega^4 - j \cdot c_2\omega^3 - k_2\omega^2}{a_4\omega^4 - j \cdot a_3\omega^3 - a_2\omega^2 + j \cdot a_1\omega + k_1k_2 - k_3^2} \quad (27)$$

To retain the same proportionality for the soundpressure of the passive radiator it is necessary to correct for the much smaller radiating surface for which reason the transfer function is multiplied with the ratio S_2/S_1 :

$$\begin{aligned} P_{a,2}(\omega) &\propto \frac{-k_3 \frac{S_2}{S_1} \omega^2}{a_4\omega^4 - j \cdot a_3\omega^3 - a_2\omega^2 + j \cdot a_1\omega + k_1k_2 - k_3^2} \\ &= \frac{-k_2\omega^2}{a_4\omega^4 - j \cdot a_3\omega^3 - a_2\omega^2 + j \cdot a_1\omega + k_1k_2 - k_3^2} \end{aligned} \quad (28)$$

The total sound pressure is then equal to the difference of these equations as being caused by the motion difference between the driven loudspeaker and the moving air in the bassreflex port.

As an example the applied loudspeaker of the previous part is used in the same enclosure cabinet while the passive radiating diaphragm is replaced by a tube with a diameter of 50 mm and a length $\ell = 100$ mm. The air volume is then approximately 0.00027 m^3 because, as mentioned before, also the air just outside the port has to be taken into account, giving an effective length $\ell_{\text{eff}} \approx 140$ mm. The resulting moving mass of the passive radiator is then approximately $0.33 \cdot 10^{-3} \text{ kg}$. This is more than a factor 100 below the moving mass of the active driven loudspeaker diaphragm and one would expect little effect. The frequency response functions for the little damped and optimally damped situation are calculated in MATLAB and shown in Figure 10, unexpectedly indicating a comparable dynamic characteristic as with the passive radiating diaphragm. As will be showed with the modal analysis this is caused by the ratios between the active surfaces of the air in the tube and the loudspeaker diaphragm. This is very prominently shown with the first eigenfrequency which is significantly below the 20 Hz resonance frequency of the unmounted loudspeaker. When looking at the modal mass analysis in the next section it is showed that the small mass of the air is perceived as a large mass on the driven loudspeaker diaphragm to the ratio of the radiating surfaces squared.

6.2 Stepresponse of Air-Port Resonator

The stepresponse from Figure 10 is calculated for four different settings of the mainly resistive amplifier output impedance. A true voltage source amplifier which almost all modern amplifiers are, shows a better damped stepresponse than the passive radiator with only one period of delayed response. This difference in dynamic behaviour is related to a somewhat higher damping of the air-port by the high velocity

