Some variants of Elvee's single op-amp filter

Version 2, Marcel van de Gevel, 11 February 2023

In the thread https://www.diyaudio.com/community/threads/any-idea-to-improve-this-electronic-filter.395321/ Elvee looks for ways to filter a -5 kV, low current supply without needing very large series resistors or very large high-voltage capacitors. His filter configuration is shown in Figure 1.

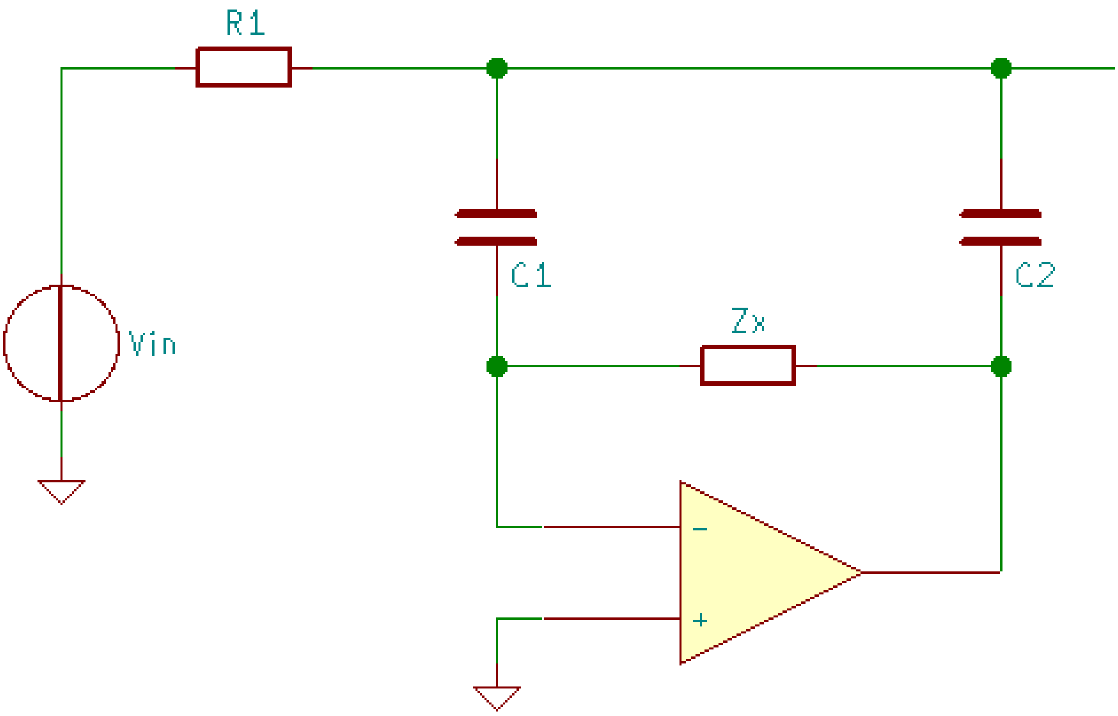

Figure 1: Elvee's single-op-amp filter configuration. In the original, Z_x is a 22 MΩ *resistor,* R_1 = 1 MΩ, C_1 = C_2 = 1.5 nF.

A problem with the circuit is its ringing when the feedback resistance is much greater than R_1.

Later in the thread, he explains that the circuit should ideally mimic a first-order low-pass filter with a capacitance of about 300 nF, which is 200 times as large as each of the two filter capacitors. However, to get exactly that behaviour (exact when all components including the op-amp behave ideally), Z_x should be a capacitor of 1.5 nF/198 = 7.57575757... pF. The voltage variations across C_2 are then 199 times as large as those across C_1, so the total capacitance is amplified by 199 + 1 = 200. This leads to the impractical circuit of Figure 2; impractical, because it won't bias properly.

