modelling and calculation program that forms part of the standard toolkit of persons skilled in the art. The format of the BSPLIN command is as follows:

BSPLIN, P1, P2, P3, P4, XV1, YV1, ZV1, XV4, YV4, ZV4

where in this embodiment the four points P1, P2, P3 and P4 correspond to the points **230**, **250**, **280** and **210** and (XV1, YV1, ZV1) corresponds to the initial slope vector at P1 and (XV4, YV4, ZV4) corresponds to the final slope vector at P4. The slope vector is defined generally as being parallel to a vector pointing from the origin of the active coordinate system to the position in space that (XV, YV, ZV) represents in that system (see P. Kohnke, editor *ANSYS* 5.7 *Theory Manual*, Ansys Inc, Canonsburg, Pa., 8th edition, 2001 whose disclosure is herein incorporated by reference).

Although a BSPLIN representation has been employed to parameterise the acoustic waveguide surface in this illustrative embodiment, it will be clear to those skilled in the art that other parameterisations applicable to more general non symmetric surfaces are contemplated to be within the scope of the invention. As an example, the acoustic waveguide surface might be represented by a bi-cubic spline representation, thereby providing a high degree of freedom of surface manipulation. As would also be appreciated by those skilled in the art, there exist a number of commercial finite element and/or numerical modelling packages that may be employed to generate a horn profile including but not limited to Mathematica™, IMSL™, MathCad™, MathCad™ or IDL™. Equally, the horn profile or parametric model of the sound waveguide surface may be explicitly programmed in a high level language accepting at least one input parameter to parameterise the model.

Referring once again to FIG. **1**, the step of simulating the sound field **130** may involve the use of any suitable numerical method that solves the Helmholtz (or Wave) equation including but not limited to techniques such as finite element analysis (FEA) or boundary element methods (BEM). As would be appreciated by those skilled in the art, there are a number of commercial acoustics analysis packages including but not limited to LMS Virtual.Lab™, ARCTRAN™ and Comet Acoustics™, that can be adapted to perform this task or alternatively the chosen numerical method may be explicitly programmed in a high level language.

The geometry of the horn surface or more generally the sound waveguide surface as described by parametric model **120** forms the input surface to the sound field simulator. Suitable boundary conditions representing the driver form further input into the sound field simulator and the relevant equations are then solved to give the sound field at a specified distance and angles located from the horn.

As described previously, FEA is one of a number of general numerical methods that can be used to solve a partial differential equation with appropriate boundary conditions. FEA techniques have been employed in a wide variety of areas to address computational problems in areas such as heat transfer, linear and non-linear solid mechanics, and fluid flow. According to one embodiment of the present invention, the Helmholtz harmonic wave equation, which governs the propagation of linear sound waves, is solved in arbitrary domains with respect to the sound waveguide surface by this method. According to FEA techniques the domain of interest is discretised or "broken up" into a number of smaller "finite" elements, and the underlying differential equation is then approximated over these elements. This process leads to a system of linear equations which are solved after the application of the appropriate boundary conditions to provide a composite solution for the whole domain.

One drawback of FEA in relation to simulating sound fields is that in order for this technique to give accurate results for acoustic problems, the individual finite elements need to be a small fraction of an acoustic wavelength in size. As the frequency considered in any sound field analysis increases the acoustic wavelength decreases. As a consequence, the corresponding number of elements required to accurately model a certain size component increases approximately as the cube of frequency. This results in a significant increase in the computational time required to simulate the sound field.

Accordingly, another technique that may be employed to simulate the sound field is the boundary integral equation method or boundary element method (BEM) as referred to earlier. This is once again a general numerical method for solving the Helmholtz harmonic wave equation that governs the linear acoustic field in arbitrary domains. This technique involves solving a surface integral equation that only requires the bounding surface to be discretised into elements rather than the whole volume being analysed, thereby resulting in a reduced computation burden when compared to FEA techniques. Unlike FEA, which requires the truncation of the discretised domain at a suitable distance away from the horn and the application of suitable radiation boundary conditions to stop reflection occurring at this interface, BEM by its formulation deals implicitly with radiation boundary conditions and there are no special modelling requirements for these conditions.