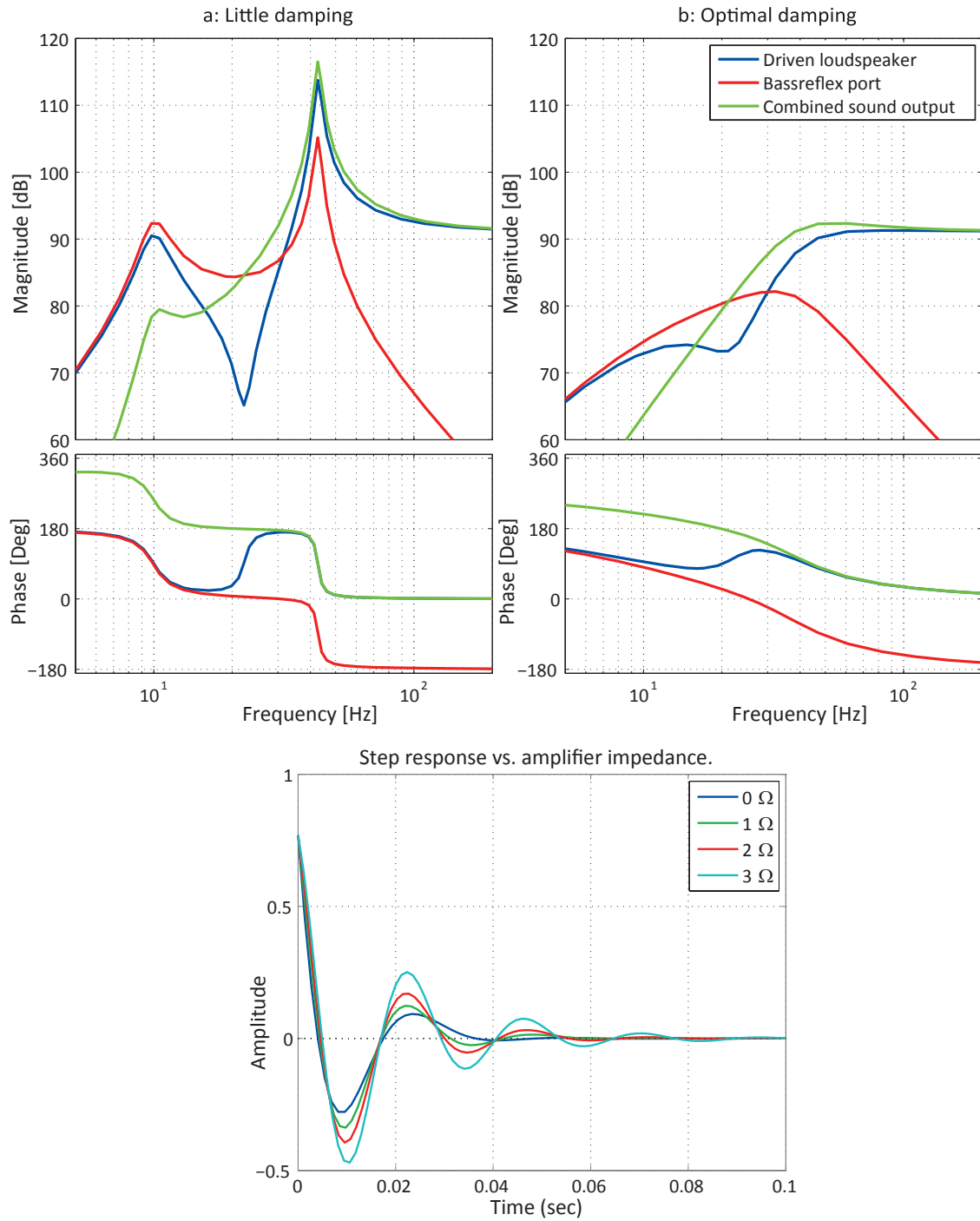


Figure 10: The Bode plots with little (a:) and optimal (b:) damping and step response with different levels of damping of the bassreflex system designed with the same 60 L enclosure as the passive loudspeaker diaphragm system of the previous section, where the passive radiating diaphragm is exchanged by an air volume in a pipe of 50 mm diameter and 100 mm length. When provided with the damping caused by a voltage source amplifier the frequency responses look almost equal as with the passive radiating diaphragm. The step response is improved although it is clearly seen that an amplifier with a non-zero output impedance rapidly results in a deterioration with a strong delayed resonance after a transient.