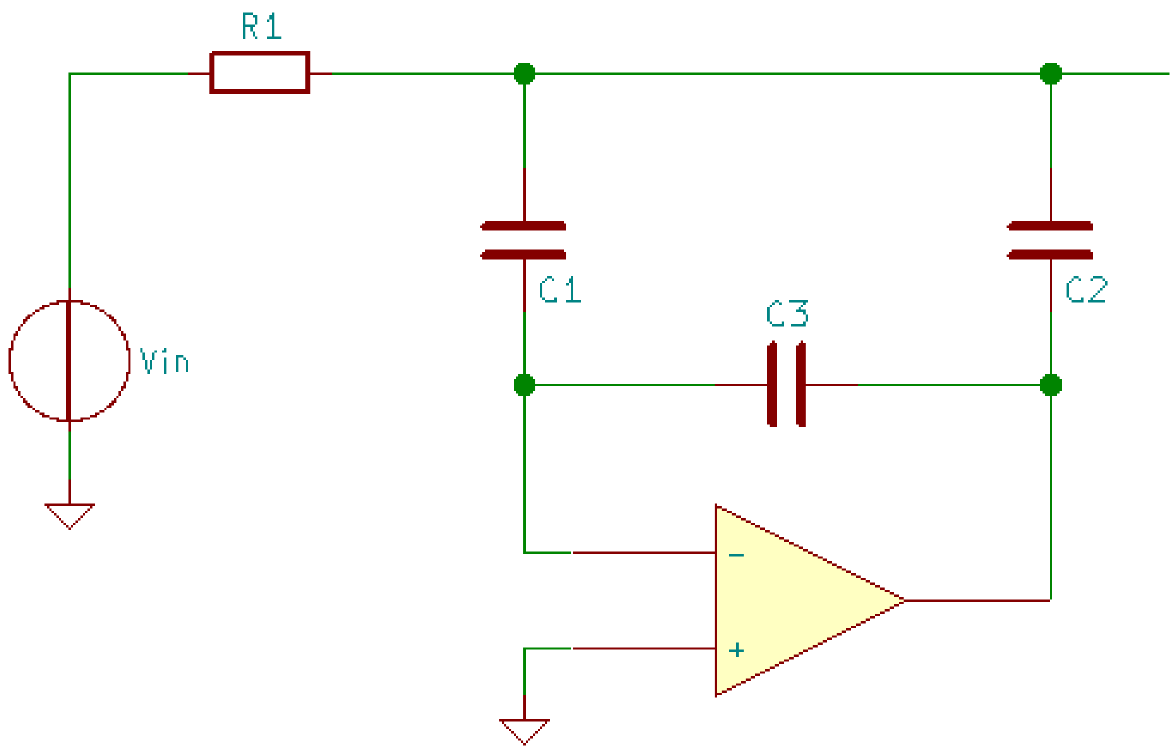

Figure 2: Impractical version that would behave as a first-order low-pass with increased capacitance, if it were biased correctly.

As a first attempt at a solution to the biasing issue, one could connect a resistor in parallel with the feedback capacitor. However, the feedback capacitor is 7.5757575757... pF and the time constant of the intended 300 nF capacitance with the 1 MΩ filter resistor is 300 ms. If the circuit is to behave like 300 nF over a time scale of the order of 300 ms, the parallel resistor has to be so large that its time constant with 7.5757575757... pF is well above 300 ms, corresponding to a resistance greater than 39.6 GΩ. Instead of using a resistor of that value, one could also use a T-network having the same voltage-to-current transfer; for example, a voltage divider that attenuates more than 1800 times followed by a 22 MΩ resistor. Needless to say, offset voltages will then be amplified more than 1800 times and a few hundreds of picoamperes of bias current will clip the op-amp output.

To solve this, one could connect some circuit that behaves inductively across the capacitor - not necessarily a real inductor (which would need to have an impractically large inductance), but something that has a relatively large voltage-to-current transfer at low frequencies that drops with a first-order slope at higher frequencies. For example, an RCR T-network. However, something inductive in parallel with something capacitive can lead to ringing/peaking again. In fact, when you look at the voltage-to-current transfer of the feedback path via C_2, R_1 and C_1, it initially increases with the square of the frequency. It therefore has FDNR-like behaviour, which further aggravates ringing and oscillation issues.