One approach to employing BEM in sound field simulation tasks is based on numerically approximating the Kirchoff-Helmholtz (K-H) integral equation which is derived from the Helmholtz equation. The K-H integral equation is defined as:

$$c(\vec{x})p(\vec{x}) = -\int_s i\rho\omega v_n(\vec{x}_s)g(\vec{x}_s \mid \vec{x}) + p(\vec{x}_s)\frac{\partial g(\vec{x}_s \mid \vec{x})}{\partial n}\,ds$$

where

$c(\vec{x})$ is a position dependent constant;

$p(\vec{x})$ is the complex acoustic pressure at the field point $\vec{x}$;

$i=\sqrt{1}$;

ρ is the density of the fluid;

ω is the circular frequency;

$v_n(\vec{x}_s)$ is the normal velocity on the surface at source position $\vec{x}_s$;

$$g(\vec{x}_s \mid \vec{x}) = \frac{e^{\frac{i\omega R}{c}}}{4\pi R}$$

is the free space Green's function;

c is the speed of sound; and

R is the distance between the source point $\vec{x}_s$ and field point $\vec{x}$.

The K-H integral equation is the fundamental equation of direct BEM, and shows that the pressure at any point can be represented by the surface integral of a combination of monopoles and dipoles. In this equation, the dipole source strength is weighted by the surface pressure. Given a distribution of surface normal velocity, once the surface pressure is found, any pressure field can be calculated.

In the direct BEM method, the variation of pressure on the exterior surface of a volume is discretised with shape functions similar to those used in FEA. If the field point is positioned at each surface node (or "collocated") then a series of n_n equations for the n_n surface pressures can be found for a given velocity distribution. The equations are generated by numerical integration over each element, and the integration technique used must be capable of dealing with the singularities found at the locations of the monopoles and dipoles. The equations can be formed into a matrix and inverted using standard linear algebra techniques. Once the matrix is inverted, and the surface pressures known, the field pressures can be calculated.

In this illustrative embodiment, the technique used to simulate the sound field of the horn is the "source superposition technique" (see for example G. H. Koopman and J. B. Fahnline, *Designing Quiet Structures: A Sound Power Minimization Approach*, Academic Press, 1997, whose disclosure is herein incorporated by reference). This technique is related to traditional BEM, but has the advantage of being very computationally efficient because it requires less computationally expensive numerical integration techniques, and is able to model the thin surfaces typical of horn loaded loudspeakers efficiently, unlike the direct BEM. A detailed description of the BEM employed in this illustrative embodiment is set out in Chapter 5 of R. C. Morgans, *Optimisation Techniques for Horn Loaded Loudspeakers*, PhD Thesis, University of Adelaide, 2004, wherein the disclosure of this PhD thesis is herein incorporated by reference.

Referring once again to FIG. **1**, the step of determining a beamwidth measure **140** involves specifying an objective function that quantifies the difference between the desired beamwidth frequency and position dependence properties and those that result from the simulated sound field **130**. In this illustrative embodiment, the desired beamwidth behaviour or target criterion is a substantially constant beamwidth with respect to frequency. However, as would be apparent to those skilled in the art, any arbitrary beamwidth function may be employed including a general spatial distribution. Such a general spatial distribution may be directed to a sound field having enhanced coverage properties or alternatively having the sound field incorporate a beamwidth that changes smoothly with frequency in a predetermined manner.

One embodiment of the objective function could be the commonly used least squares objective function, defined as

$$S=\Sigma(B(f>f_{min})-B_{nom})^2$$

where B(f) represents the vector of beamwidths calculated using the source superposition technique described above over a range of frequencies defined by the vector f. The operator $f>f_{min}$ selects only those frequencies above f_{min} and B_{nom} is the nominal or desired beamwidth. The minimum frequency f_{min} is chosen to exclude the omnidirectional beamwidth of horns at very low frequencies where the horn mouth will appear essentially as a point source of sound. However, it has been found that this objective function is not smooth and produces many local minima which make optimisation difficult.

In this illustrative embodiment, the objective function S to be minimised is defined as:

$$S=\frac{\Phi_1}{\Phi_2}$$

where

$$\Phi_1=\mathrm{std}(B(f>f_{min})),$$

and

$$\Phi_2=\mathrm{mean}(B(f>f_{min})),$$

and where the functions mean(x) and std(x) are the mean and standard deviation of a vector x respectively. This objective function has been shown to vary much more smoothly with input parameters than the least squared objective function and have less local minima. Objective function S can be used in isolation to produce a sound field and hence a horn profile having the least variation of beamwidth over a range of frequencies, this beamwidth also being determined by the optimisation procedure.

