The Problems of Low-frequency Reproduction

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A discussion of the characteristics which must be built into a low-frequency loudspeaker in order to maintain good efficiency with as smooth a response curve as possible.

A LOUDSPEAKER capable of extended low-frequency range without compromise of other values does not exist today. So-called "woofers" are often merely large cone speakers in which the frequencies above about 2000 cps are mechanically attenuated in the moving system, by mass or by compliance in the body of the cone. All that may be expected in most conventional woofers is this high-frequency loss, while a significant downward extension of frequency is rare, hence there is gained only some assistance to the work of the crossover network. This statement, however, excludes certain applications such as motion picture and professional installations, but here, too, there is room for improvement.

There are many antagonistic factors that harass the designer. The ideal low-frequency loudspeaker should fulfill the following requirements. It should have:
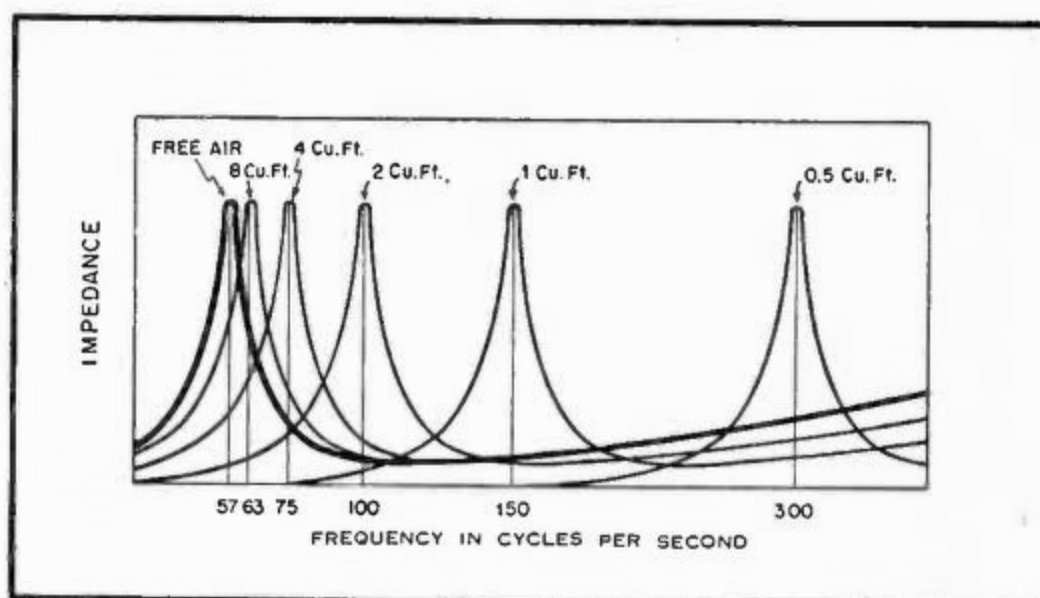
1. Uniform acoustic output from 20 cps up to any arbitrarily chosen cutoff.
2. High conversion efficiency throughout this transmission range, i.e., 50 to 100 per cent.
3. High power-handling capabilities.
4. Low distortion.
5. Excellent transient response.
6. Achieve above characteristics in an enclosure of reasonable volume.

While these specifications may appear reasonable to the average listener, the loudspeaker engineer is aware of the intensely conflicting operations of these objectives. No single transducer has been designed capable of satisfying all of the above requirements. As examples of the recalcitrant nature of the prob-

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Fig. 1. Cabinet volume vs. resonance, showing typical responses from cabinets of different sizes.

lem, consider that high conversion efficiency requires, generally, small mass, large compliance and negligible damping, but these qualities give poor response and poor power handling ability. Small mass demands a small area cone, hence inadequate radiation resistance and poor low-frequency output. Large-mass vibrating systems produce lower values of frequency, but in order to suspend this weight adequately, the suspension is made stiffer, thus partially cancelling the low-frequency trend.

"Reasonable size" means that the speaker and its enclosure can be moved readily through the doors of the average home, yet low frequencies are associated with massive speakers, enclosures, or horns. The dimensions for 20-cps reproduction using a piston or a horn, are fixed by physical law. These dimensions reach terrifying proportions if the instrument is designed for a living room. Other problems and contradictions that stagger the designer will be brought out later, but as implied in the opening sentences, practical considerations impose many compromises upon the present day type of low-frequency loudspeakers.

