Figure 1 shows options for both a full wave voltage doubler and tripler; the blue

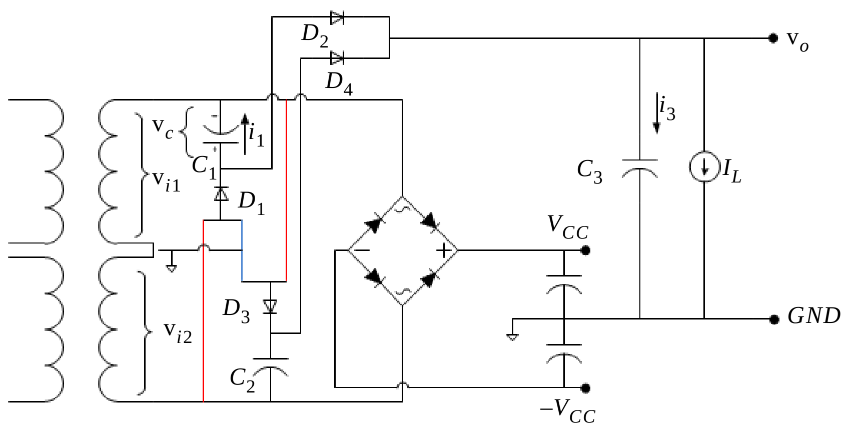

Figure 1 - Diagram of

wiring option results in the doubler whereas the red options will result in the tripler. The circuit can be transformed into a half wave doubler or tripler by eliminating C_2, D_3, and D_4. The voltage on the transformer's secondaries is $v_{i1} = -v_{i2} = v_i \sin\omega t$ where ω is the line frequency (i.e., 50 or 60 Hz). The doubler/ tripler output voltage, v_o, will contain ripple and will actually be less than 2 or 3 times v_i due to the load current, I_L, and the forward drop, v_D, of the diodes.

The morphology of the output waveforms with respect to the input wave form is depicted in Figure 2. Note that, for a given capacitor size and load current, the half wave arrangement results in more ripple and a lower v_{omin} than the full wave

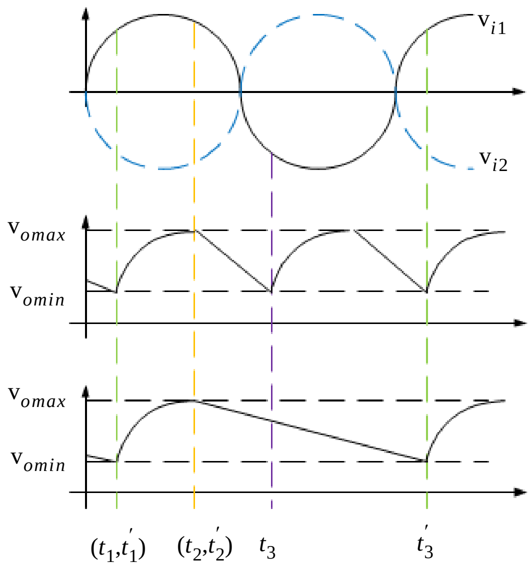

Figure 2 - Illustration of output waveforms for the full (middle) and half multipliers (bottom) versus the transformer secondaries (top).

version due to the fact that C_3 is replenished more frequently by the full wave version.

Assuming that C_1 has been charged to $nv_i - v_D$, where $n = 1$ for the doubler and $n = 2$ for the tripler prior to t_1, it follows that, at $t = (t_1,t_1')$, D_2 forward biases and C_3 starts to charge from C_1. Thus,

$$v_{i1} + nv_i - 2v_D = v_{omin} \quad \text{Eq. (1)}$$

at t_1. C_1 will transfer charge to C_3 until $t = (t_2,t_2')$ when

$$v_{i1} + v_c - v_D = v_{omax} \quad \text{Eq. (2)}$$

or when $i_1 = 0$.

From the condition at $t = (t_1, t_1')$ we have that

$$(t_1, t_1') = \frac{1}{\omega}\operatorname{asin}\left\{\frac{\mathrm{v}_{omin} + 2\mathrm{v}_D - n\mathrm{v}_i}{\mathrm{v}_i}\right\} = \frac{1}{\omega}\operatorname{asin}\{y\}\,. \qquad \text{Eq. (3)}$$

During the interval $(t_1, t_1') \le t \le (t_2, t_2')$ we have

$$\mathrm{v}_c = \mathrm{v}_o - \mathrm{v}_{i1} + \mathrm{v}_D \qquad \text{Eq. (4)}$$

and the current through C_1 is (assuming that v_D is a constant and noting the introduction of the negative sign to account for discharging C_1)

$$i_1 = -C_1\frac{\mathrm{dv}_c}{\mathrm{d}t} = -C_1\left(\frac{\mathrm{dv}_o}{\mathrm{d}t} - \mathrm{v}_i\omega\cos\omega t\right). \qquad \text{Eq. (5)}$$