Alternatively, as in the case of the least squared objective function described above, further constraints such as requiring $\Phi_2=B_{nom}$ can be applied to design a horn profile having a predetermined beamwidth which itself varies smoothly over a range of frequencies.

A detailed description of the objective function employed in this illustrative embodiment is set out in Chapter 6 of R. C. Morgans, *Optimisation Techniques for Horn Loaded Loudspeakers*, PhD Thesis, University of Adelaide, 2004.

In another illustrative embodiment, the desired beamwidth may be specified independently in both the vertical and horizontal directions. In this example, the parametric model **120** of the sound waveguide surface may include independent parameterisation for each direction, thereby defining a non-axially symmetric surface. Similarly, the beamwidth measure **140** would include an objective function formed from the combination of individual objective functions S_i calculated for each of the horizontal and vertical directions.

In another embodiment, the beamwidth measure **140** would include an objective function formed from individual objective functions S_i defining the desired beamwidth for a given frequency at different elevations in the sound field. In this manner, a general frequency dependent spatial distribution measure may be defined for the sound field. As would be appreciated by those skilled in the art, as the generality of the frequency dependent spatial distribution measure is increased the number of input parameters required to parameterise the sound waveguide surface will also generally increase.

The next step in designing the sound waveguide surface is the step of varying the input parameters **150** until the desired characteristics of the sound field are achieved by minimising S. In practice this process may use any suitable method of numerical optimisation to systematically vary the input parameters until objective function S is minimised. In general, a function $f(x)$ is said to be globally minimised if a value of $x=x^*$ is found such that $f(x^*)\leqq f(x)$ for all x. Most optimisation techniques strive to find a local minimum. This is a point $x=x^*$ defined such that $f(x^*)\leqq f(x)$ for $|x-x^*|\leqq\delta$ where $\delta>0$ (i.e. for all x in a bounded region near x*). As the objective function S in this illustrative embodiment may contain multiple local minima, a global optimisation technique is preferred.

Standard gradient based optimisation methods such as Sequential Quadratic Programming (SQP) are local optimisation methods and as such often have to be run many times from different starting positions and even then a globally optimum solution is not guaranteed. In addition, gradient information in the form of the derivative of the objective function S with respect to the input parameters is required. In this illustrative embodiment, gradient calculation is difficult for a number of reasons: no simple analytical gradient calculation is possible, a finite difference approximation to this gradient is problematic because of the discrete nature of the meshing used in the source superposition method (a small change in horn profile could lead to a jump in the objective function) and a finite difference gradient evaluation, which can require many calls to the function, is relatively computationally expensive. Accordingly, a gradient free surrogate based global optimisation technique is preferred, such as the Efficient Global Optimisation (EGO) technique.

The EGO technique proceeds as follows. A number of different sets of input parameters are randomly generated to give a representative sample. In this illustrative embodiment, the random samples are generated by Improved Hypercube Sampling (IHS), which attempts to sample points such that the distance between them is close to the optimal spacing for the number of points (see for example B. K. Beachkofski and R. V. Grandhi, *Improved Distributed Hypercube Sampling,* 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Denver, 2002, whose disclosure is herein incorporated by reference).

Objective function S is then evaluated for each set of input parameters and a surrogate model is fitted that describes both the variation of the mean value between the sample points and the uncertainty between them. In this embodiment a Kriging technique is used. Kriging techniques, developed in the geo-statistics and spatial statistics fields, fit a surface to values from a set of data points. It models the variation of the unknown function as a constant value plus the variation of a normally distributed stochastic variable. It is essentially a method of interpolation between known points that gives a mean prediction in addition to a measure of variability of the prediction. Another appropriate optimisation technique such as SQP, simulated annealing or the DIRECT method is then employed to find the next best place to sample for a minimum objective function. The secondary objective function used in this illustrative embodiment is the Expected Improvement (EI) objective function.