Despite the low-frequency shortcomings, the speaker art has progressed tremendously in the way of good fidelity, and truly extended high frequencies. It is possible today to obtain 12-in. cones whose response extends to 10,000 cps without the aid of tweeters, whereas not so long ago 5,000 cps was the upper limit. In order to analyze low-frequency performance, it will be helpful to consider several basic formulas and relationships. First let us take up the parameters which establish fundamental resonant frequency.

Resonant Frequency

The relationship for mechanical resonant frequency of a diaphragm type moving-coil loudspeaker is revealed by

$$f_r = \frac{1}{2\pi}\sqrt{\frac{S}{M}} \qquad (1)$$

where f_r = frequency of fundamental resonance
S = stiffness of system in centimeters per dyne
M = mass (weight) of moving system in grams.

The term S refers to the restraints against vibration which are developed in the cone-rim beads, and to a smaller extent, in the spider. Thus S denotes the "springiness" of the area which supports the vibrating mass, and is com-

monly called the stiffness of the moving system. A large value of stiffness means that there is a large mechanical restraint against the movement of the cone in one direction, and a release of a correspondingly large amount of stored energy in the opposite direction. Thus an increase in stiffness, which gives certain desirable qualities to a speaker, raises the fundamental resonant frequency—usually undesirable in a woofer.

Equation (1) shows that because of the square root relationship a rather large change in stiffness or mass is necessary to provide a substantial change in resonant frequency.

Cabinet Volume

If the speaker is enclosed in a simple non-vented cabinet, the total stiffness becomes the sum of the cone-rim stiffness and the cabinet stiffness, since the imprisoned air acts as an air spring and imparts additional mechanical impedance to the vibratory action.

Increasing the cabinet volume reduces the degree of this air stiffness, tending to lower the resonant frequency. But this effect grows progressively smaller because the stiffness of the cone rim remains large and is unaffected by cabinet volume. This explains why, beyond a certain cabinet volume, there is insignificant increase in bass. See *Fig. 1*. There can be no lower fundamental resonance than the free air resonance. The writer knows a number of experimenters who removed a speaker from a 4- or 5-cubic foot cabinet and built it into a wall, with disappointing results. The reason for this disappointment is the relatively high stiffness which is built into the cone suspension and over which there is no control. The cabinet stiffness can be relieved by increasing its volume. However, once all the stiffness is removed, as for example by using a true infinite baffle, then only the cone stiffness remains. The low-frequency performance is poor below the resonant frequency.