We also know that

$$i_1 = i_3 + I_L = C_3\frac{\mathrm{dv}_o}{\mathrm{d}t} + I_L \qquad \text{Eq. (6)}$$

or

$$\frac{\mathrm{dv}_o}{\mathrm{d}t} = \frac{i_1 - I_L}{C_3}\,. \qquad \text{Eq. (7)}$$

Substituting Eq. (7) into Eq. (5) yields

$$i_1 = \frac{C_1}{C_1 + C_3}(\mathrm{v}_i\omega C_3\cos\omega t + I_L)\,. \qquad \text{Eq. (8)}$$

Since $i_1 = 0$ at $t = (t_2, t_2^{'})$ we can solve this expression for

$$(t_2, t_2^{'}) = \frac{1}{\omega}\operatorname{acos}\left(\frac{-I_L}{v_i \omega C_3}\right) = \frac{1}{\omega}\left\{\frac{\pi}{2} + \operatorname{asin}\left(\frac{I_L}{v_i \omega C_3}\right)\right\} = \frac{1}{\omega}\left\{\frac{\pi}{2} + \operatorname{asin}(x)\right\}. \qquad \text{Eq. (9)}$$

By integrating Eq. (7) across the $(t_1, t_1^{'})$ to $(t_2, t_2^{'})$ time interval we get

$$\int_{v_{omin}}^{v_{omax}} dv_o = \int_{(t_1, t_1^{'})}^{(t_2, t_2^{'})} \frac{i_1 - I_L}{C_3} dt = \int_{(t_1, t_1^{'})}^{(t_2, t_2^{'})} \left(\frac{C_1}{C_3(C_1 + C_3)}(v_i \omega C_3 \cos\omega t + I_L) - \frac{I_L}{C_3}\right) dt. \qquad \text{Eq. (10)}$$

Using the trig identities $\sin\omega(t_2, t_2^{'}) = \sin\left(\frac{\pi}{2} + \operatorname{asin}(x)\right) = \sqrt{1 - x^2}$ and $\sin\omega(t_1, t_1^{'}) = \sin(\operatorname{asin}(y)) = y$ the integral results in the expression

$$v_{omax} - v_{omin} = \frac{C_1}{C_1 + C_3} v_i \left\{\sqrt{1 - x^2} - y\right\} - \frac{I_L}{C_1 + C_3}((t_2, t_2^{'}) - (t_1, t_1^{'})). \qquad \text{Eq. (11)}$$

During the time interval $(t_2, t_2^{'}) \le t \le (t_3, t_3^{'})$ the voltage drop on the output will be

$$v_{omax} - v_{omin} = \frac{I_L}{C_3}((t_3, t_3^{'}) - (t_2, t_2^{'})). \qquad \text{Eq. (12)}$$

For the full wave tripler the time t_3 will occur a time equivalent to t_1 after v_{i2} crosses zero and C_2 starts to charge C_3. This situation can be expressed as

$$t_3 = \frac{1}{\omega}\operatorname{asin}\{-y\} = \frac{1}{\omega}\{\pi + \operatorname{asin}(y)\} \qquad . \qquad \text{Eq. (13)}$$

If the tripler is a half wave configuration then t_3' will occur a time equal to t_1 after v_{i1} crosses zero in a positive direction and C_1 again starts to charge C_3. This situation can be expressed as

$$t_3' = \frac{1}{\omega}\{2\pi + \operatorname{asin}(y)\} . \qquad \text{Eq. (14)}$$

Thus, in a general sense, the next charging cycle starts at a time

$$t_3 = \frac{1}{\omega}\{m\pi + \operatorname{asin}(y)\} \qquad \text{Eq. (15)}$$

where $m = 1$ for a full wave configuration and $m = 2$ for a half wave version.

Equating Eq. (11) and Eq. (12) yields

$$\sqrt{1-x^2} - y - x\left\{\left(1 + \frac{2m}{(2m-1)}\frac{C_3}{C_1}\right)\frac{(2m-1)}{2}\pi + \operatorname{asin}(y) - \operatorname{asin}(x)\right\} = 0 \qquad \text{Eq. (16)}$$

which, for a selected values of x and C_3/C_1 can be solved numerically for y. Also, from Eq. (12) we can deduce that the percentage ripple is given by

$$\Delta r = \frac{v_{omax} - v_{omin}}{v_i} = x\left\{\frac{(2m-1)}{2}\pi + \operatorname{asin}(y) - \operatorname{asin}(x)\right\}. \qquad \text{Eq. (17)}$$

To make use of these equations we can simply select a value for x (i.e., pick a value for C_3) and for C_3/C_1 and then calculate y from Eq. (16) and, subsequently, Δr from Eq. (17). x and C_3/C_1 can be varied, based on available component values, cost constraints, and size constraints, to obtain values for y and Δr that satisfy the design requirements. Unfortunately, Eq. (16) must be solved numerically which is not readily convenient.