The expected improvement is the expectation of the improvement over the current best point. The improvement I can be given as

$$I=\max(g_{min}-\hat{y}(x),0)$$

where g_{min} is the current best value from the set of samples and $\hat{y}(x)$ is the expected value of a normally distributed continuous random variable at point x (i.e. the mean value of the Kriging prediction at point x). The expected value of the improvement can be calculated as

$$E[I]=(g_{min}-\hat{y}(x))\Phi\left(\frac{g_{min}-\hat{y}(x)}{s(x)}\right)+s(x)\phi\left(\frac{g_{min}-\hat{y}(x)}{s(x)}\right)$$

where

$\Phi(x)$ is the standard normal cumulative density function;

$\phi(x)$ is the standard normal probability density function; and

$s(x)$ is the estimated standard deviation of the prediction at point x.

The point at which the value of the expected improvement is maximised gives the best point at which to calculate the true objective function. It is constructed to search for both local and global minima.

The surrogate model is then updated to include the newest sampled point, and the operation repeated until the sampling point does not change and the global minimum of the objective function has been found.

A detailed description of the optimisation techniques employed in this illustrative embodiment is set out in Chapter 7 of R. C. Morgans, *Optimisation Techniques for Horn Loaded Loudspeakers*, PhD Thesis, University of Adelaide, 2004, and a general discussion of optimisation is included in Appendix B.

Referring now to FIGS. **4**, **5** and **6**, there is shown the results of a series of optimisations according to an illustrative embodiment of the present invention. These results are for a sound waveguide surface which in this illustrative embodiment is a horn having a constant mean beamwidth as a function of frequency as smooth as possible for a given mean beamwidth Φ_2.

In FIG. **4** there is shown a contour map **400** of the objective function S for a range of horn lengths L and throat radii R_t in R_m normalised units, overlayed with contours of constant mean beamwidth Φ_2. Region **420** indicates where input parameters have given results that are not smoothly varying and accordingly region **420** may be ignored. Contours of constant mean beamwidth Φ_2 corresponding to 35° (**410**), 40° (**411**), 45° (**412**), 50° (**413**), 55° (**414**) and 60° (**415**) indicate the regions of contour map **400** which relate to a given constant mean beamwidth Φ_2.

As an example, to select the horn size parameters R_m, R_t and L that correspond to the best performance for a given constant mean beamwidth of 45°, the point on the 45° contour line having the minimum value of S is selected. This point is indicated by white cross **430**. The horn dimensional parameters can then be simply read from the axes of the contour map **400**.

In FIGS. **5** and **6** there are shown respective contour maps **500**, **600** of the optimum values of horn shape parameters x_1 and x_2, which define the shape of the horn as illustrated in FIGS. **2** and **3**. These values are simply read from the axes of the respective contour maps **500**, **600** for the minimum S location **430** which was previously chosen in FIG. **4**. In this case, x_1=0.54 and x_2=0.69. In this manner, both the relevant dimensional and shape parameters can be determined for a horn profile having a predetermined constant mean beamwidth. As would be apparent to those skilled in the art, various compromises will be involved in this optimisation process. As an example, a more constant beamwidth as a function of frequency may be possible for a slightly lower overall mean beamwidth than originally desired.

Referring now to FIG. **7**, there is shown a graph **700** of simulated beamwidth as a function of normalised frequency kR_m calculated using the values

$$\frac{L}{R_m}=1.87 \text{ and } \frac{R_m}{R_t}=8.4$$

as read from the axes of FIGS. **4**, **5**, and **6**. Indicated in the top right hand corner of FIG. **7** is the "Smoothness" of the beamwidth as a function of frequency with respect to the design beamwidth 45° which corresponds to the value of the objective function S. As would be appreciated by those skilled in the art, the performance of this profile represents a considerable improvement over the prior art.

A further simplification may be obtained by fitting a simplified linear relationship between the mouth to throat ratio R_m/R_t and normalised length L/R_m for a given desired beamwidth over a range of these values (as best seen in FIG. **4**) and then parameterising horn shape parameters x_1 and x_2 in terms of one of these values. In this manner, for a given throat size R_t or horn length L and desired beamwidth, the remaining size and shape parameters for this beamwidth requirement can be determined.

As an example, these relationships can be parameterised as follows for a horn having a desired beamwidth of 35°.