Force Acting on a Diaphragm

$$F = Bli \quad (classical) \qquad (2)$$

or more practically

$$F = B \times ampere\text{-}turns.$$

From the designer's standpoint,

$$F = B \times \frac{Volume\ of\ copper}{Volume\ of\ air\ gap} \times Amperes$$
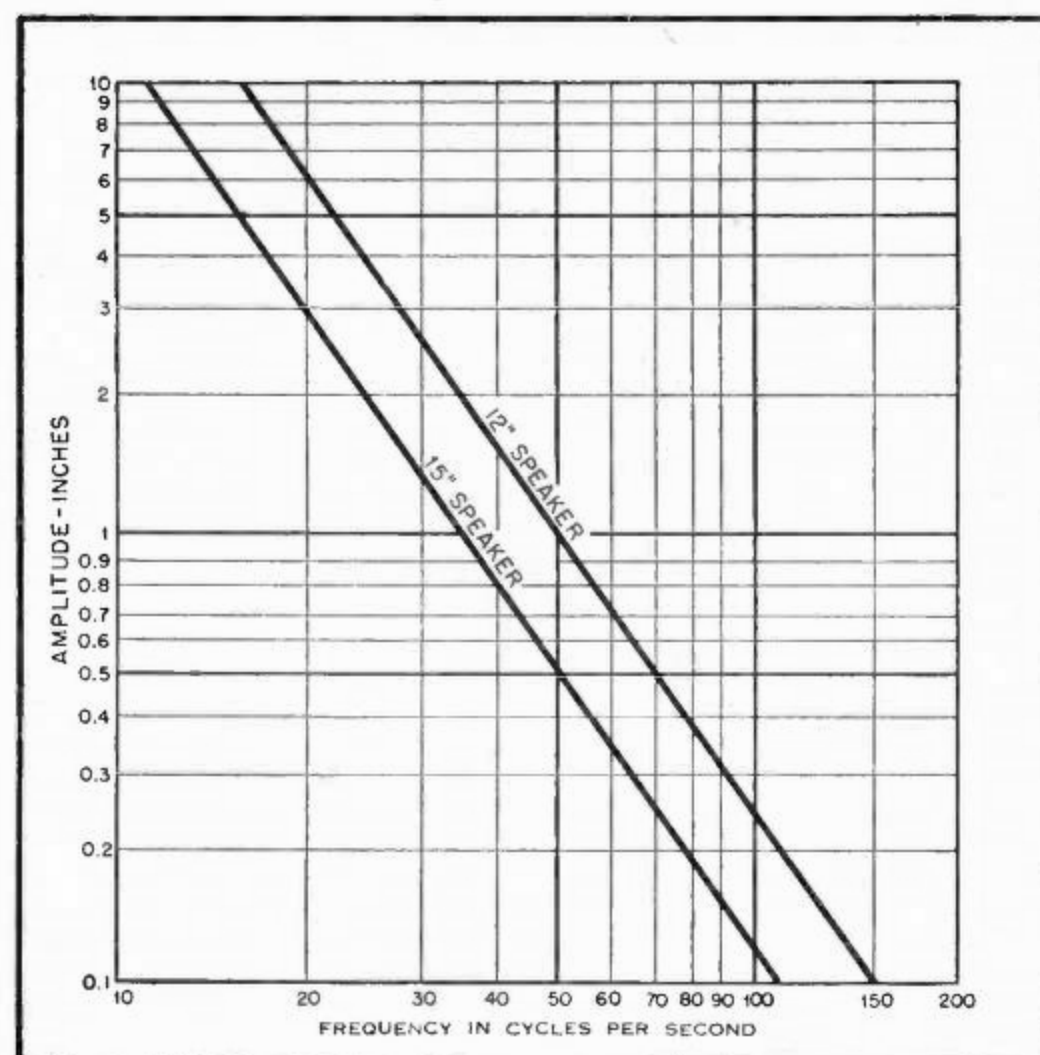
where F = force acting on voice coil
B = flux density in air gap
l = ength of copper wire in voice coil.
i = current through voice coil.

The above formulas show the force acting on the voice coil in the presence of a given signal, but do not yet represent the acoustic output. In fact, they do not necessarily indicate the force that moves the cone because of coupling losses between the voice coil and the main area of the cone. Equation (2) expresses a coupling factor between voice coil and gap flux, and is involved in the following equation for acoustic output:

$$P = \frac{D^2 R_r E^2}{10^7 Z_m^2 Z_e^2} \qquad (3)$$

where P = average acoustic output power in watts
D = electromechanical coupling factor
R_r = radiation resistance on cone
E = rms volts across voice coil
Z_m = mechanical impedance
Z_e = electrical impedance.

The term 10^7 converts ergs per second into watts. The term D is similar to the force F in equation (2) and involves gap constants such as flux and amount of copper. The term R_r, or radiation resistance, requires some explanation.

Radiation Resistance

Direct-radiator speakers are inherently poor radiators at low frequencies because of the low radiation resistance which is presented by the atmosphere. Although a cone may have a large vibration, it does not follow that it transmits sound energy to the air. It can have an amplitude sufficient to rupture it, yet the sound propagated is negligible.

This is the case when the diaphragm is small compared to the wave length of sound it is trying to propagate. For instance, at 200 cps a 12-in. cone is an efficient radiator, but at 20 cps even a violent movement produces practically no 20-cps fundamental acoustic energy. The listener hears the harmonics generated because of the large distortion present at this frequency. The fundamental frequency is unheard because there is insufficient diaphragm surface acting on the atmosphere.

The diaphragm acts in conjunction with air, which is its load. In order to deliver the necessary acoustic energy to the air it is necessary to employ a definite number of air particles to propagate the energy. This involves a definite reaction between diaphragm and atmosphere and fixes the minimum dimensions of the cone. If a diaphragm is too small for the frequency of its vibration, it has only slight resistance opposing it and it is not possible to impart all the energy of the diaphragm to the air. This is the condition of inadequate or low *radiation resistance*.