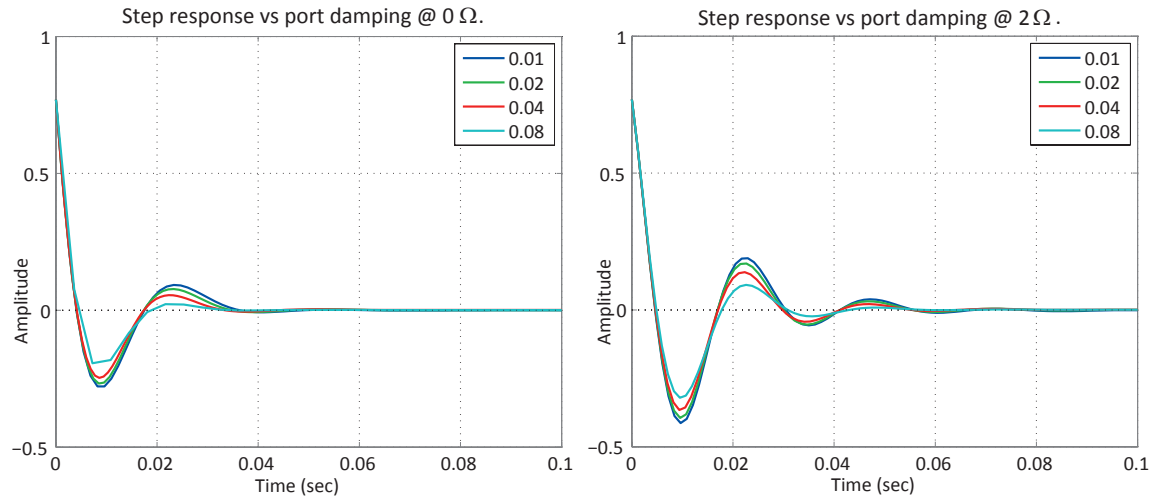


Figure 11: The impact of the damping in Ns/m by the port on two settings of amplifier impedance. With a voltage source amplifier ($0\ \Omega$) the level of port-damping influences the magnitude rather than the periodicity of the response while a high output impedance amplifier ($2\ \Omega$) benefits more of a stronger damped port.

of the air than was the case with the modelled undamped passive diaphragm of the previous case. An increasing amplifier impedance shows a significant effect on the delayed resonance with almost three periods when the impedance of the amplifier becomes equal to the resistive value of the loudspeaker impedance. Even though this is a high value especially tube amplifiers often show an output impedance in the Ohmic range due to lack of feedback and these amplifiers require a higher level of internal damping by for instance the port. This is demonstrated in Figure 11 where four different levels of port damping are calculated for two settings of amplifier impedance. Both situations show a beneficial effect of increased port damping but the effect is most prominent with the high output impedance amplifier. This indicates that a less fortunate amplifier loudspeaker combination can be improved by increasing the port damping. A reason why several people prefer to combine their tube amplifier with a bassreflex loudspeaker over a closed box loudspeaker.

The beneficial effect of the port damping on the dynamics is however at a sacrifice of noise as most of the energy is dissipated in turbulence around the edges of the port. This can be improved by rounding the edges but then the damping is decreased and one might insert a piece of fibre padding or rubber foam with open cells inside the tube to increase the damping. This again is quite unpredictable, resulting in a larger spread in performance of different loudspeakers with the same design. On the other hand it gives the possibility to tune a loudspeaker to the amplifier and with suitable measuring microphones one can even optimise the system for the listening room to a limited extent.

6.3 Modal Analysis of Air-Port Resonator

It was previously explained that the modal analysis of a “normal” mechanical system consisting of lumped bodies, springs and dampers is based on the principle that each eigenmode can exist independent of the others and will show a resonance when excited in its eigenfrequency. It also implies that no other external forces act on the system other than the excitation force by the actuator and the forces in the springs and dampers that connect the bodies to each other and to the stationary world.

The passive radiator had the same diameter as the driven loudspeaker for which reason the connecting air could be modelled straightforward as a mechanical spring acting equally on both diaphragms. The system with a bassreflex port is however quite different as the driven loudspeaker will experience another stiffness value by the enclosed air volume as the air-mass in the port, due to the diameter difference that comes squared in the equation for the stiffness value. Furthermore the volume of air acts like a compressible medium creating forces to all surfaces inside the enclosure.

The easy part is the fact that also in this case it is allowed to limit the relevant eigenmodes to just two as the non-modal analysis shows two clearly distinguishable eigenfrequencies, corresponding to a first eigenmode where both bodies (diaphragm and air-column in the bassreflex port) move in the same direction and a second eigenmode where they move opposite to each other.