Variable range:

$$4.2 \le \frac{R_m}{R_t} \le 5.6$$

$$2.0 \le \frac{L}{R_m} \le 2.4$$

Parameterisation based on mouth to throat ratio R_m/R_t:

$$\frac{L}{R_m} = 1.0 + 0.24\frac{R_m}{R_t}$$

$$x_1 = 0.35 + 0.056\frac{R_m}{R_t}$$

$$x_2 = 0.72$$

An alternative equivalent parameterisation based on normalised length L/R_m:

$$\frac{R_m}{R_t} = -4.3 + 4.2\frac{L}{R_m}$$

$$x_1 = 0.11 + 0.23\frac{L}{R_m}$$

$$x_2 = 0.72$$

As would be apparent to those skilled in the art, the parameterisation described above provides an extremely simple method to determine the optimum size and shape of a horn profile for a desired beamwidth.

The previously described embodiments have been directed to audio applications which include those frequencies generally considered to be within the range of human hearing (i.e. 20 Hz to 20 kHz). However, the present invention may also be applied to sound frequencies not necessarily in the audible range. The range of sound frequencies above the audible hearing range (i.e. greater than 20 kHz) is termed the ultrasonic frequency range and there are many applications including medical diagnostics and non destructive testing (NDT) involving the emission and detection of sound in this frequency range.

In one illustrative embodiment, the present invention may be applied to the design of ultrasonic air coupled transducers which are commonly used in NDT applications. As is known in the art, one of the major problems associated with the design of ultrasonic air coupled transducers is the impedance mismatch between air (in the order of 100 Rayl) and any sample liquid or solid that is being investigated which will have an impedance typically in the MegaRayl range. Accordingly, couplants are used to reduce the impedance mismatch in an attempt to reduce the associated high reflection losses. As an example, the total reflection losses from the combined transducer/air interface and air/sample interface may be in the range of 120 dB or greater.

However, the use of couplants involves added complexity in any NDT activity. According to the present invention, the shape of an ultrasonic horn may be designed with the optimisation criterion of minimising the reflection losses at the transducer interface by impedance matching at the horn mouth, thereby increasing the overall efficiency of the associated ultrasonic transducer. These designs can be optimised assuming an air interface or a specified couplant as desired. Furthermore, the horn profile may be optimised to improve the broadband nature of the ultrasound field being generated, in the process allowing a single transducer horn design to be applied over a large frequency range. In this manner, only the piezo element whose resonant mode is typically excited to achieve large sound pressures in an ultrasonic transducer, would need to be changed or actively controlled for different ultrasonic frequency ranges whilst maintaining the same horn design.

Similar considerations apply to ultrasonic transducers used in detection systems such as car alarms, motion detectors, distance sensors and automated car parking systems in that they will also be subject to efficiency losses due to impedance mismatch and internal reflections and be limited to a fixed frequency of operation if used in resonant mode. According to the present invention, the shape of an ultrasonic horn or sound waveguide surface may be designed with the optimisation criterion of minimising the reflection losses at the transducer air interface by impedance matching at the horn mouth, thereby increasing the overall efficiency of the associated ultrasonic transducer. Furthermore, the horn profile may be optimised to also improve the broadband nature of the ultrasound field being generated. This results in a single transducer horn design being able to be applied over a large frequency range and once again only the piezo element whose resonant mode is typically excited to achieve large sound pressures in an ultrasonic transducer, would need to be changed or actively controlled for different ultrasonic frequency ranges whilst maintaining the same horn design.

In another illustrative embodiment of the present invention, the method of designing a sound waveguide surface may be applied to create an anechoic duct termination. These are particularly useful for impedance testing in general and specifically for testing the transmission losses of muffler elements. This will obviate the requirement of having sound absorbent material at the pipe termination of the muffler element which impedes the exhaust flow down the muffler and interferes with testing.

In yet another illustrative embodiment directed to active noise control, horn loaded loudspeakers and/or transducers can be designed with lower reflection losses and improved directional control of the sound field. This combination of features will provide improvements in the capabilities of these devices both in the audio and ultrasonic ranges which can also be extended to arrays of small horn loaded loudspeakers and/or transducers as well.