To facilitate the design process, Eq. (16) has been solved for various values of x and C_3/C_1 and plotted in Figure 3 along with the corresponding values of Δr. As

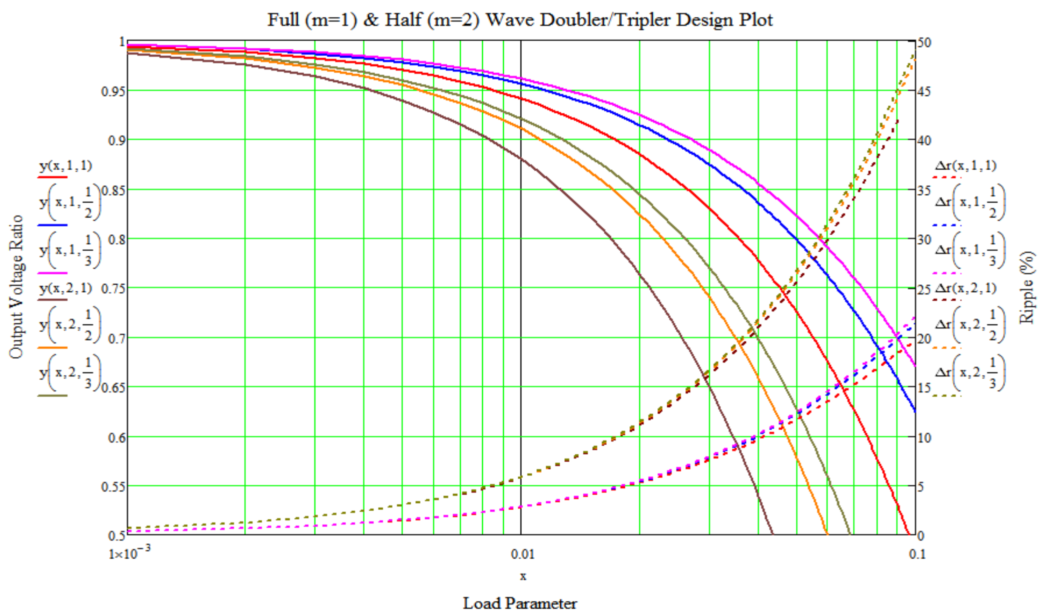

Figure 3 - Graph of y and Δr for various values of x and C_3/C_1.

can be seen from the graph the full wave version results in less ripple and regulates the voltage better (i.e., does not fall off as quickly as x increases) than does the half wave version. Also, it can be noted that doubling C_3/C_1 can have a substantive improvement in regulation for $x > 0.01$, but tripling C_3/C_1 does not provide a significant further improvement in this range. However, in general, one would not want to operate in this regime as the regulation rapidly degrades. Thus, when $x < 0.01$, there does not appear to be a significant advantage to using a large value of C_3/C_1. Furthermore, it is also apparent from the plot that C_3/C_1 does not significantly affect the ripple as one would expect from Eq. (17).

Based on the plot it would appear that choosing a full wave configuration with $x \le 0.01$ and $C_3/C_1 = 1$ or a half wave configuration with $x \le 0.005$ and $C_3/C_1 = 1$ would both yield performance levels that would be acceptable in many instances. Once the values for x and C_3/C_1 have been established, y and Δr can be estimated from the graph (or calculated exactly from Eq. (16) and Eq. (17)) and v_{omin} calculated from

$$\mathrm{v}_{omin} = (n + y)\mathrm{v}_i - 2\mathrm{v}_D \qquad \text{Eq. (18)}$$

and

$$\mathrm{v}_{omax} = \mathrm{v}_{omin} + \mathrm{v}_i\Delta r \quad . \qquad \text{Eq. (19)}$$

Table 1 conveniently presents exact values of y and Δr for explicit values of x and C_3/C_1.

Table 1 - Table of design parameters.

x	Full Wave ($m = 1$)				Half Wave ($m = 2$)			
	$C_3/C_1 = 1$		$C_3/C_1 = 1/2$		$C_3/C_1 = 1$		$C_3/C_1 = 1/2$	
	y	Δr (%)	y	Δr (%)	y	Δr (%)	y	Δr (%)
0.001	0.994	0.303	0.995	0.304	0.988	0.612	0.991	0.615
0.002	0.988	0.597	0.991	0.601	0.975	1.212	0.982	1.218
0.003	0.982	0.884	0.986	0.892	0.963	1.802	0.972	1.813
0.004	0.976	1.167	0.982	1.179	0.951	2.386	0.963	2.403
0.005	0.970	1.445	0.978	1.462	0.939	2.963	0.954	2.988
0.006	0.964	1.720	0.973	1.742	0.927	3.536	0.945	3.567
0.007	0.958	1.991	0.969	2.019	0.915	4.103	0.937	4.143
0.008	0.952	2.259	0.964	2.293	0.903	4.665	0.928	4.714
0.009	0.946	2.524	0.960	2.565	0.891	5.223	0.919	5.282
0.01	0.941	2.786	0.956	2.834	0.879	5.777	0.910	5.846