

The filter for bass extension is shown in figure 1. It is clear from the picture that it has an output impedance of $R_4R_5/(R_4+R_5)$, that the DC gain is 1 and the gain at high frequencies is $R_4/(R_4+R_5)$.

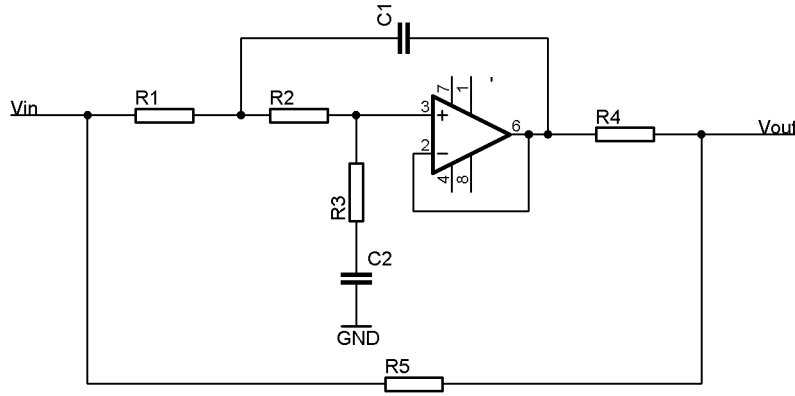


Figure 1. Bass extending correction filter based on a modified Sallen and Key low-pass filter

The transfer function can be written as:

$$H(s) = \frac{a_2 s^2 + a_1 s + 1}{b_2 s^2 + b_1 s + 1}$$

A straightforward but tedious calculation shows that:

$$a_2 = R_1 R_2 C_1 C_2 \frac{R_4}{R_4 + R_5}$$

$$a_1 = R_3 C_2 + (R_1 + R_2) C_2 \frac{R_4}{R_4 + R_5}$$

$$b_2 = R_1 R_2 C_1 C_2$$

$$b_1 = (R_1 + R_2 + R_3) C_2$$

After solving the component values for given a_2 , a_1 , b_2 and b_1 , this results in the following design procedure:

-Calculate the desired values of a_2 , a_1 , b_2 and b_1 from the desired pole and zero positions, or from the desired quality factors and natural frequencies.

The relation between the zeros z_1 and z_2 and the coefficients a_2 and a_1 is:

$$a_2 = \frac{1}{z_1 z_2}$$

$$a_1 = -\frac{1}{z_1} - \frac{1}{z_2}$$

The relation between the poles p_1 and p_2 and the coefficients b_2 and b_1 is:

$$b_2 = \frac{1}{p_1 p_2}$$

$$b_1 = -\frac{1}{p_1} - \frac{1}{p_2}$$

In the case of a closed-box loudspeaker with quality factor Q_{TC} , resonant frequency f_C , desired quality factor after correction Q_{TC}' and desired resonant frequency after correction f_C' , this becomes:

$$a_2 = \frac{1}{(2\pi f_C)^2}$$

$$a_1 = \frac{1}{2\pi f_C Q_{TC}}$$

$$b_2 = \frac{1}{(2\pi f_C')^2}$$

$$b_1 = \frac{1}{2\pi f_C' Q_{TC}'}$$

-Choose values for R_5 , C_1 and C_2

-Calculate R_4 :

$$R_4 = R_5 \frac{a_2}{b_2 - a_2}$$

-Calculate R_3 :

$$R_3 = \frac{a_1 b_2 - a_2 b_1}{(b_2 - a_2) C_2}$$

-Calculate R_1 :

$$R_1 = \frac{\frac{b_1}{C_2} - R_3 \pm \sqrt{\left(R_3 - \frac{b_1}{C_2}\right)^2 - 4 \frac{b_2}{C_1 C_2}}}{2}$$

-Calculate R_2 :

$$R_2 = \frac{b_2}{C_1 C_2 R_1}$$

The following are necessary requirements to get real and positive solutions (assuming a_1 , a_2 , b_1 and b_2 to be positive, as they normally are):

$$\frac{C_2}{C_1} \leq \frac{b_2 (a_1 - b_1)^2}{4(b_2 - a_2)^2}$$

To avoid extreme resistor ratios, it is advisable to choose the ratio C_2/C_1 as large as possible without violating this constraint (and without spending fortunes on parallel connections). That is, given a chosen value for C_1 , take the largest standard value for C_2 that meets the constraint.

$$b_2 > a_2$$

$$b_1 > a_1$$

$$\frac{a_1}{a_2} \geq \frac{b_1}{b_2}$$

Example: bass extension from 80 Hz Butterworth to 40 Hz Butterworth for a closed box

As an example, suppose you have a closed box with $f_C = 80$ Hz and $Q_{TC} = \sqrt{2}/2 \approx 0.7071068$ (second order Butterworth high-pass). Say you want to extend the bass response by an octave using the circuit of figure 1.