The first eigenmode will have a mode-shape where the air in the bassreflex port will show a higher amplitude than the loudspeaker diaphragm in the ratio of the cross-section of the loudspeaker and the bassreflex port. When assuming the air to be incompressible, the corresponding shape function would equal $\begin{bmatrix} 1 & \frac{S_1}{S_2} \end{bmatrix}^T$. The assumption of incompressibility at the low frequency is based on the understanding that at this frequency the air in the port will not yet receive much motion resistance, hence not exert large forces. It will be shown that this assumption is only allowed for a very rough approximation and in any case the mass of the air is accelerated with a large factor higher than the first body and this has a very interesting effect on the equivalent modal mass as observed at the point of excitation, which is the first moving mass. This is best explained with mathematics, starting with Newton's second law on inertia ($F = ma$) with the variables as defined in Figure 9:

$$F_a = m_m a_1 = m_1 a_1 + F_r \implies m_m = m_1 + \frac{F_r}{a_1} \quad (29)$$

where a_1 equals the acceleration of the driven loudspeaker with mass m_1 and F_r equals the reactive force by the mass m_2 of the air in the bassreflex port. This reactive force is equal to the pressure P_e that is created by the force $F_2 = m_2 a_2$ that accelerates the air in the port.

$$F_r = P_e S_1 = F_2 \frac{S_1}{S_2} = m_2 a_2 \frac{S_1}{S_2} = m_2 a_1 \left(\frac{S_1}{S_2} \right)^2 = m_2 a_1 \left(\frac{d_1}{d_2} \right)^4 \quad (30)$$

Note that in the relation between the acceleration levels the condition of incompressibility is assumed. Both equations combined give the following value for the modal

mass:

$$m_m = m_1 + m_2 \left(\frac{S_1}{S_2} \right)^2 = m_1 + m_2 \left(\frac{d_1}{d_2} \right)^4 \quad (31)$$

With $S_1 = 0.0466 \text{ m}^2$ for the driven loudspeaker, $S_2 = 0.25\pi 0.05^2 = 0.002$ for the port and $m_2 = 0.33 \cdot 10^{-3} \text{ kg}$, the modal mass for the first eigenmode becomes equal to $m_m = m_1 + 0.18 = 0.13 + 0.18 \approx 0.3 \text{ kg}$. This means that the mass of the air is even more dominant than the mass of the driven loudspeaker for this eigenmode. In reality it is necessary to take into account the real finite stiffness for the connecting air spring. It is well imaginable that a lower stiffness value than infinite will create a smaller movement of the mass of the air in the bassreflex port, thus reducing the reactive force. Most probably the modal mass component of the air in the bassreflex port is approximately equal to the mass of the loudspeaker diaphragm like is the case for the second eigenmode as will be shown further on.

The stiffness of the connection of the first eigenmode to the stationary enclosure is equal to the stiffness of the suspension of the driven loudspeaker. With this stiffness the more than doubled mass will result in a significantly lower eigenfrequency at $\approx 1/\sqrt{2}$ times the eigenfrequency of the unmounted loudspeaker which was $\approx 18 \text{ Hz}$. The resulting resonance at around 12 Hz corresponds with the value shown in Figure 10.a:. Unfortunately this lower frequency does not mean that the loudspeaker will reproduce sound at this frequency as the sound pressure is not produced by the first eigenmode, which was also the case with the passive diaphragm version. Even though the air in the port moves faster in the ratio of the radiating surfaces, the same ratio compensates the effect on sound pressure as it is linear proportional to both surface and excursion ($P_a \propto Sx$).

For the sound radiation the second eigenmode is the determining factor and this analysis is even more complicated because now the compressibility of the air must be taken into account. The starting point for this modal analysis is the assumption that the enclosure is small in respect to the wavelength of the sound at the eigenfrequency of the second eigenmode. From Figure 10.a: this eigenfrequency is expected around 45 Hz where the second resonance is shown. This corresponds to a wavelength of several metres so the condition is met and as a consequence the air pressure can be assumed homogeneous inside the enclosure. This means that the forces acting on both moving masses will relate to the radiating surfaces S_1 and S_2 . At the eigenfrequency of the second eigenmode the system is in full equilibrium and assuming no energy is dissipated it will keep resonating at this frequency. In that case the relative periodic accelerations and the directly proportional relative periodic displacements of both bodies can then be calculated as follows using Newton's second law with the necessary equilibrium in pressure inside the enclosure:

$$\begin{aligned} F_1 = S_1 P_e = m_1 a_1 = m_1 s^2 x_1 &\Rightarrow x_1 = \frac{S_1 P_e}{m_1 s^2} \\ F_2 = S_2 P_e = m_2 a_2 = m_2 s^2 x_2 &\Rightarrow x_2 = \frac{S_2 P_e}{m_2 s^2} \end{aligned} \quad (32)$$