The steps of a method or algorithm described in connection with the embodiments disclosed herein may be embodied directly in hardware, in a software module executed by a processor, or in a combination of the two. The software module may contain a number of source code or object code segments and may reside in any computer readable medium such as a RAM memory, flash memory, ROM memory; EPROM memory, registers, a hard disk, a removable disk, a CD-ROM, a DVD-ROM or any other form of computer readable medium. In the alternative, the computer readable medium may be integral to the processor. The processor and the computer readable medium may reside in an ASIC.

Although a number of illustrative embodiments of the present invention have been described in the foregoing detailed description, it will be understood that the invention is not limited to the embodiment disclosed, but is capable of numerous rearrangements, modifications and substitutions without departing from the scope of the invention as set forth and defined by the following claims.

The invention claimed is:

1. A method for designing a sound waveguide surface, the method including the steps of:

forming a parametric model of the sound waveguide surface, the parametric model having at least one input parameter;

simulating a sound field that is formed by the sound waveguide surface;

determining a frequency dependent spatial distribution measure for the sound field associated with the sound waveguide surface,

varying the at least one input parameter to change the sound waveguide surface to adjust the value of the frequency dependent spatial distribution measure.

2. The method for designing a sound waveguide surface as claimed in claim **1**, wherein the step of varying the at least one input parameter includes varying the at least one input parameter to adjust the frequency dependent spatial distribution measure towards a target criterion.

3. The method for designing a sound waveguide surface as claimed in claim **2**, wherein the step of varying the at least one input parameter includes determining an objective function characterising the difference between the frequency dependent spatial distribution measure and the target criterion.

4. The method for designing a sound waveguide surface as claimed in claim **3**, wherein the step of varying the at least one input parameter further includes minimising the objective function to generate a resultant value for the at least one input parameter thereby defining the sound waveguide surface having the frequency dependent spatial distribution measure approaching the target criterion.

5. The method of designing a sound waveguide surface as claimed in claim **4**, wherein the target criterion for the frequency dependent spatial distribution measure is based on a beamwidth variation of the sound field as a function of frequency.

6. The method of designing a sound waveguide surface as claimed in claim **5**, wherein the target criterion for the frequency dependent spatial distribution measure is a predetermined variation in the beamwidth as a function of frequency.

7. The method of designing a sound waveguide surface as claimed in claim **5**, wherein the target criterion for the frequency dependent spatial distribution measure is a substantially constant beamwidth as a function of frequency determined by limiting the objective function to vary less than or equal to 5% from a desired beamwidth.

8. The method of designing the sound waveguide surface as claimed in claim **5**, wherein the target criterion for the frequency dependent spatial distribution measure is a predetermined constant beamwidth as a function of frequency.

9. A sound waveguide surface designed and constructed in accordance with the method of claim **1**.

10. A method for designing an acoustic horn, the method including:

forming a parametric model of a size and shape of the acoustic horn, the parametric model dependent on at least one input parameter;

simulating a sound field corresponding to the size and shape of the acoustic horn;

determining a beamwidth measure of the sound field, the beamwidth measure dependent on frequency and position; and

optimising the beamwidth measure with respect to a target criterion by varying the at least one input parameter.

11. The method for designing the acoustic horn of claim **10**, wherein the target criterion is a predetermined variation in the beamwidth as a function of frequency.

12. The method for designing the acoustic horn of claim **10**, wherein the target criterion is a substantially constant beamwidth as a function of frequency determined by limiting the objective function to vary less than or equal to 5% from a desired beamwidth.

13. The method for designing the acoustic horn of claim **10**, wherein the target criterion is a predetermined constant beamwidth as a function of frequency.

14. The method for designing the acoustic horn of claim **10**, wherein the at least one input parameter includes a horn throat radius, a horn length, and a horn mouth radius.

15. The method for designing the acoustic horn of claim **11**, wherein the at least one input parameter includes a horn throat radius, a horn length, and a horn mouth radius.

16. The method for designing the acoustic horn of claim **12**, wherein the at least one input parameter includes a horn throat radius, a horn length, and a horn mouth radius.

17. The method for designing the acoustic horn of claim **14**, wherein the at least one input parameter furthers include a horn profile.

18. The method for designing the acoustic horn of claim **17**, wherein the horn profile is represented as a spline.

19. The method for designing the acoustic horn of claim **10**, wherein the acoustic horn is axially symmetric.

20. An acoustic horn designed and constructed in accordance with the method of claim **10**.

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