I think you can solve this by combining an FDNR-like, a capacitive, a resistive and an inductive voltage to current transfer in the feedback, see Figure 3. I've called the proportionality constant of the admittance of the FDNR *D*, I haven't a clue what it is usually called.

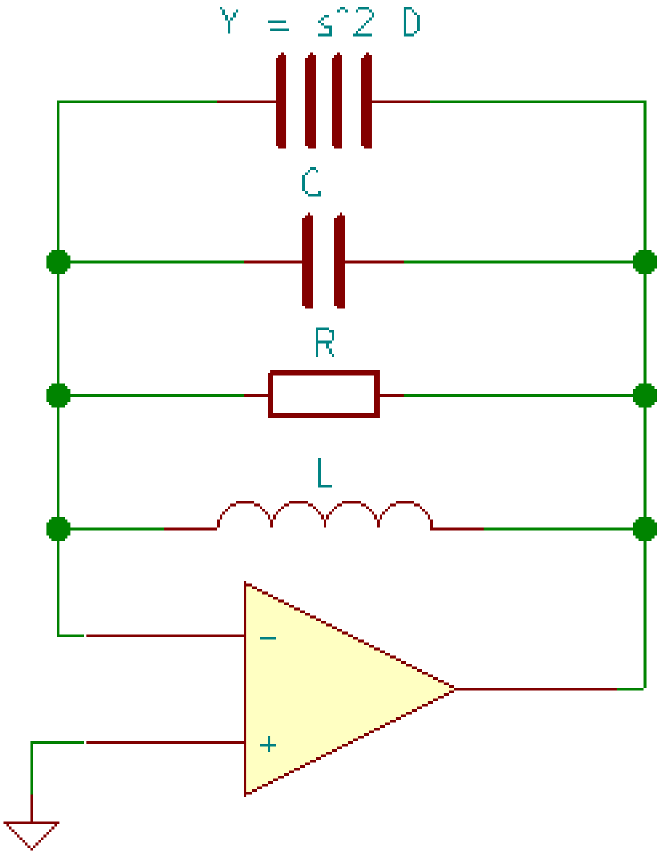

Figure 3: Op-amp with FDNR, capacitive, resistive and inductive feedback

When the op-amp is ideal, the current-to-voltage transfer of the whole circuit is the reciprocal of the voltage-to-current transfer of its feedback network. The zeros of the voltage-to-current transfer of the feedback network are then the poles of the whole circuit.

The voltage-to-current transfer of the feedback network is simply the sum of the admittances of the four branches:

$$Y=\frac{1}{sL}+\frac{1}{R}+sC+s^2D=\frac{1}{sL}\left(1+s\frac{L}{R}+s^2LC+s^3LD\right)$$

The term between parenthesis is the characteristic polynomial of the whole circuit. To get Butterworth pole positions,

$$\frac{L}{R}=2\sqrt[3]{LD}$$

$$LC=2\left(\sqrt[3]{LD}\right)^2$$

D is known, as $D=C_2R_1C_1$, and C is the desired feedback capacitance (7.575757... pF). For Butterworth pole positions, one can now derive that

$$LC=2L^{2/3}D^{2/3}\rightarrow L^{1/3}=\frac{2D^{2/3}}{C}\rightarrow L=\frac{8D^2}{C^3}$$

and

$$R = \frac{L}{2\sqrt[3]{LD}}$$

The resistive and inductive terms could be realized with a common T-network, as shown in Figure 4.