The atmosphere is capable of absorbing a large amount of cone energy if the cone is large relative to the wavelength, and efficient transfer of energy takes place. Thus more sound is produced with smaller cone excursions. This is the condition of *adequate or high radiation* resistance. Therefore, loudness at low frequencies can be increased by forcing more air particles in contact with the diaphragm. This is accomplished by using a large diameter cone, in the case of direct radiators, and by using a slow flare and large mouth in the case of horn speakers.

Looked at in another light, the entire atmosphere is the load for the loudspeaker. Where there is no load, there can be no absorption of energy. Incidentally, the listener's ears are a part of, and within this load.

If the acoustic output is to be independent of frequency, which is required for uniform response, it is of course required that the various terms of the expression in equation (3) be independent of frequency. Unfortunately, R_r and Z_m are both frequency dependent, and thus the requirement for linear output is difficult to attain at low frequencies. For frequencies *at* and *below* resonance, R_r, the radiation resistance is proportional to the square of the frequency. This term shrinks as frequency drops, hence efficiency falls off rapidly. This is seen from *Fig. 2*, which shows the peak-to-peak excursion for 1 acoustic watt output. The chart indicates that for a given cone diameter, a decrease of one octave increases the excursion by a factor of four.

The Z_m term for mechanical imped-

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ance grows larger as frequency is lowered, being inversely proportional to frequency below resonance. This quantity further cuts into efficiency at low frequencies. Z_m increases as frequency is lowered because the stiffness of the suspension system is operating close to its elastic limits. The suspension is the source of much low-frequency distortion.

One method of increasing the output at very low frequencies is to increase the radiation resistance, R_r, by increasing the diameter of the cone. R_r increases as the square of the cone area and raises the weight of the cone. The effect is in the right direction since the mass is increased, and resonance lowered. But in order to carry this additional weight and permit movement over a long period of use, the compliant rim is made stiffer, or stronger, thus increasing Z_m, and offsetting to some degree the full benefit of the larger diameter. Thus performance does not confirm the theoretical improvement attainable with the increase in diaphragm area.

At this point the designer attempts to lower the stiffness of the rim; that is, to make it more compliant. This would be a simple and excellent aid to low-frequency improvement if it were possible to disregard high power, long life, and good transient performance. Therefore, the best that can be attained is a compromise at the point where no further distortion can possibly be tolerated at the rated maximum input power. The proud owner of the most expensive speaker system would be staggered at the amount of harmonic and inharmonic distortion in his speaker at, say 40 cps, a full octave above the low limit of our ideal speaker.

With a large cone and a compromise suspension, a lower resonant frequency is reached resulting in a larger displacement for a given input signal. Here is the antagonism between the displacement and the restoring force. At moderate excursions there is insufficient reaction or stored energy in the suspension to drive the cone back to rest quickly at the termination of the signal. This becomes a source of distortion. Under extreme excursion, the displacement close to the elastic limits of the suspension is small for a given signal increment. However, the displacement is great in the relaxed direction for the same signal as a decrement, because of the release of the stored rim energy in this extreme position. This develops enormous harmonic distortion, and the dubious satisfaction of preserving some power handling ability and reasonably long life.

Suppose there had been no compromise with the suspension, and this permitted extremely free movement to wide limits by making the rim corrugations deep and the paper stock very thin. The cone is flabby and delivers a satisfyingly deep thump when struck with the fingers. But, unfortunately, a new set of problems arises. The thin rim will soon wear through by the action of friction among its fibres. Insufficient physical support is given to the body of the cone, which may go out-of-round at the edges where it blends into the thin compliant rim. This causes distortion and possible loss of annular centering within the air gap. Further, there can be no assurance that the rest position will always accurately align the voice coil axially with the gap. Because the forces in the rim are removed, the cone lacks tranquil stability and may fail to find a definite rest position. The cone becomes sensitive to external mechanical vibrations. A shift of voice coil position with respect to gap alignment seriously lowers the conversion efficiency and introduces further distortion. Varying humidity conditions may cause an axial shift of the voice coil with respect to gap alignment since paper cones are hygroscopic to a certain extent. The freer the rim compliance, the greater is the amount of voice-coil shift. These are some of the conflicting effects of attempts to reduce mechanical impedance, Z_m, by varying rim freedom.