This enables to write down the following ratio between x_1 and x_2 :

$$\frac{x_1}{x_2} = \frac{m_2 S_1}{m_1 S_2} \quad (33)$$

The sound pressure is a function of the excursion and the radiating surfaces ($P_a \propto xS$) giving:

$$\frac{P_{a,1}}{P_{a,2}} = \frac{x_1 S_1}{x_2 S_2} = \frac{m_2 S_1^2}{m_1 S_2^2} \quad (34)$$

With the numbers from the example this ratio equals approximately 1.4, meaning both radiating surfaces act almost equal on the sound pressure at the eigenfrequency of the second eigenmode. The determination of the eigenfrequency is based on the fact that there should be a neutral zone in the air spring as both bodies move opposite to each other. This means that there is a dividing plane inside the enclosure where the molecules of air stand still. This plane is determined by the volume change which is related to a displacement of both diaphragms of which the relation is given by Equation (33).

$$\frac{V_1}{V_2} = \frac{x_1 S_1}{x_2 S_2} = \frac{m_2 S_1^2}{m_1 S_2^2} \quad (35)$$

which is not unexpectedly the same relation as between the sound pressures with a value of 1.4 for the practical example, meaning that about 60% of the enclosure volume is used by the loudspeaker and 40% by the bassreflex-port. These findings all point stronger and stronger to the previously found similarity between the passive radiator and the port-loaded bassreflex system and indeed this is a true finding. If the ratio value was equal to one the systems would be exactly the same and this can be arranged in this example by increasing the air mass with a longer bassreflex port. The eigenfrequency of the second eigenmode is then equal to the value found with the passive radiator with ≈ 45 Hz but even with the given dimensions the system acts almost the same. As a check whether this reasoning is true the eigenfrequency of the second eigenmode can be calculated on both the loudspeaker mass m_1 and bassreflex port mass m_2 each with the stiffness of their own part of the enclosure volume using Equation (30) of the paper on “Low Frequency Sound Generation by Loudspeaker Drivers”. Using the values for the loudspeaker with $S_{d,1} \approx 0.0466 \text{ m}^2$ and $V_{e,1} = 0.036 \text{ m}^3$, and for the moving air with $S_{d,2} \approx 0.002 \text{ m}^2$ and $V_{e,2} = 0.024 \text{ m}^3$ and taking for both stiffness values $\lambda = 1.2$ and $P_0 = 10^5 \text{ Pa}$, the following is obtained:

$$\begin{aligned} k_1 &= \lambda S_{d,1}^2 \frac{P_0}{V_{e,1}} = 1.2 \cdot 0.0466^2 \frac{10^5}{0.036} = 7.2 \cdot 10^3 \text{ [N/m]} \\ k_2 &= \lambda S_{d,2}^2 \frac{P_0}{V_{e,2}} = 1.2 \cdot 0.002^2 \frac{10^5}{0.024} = 20 \text{ [N/m]} \end{aligned} \quad (36)$$

Adding the stiffness of the diaphragm suspension ($k_s = 1.88 \cdot 10^3 \text{ N/m}$) results in the total stiffness for the loudspeaker of $k_1 \approx 9.1 \cdot 10^3 \text{ N/m}$. With the moving mass values

of $m_1 = 0.13$ and $m_2 = 0.33 \cdot 10^{-3}$ kg this results respectively in a natural frequency of:

$$\begin{aligned} f_1 &= \frac{\omega_1}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k_1}{m_1}} = \frac{1}{2\pi} \sqrt{\frac{9.1 \cdot 10^3}{0.13}} \approx 42 \text{ [Hz]} \\ f_2 &= \frac{\omega_2}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k_2}{m_2}} = \frac{1}{2\pi} \sqrt{\frac{20}{0.33 \cdot 10^{-3}}} \approx 38 \text{ [Hz]} \end{aligned} \quad (37)$$

The difference in these calculated values, which should have been equal, is small enough to prove the assumption, while it is easily caused by the approximation of the factor 1.4 between the volume parts of the enclosure, having a large impact on the stiffness. A 10% larger part of the enclosure volume for the loudspeaker and a corresponding smaller part for the moving air in the bassreflex port would result in a 5% (≈ 2 Hz) different frequency thus equalising the values.