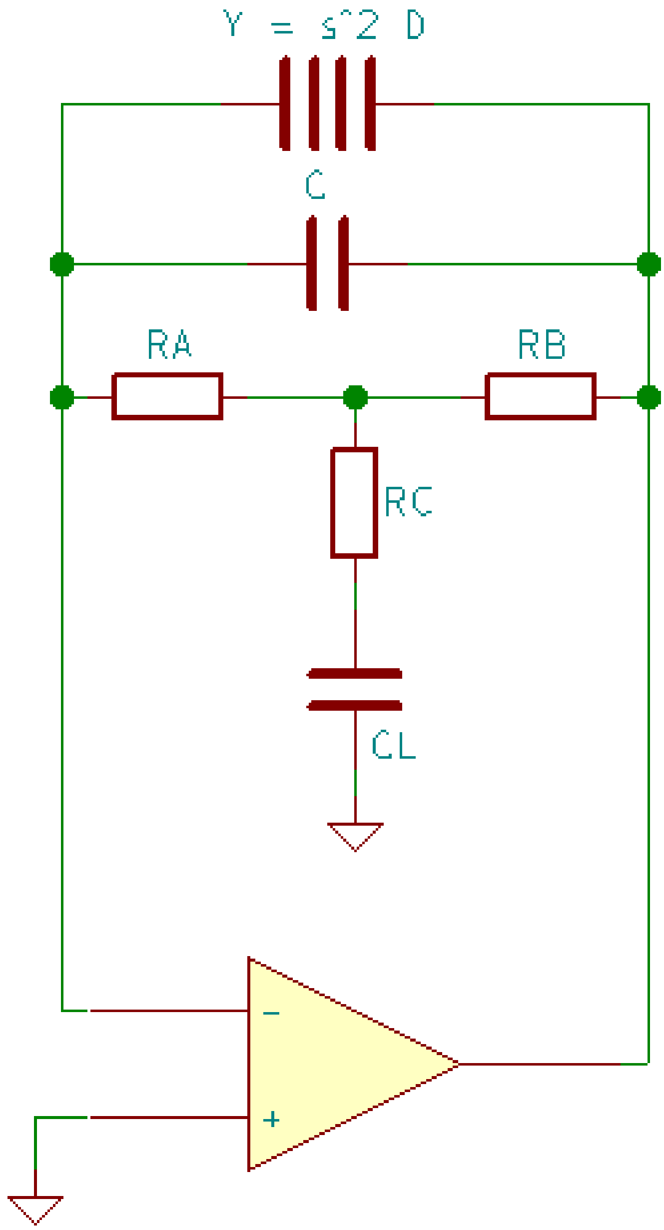

Figure 4: Practical implementation of the resistive and inductive branches. As mentioned in the text, the FNDR models C_2, R_1 and C_1, see Figure 5.

The whole filter then looks as shown in Figure 5.