Peak-to-Peak Excursion

Figure 2 shows the length of path which a cone must traverse in order to radiate one acoustic watt. While this is much more power than a person needs in a living room, it is about right for an average theatre. For a 12-in. speaker to radiate one acoustic watt at 200 cps requires a maximum displacement of 1/16 in., which is easily attainable. But to deliver the same acoustic watt at 50 cps requires a total displacement of one inch. This is *not* easily attainable. However, with larger cones, the movement becomes smaller. A 15-inch cone must move 0.6 in., and an 18-inch cone (not shown) must move only 0.4 in. These excursions are within design possibilities. But at 20 cps the problem is formidable. Notice the distances involved for the 12- and 15-inch cones. These displacements are for direct radiating cones, and are not affected by the baffle or cabinet, except when fitted to horns.

Other Causes of Distortion at Low Frequencies

1. Input distortion at *high signal peaks*. Radio transmitter and receiver signal peaks of 100 per cent modulation contain significant distortion. Similarly, high-level peaks on phonograph recordings contain large values of distortion. The non-linear conditions characterizing loudspeaker distortion at large cone amplitudes, likewise exist in recording heads, microphones, and some pickups. By adding to this the fact that the amplifier may be operating close to maximum output, it is seen that the loudspeaker is not the only offender.

2. *Cabinet Vibration.* A poorly constructed cabinet asserts its independence by taking over the function of the loudspeaker through a limited frequency range; various flexible members are excited into resonance. The lower the desired operating frequency, the more solid and massive must be the construction of the cabinet.

3. *Lack of Cone Homogeneity.* Although there is usually a smooth reduction in cone thickness from apex to periphery, there must be a similar density and unit mass at any given radius from the center. If there are irregular spots of heavy and light areas, inharmonic distortion may result. This is usually serious because the driving frequency is then unrelated to these harmonics. The effect of a heavier mass in one spot would be to act independently of the remainder of the cone by failing to keep up with it during the accelerating portion of the cycle and to push ahead during the deceleration periods. At high signal levels it may introduce its own resonance.

4. *Irregular Response Due To Low Efficiency.* Aside from some of the previously mentioned shortcomings of poor conversion efficiency, another important disadvantage must be mentioned, namely that of irregular and erratic frequency response. A run-of-the-mill speaker, such as used in commercial radio sets, may have a conversion efficiency around 2 per cent. Such a speaker may easily have a number of spiked resonances at various frequency points, where the efficiency can rise to a theoretical maximum of 100 per cent. This means a possible variation of 20 to 1 in output along its spectrum or a change of 13 db. This does not consider any dips or cancellation effects which may give sharp holes with zero output. A quality speaker with an average efficiency of 50 per cent, also may possess resonant peaks and spots where the efficiency suddenly hits 100 per cent. But this is only a 2 to 1 change in output, or a 3 db variation in pressure. This is a very small change indeed and the speaker may be considered almost flat.

ERRATA

The Simple Preamplifier and Tone-Control Unit described under that title by David H. O'Brien in the November, 1951, issue has occasioned several inquiries from readers, and Mr. O'Brien has provided a few suggestions, which are here presented.

R_{10} should be connected to the junction of C_{13} and R_{16}, rather than to the plate of V_3. This occasions the need to reverse the polarity of C_6, since the cathode of V_2 is positive with respect to the output lead, which is normally at d.c. ground potential. Since C_{13} is then included in the feedback loop, its effect is nullified by the feedback and the low bass frequencies are improved.

R_{16} should be increased to 0.1 meg; R_9 should be changed to 330 ohms, which would decrease the tendency for over-equalization of the high frequencies when the tone controls are set for "flat." In some constructions, there has been a tendency to "motorboat," and this may be eliminated by changing R_{13} to 0.1 meg, R_{18} to 39,000 ohms, and by changing C_{13A} to 80 µf, C_{13B} to 40 µf, and C_{11C} to 40 µf.

With the new value of R_{16}, closing the switch S_2 will provide the correct roll-off for LP records without the necessity of adjusting the treble tone control to effect this correction.