Based on the found similarity with the passive radiator it is not without logic to expect that the modal mass, effective on the point where the actuator drives the system is most probably approximately equal to the modal mass of the first eigenmode. More exactly it can be determined starting with the first part of Equation (30):

$$F_r = P_e S_1 = F_2 \frac{S_1}{S_2} = m_2 a_2 \frac{S_1}{S_2} \quad (38)$$

Using Equation (33) the following relation between the accelerations is obtained

$$\frac{x_1}{x_2} = \frac{a_1}{a_2} = \frac{m_2 S_1^2}{m_1 S_2^2} \Rightarrow a_2 = a_1 \frac{m_1 S_2}{m_2 S_1} \quad (39)$$

which leads to a simple expression for the reactive force:

$$F_r = \cancel{m_2} a_1 \frac{m_1 \cancel{S_2} \cancel{S_1}}{\cancel{m_2} \cancel{S_1} \cancel{S_2}} = m_1 a_1 \quad (40)$$

This means that the reactive force by the coupled mass on the point of insertion of the driving force is equal to force needed for the acceleration of the driven loudspeaker diaphragm alone and the coupled mass just doubles the perceived mass of the driven loudspeaker for the second eigenmode. This fully complies with the “gutfeel” that the second eigenmode should be in mass balance. It is also logical to state then that the modal stiffness is twice the perceived stiffness of the enclosure by the driven loudspeaker. With these findings the frequency response can be derived using Matlab with the following steps.

- The first step is to model the first eigenmode for the driven loudspeaker which can be based on the modal mass $m_m = 0.3$ kg with the stiffness of the suspension of the driven loudspeaker.
- The acoustic radiation by the first eigenmode of the air-mass in the bassreflex tube is equal and with opposite sign to the acoustic radiation by the first eigenmode of the driven loudspeaker.