The required values for a_2 , a_1 , b_2 and b_1 are:

$$a_2 = 1/(2\pi f_C)^2 \approx 3.957858736 \cdot 10^{-6} \text{ seconds}^2$$

$$a_1 = 1/(2\pi f_C Q_{TC}) \approx 2.813488488 \cdot 10^{-3} \text{ seconds}$$

$$b_2 = 1/(2\pi f_C')^2 \approx 15.83143494 \cdot 10^{-6} \text{ seconds}^2$$

$$b_1 = 1/(2\pi f_C' Q_{TC}') \approx 5.626976976 \cdot 10^{-3} \text{ seconds}$$

C_2 and C_1 must be chosen according to the following inequality:

$$\frac{C_2}{C_1} \leq \frac{b_2 (a_1 - b_1)^2}{4(b_2 - a_2)^2}$$

That is, $C_2/C_1 \leq 0.2222...$

Take $C_1 = 1 \mu\text{F}$ and $C_2 = 220 \text{ nF}$ and choose $R_5 = 10 \text{ k}\Omega$.

$$R_4 = R_5 \frac{a_2}{b_2 - a_2} = 3.33333... \text{ k}\Omega$$

$$R_3 = \frac{a_1 b_2 - a_2 b_1}{(b_2 - a_2) C_2} \approx 8.52572269 \text{ k}\Omega$$

$$R_1 = \frac{\frac{b_1}{C_2} - R_3 \pm \sqrt{\left(R_3 - \frac{b_1}{C_2}\right)^2 - 4 \frac{b_2}{C_1 C_2}}}{2}$$

Arbitrarily choosing the solution with the plus sign:

$$R_1 = \frac{\frac{b_1}{C_2} - R_3 + \sqrt{\left(R_3 - \frac{b_1}{C_2}\right)^2 - 4 \frac{b_2}{C_1 C_2}}}{2} \approx 9.378294976 \text{ k}\Omega$$

$$R_2 = \frac{b_2}{C_1 C_2 R_1} \approx 7.673150407 \text{ k}\Omega$$

Rounding the resistances to the nearest E96 values gives:

$$R_1 = 9.31 \text{ k}\Omega$$

$$R_2 = 7.68 \text{ k}\Omega$$

$$R_3 = 8.45 \text{ k}\Omega$$

$$R_4 = 3.32 \text{ k}\Omega$$

$$R_5 = 10 \text{ k}\Omega$$

The final circuit is shown in figure 2. Linda simulations indicate that the pole and zero positions are:

poles $(-177.9 \pm 178.672 j) \text{ rad/s}$

zeros $(-355.883 \pm 358.332 j) \text{ rad/s}$

Ideally, the poles should be at $(-177.715317... \pm 177.715317... j) \text{ rad/s}$ and the zeros at $(-355.430635... \pm 355.430635... j) \text{ rad/s}$. The small differences are due to rounding all resistances to E96 values; when the resistances are not rounded, all six digits of the Linda output are according to expectation.

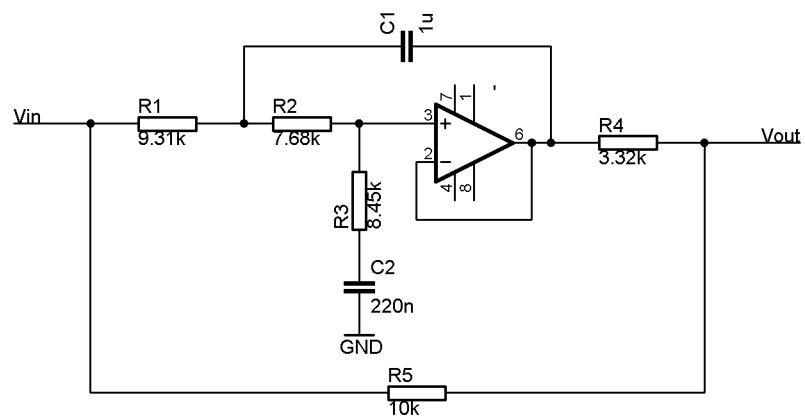


Figure 2. Example bass extension circuit, that extends the bass from 80 Hz to 40 Hz