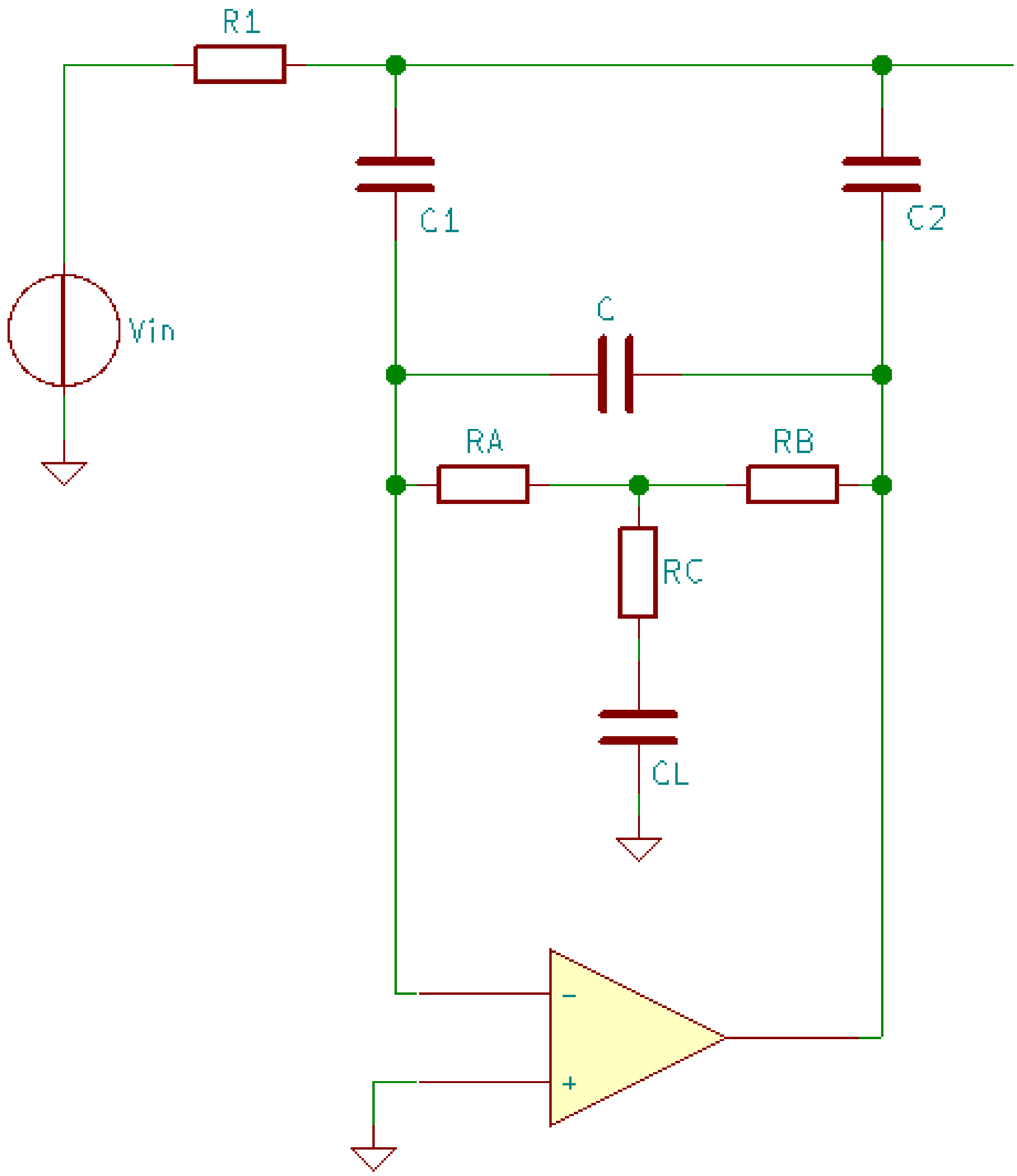

Figure 5: Filter circuit corresponding to the simplified model of Figure 4.

At the frequencies where the DC biasing stuff kicks in, the network consisting of C_2, R_1 and C_1 has a gain that is almost proportional to frequency squared. The fact that it is not an exact proportionality will therefore be neglected for the DC biasing stuff calculations, that is, we assume that modelling C_2, R_1 and C_1 with an ideal NFDR is accurate enough. Capacitor C and the op-amp will also be assumed to be ideal.

With these simplifications, we only need to approximate L and R with the network R_B, C_L, R_A. This network is depicted in Figure 6.