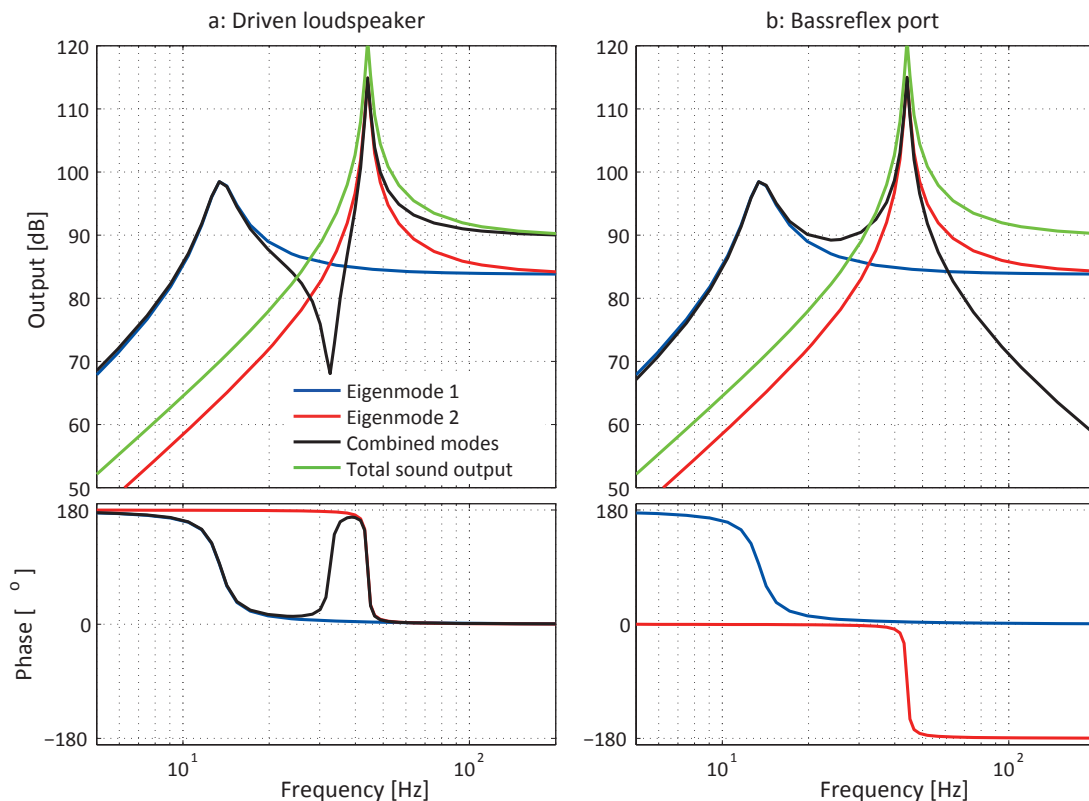


Figure 12: The construction of the frequency response by means of eigenmodes with a bassreflex system with air port gives comparable results as with the analytical solution. The deviations are larger due to the many assumptions on the system.

- The second eigenmode is calculated first on the driven loudspeaker with the modal mass equal to the double mass of the moving diaphragm acting on the double stiffness value of the air-spring defined by the volume part of the enclosure that is compressed/expanded by the driven loudspeaker.
- Finally the responses are combined to give the result.

Figure 12 shows the result of this exercise and when comparing with Figure 10 several deviations are visible. First of all the anti-resonance is not at the same frequency. This is due to the large impact of relative gains of both eigenmodes on the point where they intersect. Secondly the small remaining resonance in the sound output at the first eigenfrequency is not seen. But for the remainder the results are qualitatively comparable. Certainly this all points out that these simplified calculations have to be seen as not better than rough approximations to obtain qualitative indications of the phenomena that can be expected in reality as a large series of assumptions are made which are all not completely true:

- The air is assumed to flow frictionless for both modes without turbulence.
- All is modelled linear.
- The moving mass in the bassreflex port is estimated with a "rule of thumb" correction factor for the air just outside the port.

- The mass of the air in the enclosure is not taken into account.
- The mass of air that is driven to produce sound is neglected for reason of the low coupling between a diaphragm and the surrounding air but it is not zero.
- The wavelength and speed of the sound can be neglected in the enclosure.
- etc.

Nevertheless the shown dynamic responses are sufficiently representative for real bassreflex systems to be conclusive.

7 Conclusions on Bassreflex for Very Low Frequencies

When observing the resulting responses from Figure 5 and Figure 10 it is clear that even with a good sized enclosure of 60 litres and a large loudspeaker the achieved lowest -3dB frequency is around 35 Hz, with a very steep 18-24 dB/octave slope below this frequency depending on the applied damping. One can in principle extend this range and reduce the dynamic effect of the resonance by a compensation filter at the input signal. This will however drive the driven loudspeaker diaphragm in extreme excursions, while the entire purpose of bass reflex was to prevent this. Furthermore, increasing the movement of large volumes of air below the frequency, where the port takes over the main part of the sound reproduction, will increase flow noise.

The only way to really extend the response to 20 Hz is to decrease all resonance frequencies with a factor two. Due to the square root relation with mass or stiffness this means a factor four less stiffness or higher mass or a factor two in both. While a higher mass will further decrease efficiency, only a four times lower stiffness would work as long as all stiffness values are decreased that much, so including the surround, spider and the volume of the enclosure. The last one can also be decreased by means of a smaller loudspeaker but then the moving mass is also decreased.

From this reasoning it can be concluded that only extremely large bass-reflex systems can produce frequencies around 20 Hz. And even then the stepresponse will always show a delayed reaction, giving the impression of uncontrolled “woolly” bass.

Finally there have been times that people believed and seriously stated that a bass-reflex system has a higher efficiency. They used as argument that the pipe is open and transfers the sound from the back like in a delay-line. The fact is, however, that a higher output for the same input power only occurs, when the bassreflex resonance on the second eigenmode (45 Hz in the examples) is insufficiently damped and then only around that frequency. Below ≈ 35 Hz the roll-off is 4th order, so the power output and efficiency is lower than with the second order roll-off of a closed-box enclosure with 35 Hz bandwidth. Above ≈ 50 Hz the air in the pipe or the passive diaphragm hardly moves and does not transfer anything. This means that it

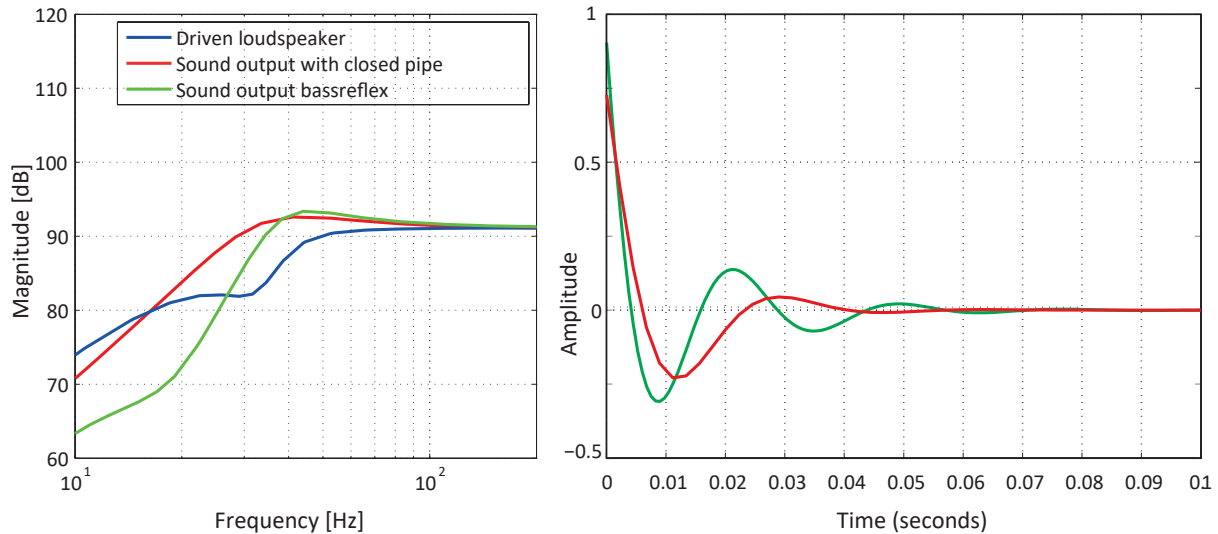


Figure 13: By closing off the pipe or passive membrane the effect of the bassreflex principle on both frequency and time response is made clear. In a closed-box enclosure the sound output matches the diaphragm motion of the driven loudspeaker.

effectively closes off the enclosure at those frequencies and consequently the sound output is then equal to the situation with a closed-box enclosure.

The only exception to this statement is the effect of occurring standing waves in the enclosure at higher frequencies as these will be transferred by the port. For a subwoofer this is, however, not relevant as even in a sizeable enclosure frequencies of 100 Hz and below with a wavelength of 3 metres or more a half wavelength will not fit in the enclosure. So standing waves do not occur. Figure 13 is derived from the optimally damped graphs of Figure 5 where the passive membrane part is omitted and the response of the system is added when closing off the passive membrane. It clearly shows the benefit of a smaller diaphragm excursion, however as was shown before in Figure 6 only after the transient periods are over!. It is also clear that there is hardly an increase in efficiency, while the steeper slope with less output below 35 Hz is also evident. Finally the stepresponse of the closed-box situation is better controlled. When increasing the damping of the bassreflex system this can be improved, but then the benefit on diaphragm excursion will also decrease. The really only benefit that remains is when a small size system is required to continuously produce large amounts of sound at long lasting periods of low frequencies in the 30-50 Hz band. This is often the case with heavy pop-music like dance, metal and hiphop. In those cases the increased efficiency in that frequency band is beneficial because of the reduced heat load of the voice coil, reducing the risk of failure. This kind of music is also less sensitive for the drawbacks of bass-reflex in the delayed response, while the resonant behaviour is often even appreciated.

Still the conclusion from Section 3 remains valid that for high quality transparent well-controlled low-frequency sound reproduction one should use a closed-box enclosure. In two other papers it is shown that the favourable time response can even be further improved by active velocity or acceleration feedback.