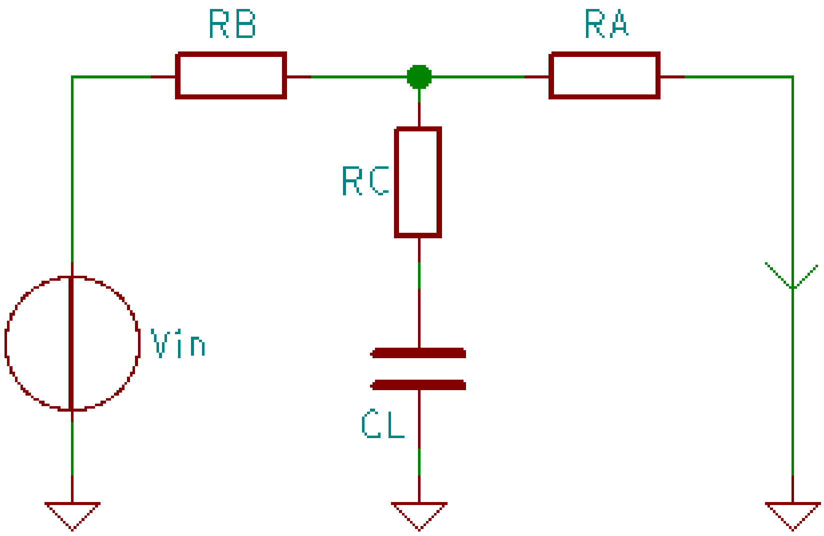

Figure 6: Subcircuit that realizes the resistive and inductive feedback

The transfer from voltage to current is

$$\frac{I}{Vin}=\frac{1}{R_A+R_B}\frac{sR_C C_L+1}{s\left(R_C+\frac{R_A R_B}{R_A+R_B}\right)C_L+1}\approx\frac{1}{R_A+R_B}\frac{sR_C C_L+1}{s\frac{R_A R_B}{R_A+R_B}C_L}=\frac{R_C}{R_A R_B}+\frac{1}{s R_A R_B C_L}$$

where the approximation holds when $R_C << R_A // R_B$ and the frequency is much greater than the corner frequency defined by the three resistors and C_L.

The first term on the right is the desired resistive term, the second the inductive term. That is,

$$L=R_A R_B C_L$$

$$R=\frac{R_A R_B}{R_C}$$

Hence,

$$C_L=\frac{L}{R_A R_B}$$

$$R_C=\frac{R_A R_B}{R}$$

That is, the design procedure is:

Choose R_1, C_1, C_2 and C.

Calculate $D = R_1 C_1 C_2$

Calculate L and R

Choose R_A and R_B

Calculate C_L and R_C.

Example:

$R_1 = 1\ \mathrm{M\Omega}$

$C_1 = C_2 = 1.5\ \mathrm{nF}$

$C = 7.57575757...\ \mathrm{pF}$ to get a total capacitance of 300 nF

$D = 2.25 \cdot 10^{-12}\ \Omega\mathrm{F}^2$

$L \approx 93.14870403\ \mathrm{GH}$ (gigahenry, so indeed too large for a physical inductor)

$R = 78.408\ \mathrm{G\Omega}$

Using $R_A = 22\ \mathrm{M\Omega}$ and $R_B = 1\ \mathrm{M\Omega}$:

$R_C \approx 280.5836139\ \Omega$

$C_L \approx 4234.032\ \mu\mathrm{F}$

Using $R_A = 22\ \mathrm{M\Omega}$ and $R_B = 10\ \mathrm{M\Omega}$ to reduce the large capacitance:

$R_C \approx 2805.836139\ \Omega$

$C_L \approx 423.4032\ \mu\mathrm{F}$

Rounded to the nearest standard values:

$R_1 = 1\ \mathrm{M\Omega}$

$C_1 = C_2 = 1.5\ \mathrm{nF}$

$C = 6.8\ \mathrm{pF}$

$R_A = 22\ \mathrm{M\Omega}$

$R_B = 10\ \mathrm{M\Omega}$

$R_C = 2700\ \Omega$

$C_L = 470\ \mu\mathrm{F}$

This circuit may be somewhat impractical for a single-supply implementation, with the positive op-amp input biased at half the supply voltage and with C_L connected with its negative terminal to ground. Even with a leakage-free C_L, such a circuit would need about an hour (10 MΩ times 470 μF times ln(2)) to settle after power on. Leakage would aggravate this further.

It should be much less of an issue for an implementation with a symmetrical supply, because the initial and final voltages across C_L are much closer. If a single supply would be required, it may be a good idea to split C_L into two 220 μF capacitors, one to ground and one to the supply.

Running the circuit through the LINDA pole-zero extraction program results in these poles and zeros:

No component value rounding:

poles: -1.675 rad/s and $(-0.837543 \pm 1.455\, j)$ rad/s

zeros: $(-0.841579 \pm 0.841579\, j)$ rad/s

The poles are quite close to third-order Butterworth locations. The asymptotic roll-off is only first order due to the two zeros. This was to be expected, because at high frequencies (if you can call frequencies above 0.3 Hz high), only C_1, C_2 and C play a role, so the circuit becomes equivalent to the first-order filter of Figure 2.

With component values rounded to standard values:

poles: -1.34 rad/s and (-0.835688 ± 1.574 *j*) rad/s

zeros: (-0.902207 ± 0.779549 *j*) rad/s