

Charge Movements on the Stretched Membrane in a Circular Electrostatic Push–Pull Loudspeaker*

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An analysis is made of the charge movements on the membrane of an electrostatic push–pull loudspeaker. It is shown that in steady-state operation (harmonic excitation) the movement of charge causes two important effects: harmonic distortion and a net change of static membrane charge. The former effect is found to be of minor interest in properly designed loudspeakers, whereas the latter causes the steady-state frequency characteristics to be input-level dependent. Measurements show that this level dependence may reach values well above 1-dB deviation from strictly linear behavior. A theoretical model is presented from which the charge distribution in a circular electrostatic push–pull loudspeaker can be calculated. The theoretical results are shown to conform with measurements.

0 INTRODUCTION

Electrostatic push–pull loudspeakers are commonly praised for their outstanding transducer performance. Very low distortion figures and extremely smooth frequency response are some of the “inherent” characteristics of the electrostatic principle of drive. Although the electrostatic loudspeakers have received a lot of theoretical attention in the past, there are still a number of problems that need to be solved in order to utilize the full potential of the system. One of these problems concerns the charge movements on the vibrating membrane. The basic analysis [1] of the push–pull system shows that the ideal situation would be a nonvarying charge distribution on the membrane, but then questions arise such as how to get that charge on the membrane and what to do about practically unavoidable leakage currents. Clearly a compromise has to be accepted, and generally accepted design rules state that the membrane should have semiconductive properties (see, e.g., [2]). It is known [2] that the small but nonzero conductivity of the membrane leads to two important effects, namely, harmonic distortion and level-dependent sensitivity. Although qualitative approaches exist [2]

to estimate the relative importance of these effects, it may be useful to study the effects in a more fundamental way. Especially the analysis of the distribution of charge on the membrane may give valuable insight into matters such as the spatial distribution of the driving force, places where ionization is most likely to occur, and so on.

In this paper we analyze the charge behavior under steady-state harmonic drive, starting from the assumption that the charge movements merely cause perturbation effects, that is, the charge variation component is small compared to the total membrane charge. This assumption is used in Sec. 1 to describe the physical background of the charge movement. In Sec. 2 we discuss a calculation model that enables us to predict the charge distribution on the membrane of a circular electrostatic loudspeaker. The harmonic distortion component is shown to be of minor importance, provided proper design rules are followed. Hence focus is on the level-dependent sensitivity and the related steady-state charge distribution on the membrane (Sec. 2.2). In this analysis we use the theory in [3]–[5] to calculate the linear vibrational behavior of the air-loaded circular membrane and then use that result as input for the charge distribution calculation. The results obtained are shown to be in good agreement with practical measurements.

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1 DESCRIPTION OF THE PHYSICAL MODEL

We consider a circular electrostatic push-pull loudspeaker (Fig. 1) consisting of an edge-clamped circular stretched membrane M of radius a and two perforated electrodes E at a distance d_0 of the membrane. The membrane is charged by means of a dc source V_0 , which causes a dc charge q_0 to appear on the membrane. Disregarding the electrical edge effects of this double-capacitor system, the value of q_0 is easily calculated to be

$$q_0 = \frac{2V_0\epsilon_0(\pi a^2)}{d_0} \quad (1)$$

where we have used the dielectric constant of vacuum ϵ_0 to approximate the dielectric constant of air. The value of q_0 has to be constant in order to ensure a linear behavior of the loudspeaker [1]. Therefore a large valued resistor R_0 is usually included in the charging circuit, physically in the form of a highly resistive membrane coating. After dc charging the ac signal sources V_1 will drive the electrodes in opposite phase by means of a transport of charge from one electrode to the other, which causes the charge q_0 , and thus the membrane, to vibrate. The electrostatic force per unit area in the positive z direction (Fig. 2), which acts upon the membrane, may be calculated from [1].

$$F_e(r, t) = 2\epsilon_0 \frac{V_0}{d_0} \frac{V_1(t)}{d_0} + 2\eta(r, t)\epsilon_0 \frac{V_0^2}{d_0^3} \quad (2)$$

where $\eta(r, t)$ is the membrane deflection in the positive z direction. Since this force acts upon the membrane charge, it will be clear that in the case of a nonzero

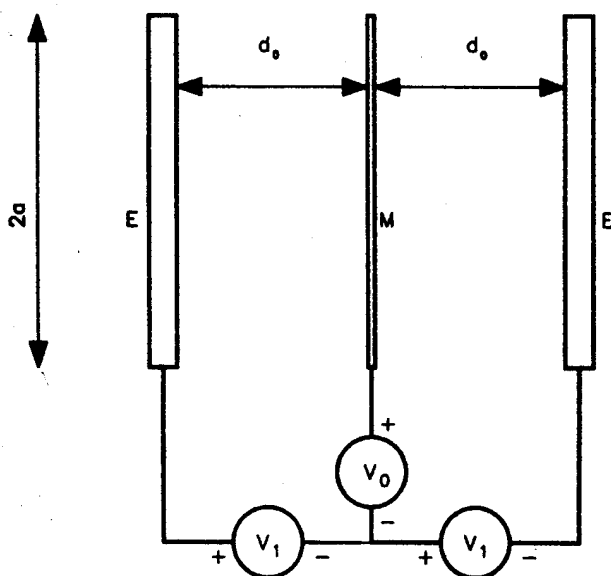


Fig. 1. Basic setup of electrostatic push-pull loudspeaker. A thin edge-clamped circular membrane M with radius a is placed between two stationary electrodes E at a distance d_0 . The membrane is charged by means of a dc voltage V_0 , whereas the electrodes are driven in opposite phase by means of the ac sources V_1 .

membrane deflection there will be a force component F_r (Fig. 2) which causes the membrane charge to move along the membrane surface. We consider this effect to be of second order, which means that we are only interested in the situation where $|\partial\eta(r, t)/\partial r| \ll 1$ and correspondingly $|\eta(r, t)| \ll a$, and thus the local charge variation is much less than q_0 . In that case the following approximation holds:

$$F_r(r, t) \approx \frac{\partial\eta(r, t)}{\partial r} F_e(t) \quad (3)$$

For time-harmonic excitation, that is, a $\cos \omega t$ dependence, we see that F_r varies as $\cos^2 \omega t$. Since $\cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$, it may be expected that this field will cause a dc effect and a second harmonic effect in the membrane charge distribution, which will ultimately lead to a third-harmonic acoustic distortion because the actual membrane deflection is determined by the product of electric field and membrane charge, hence a $\cos^3 \omega t$ dependence.

2 A CALCULATION MODEL

Fig. 3 depicts the axisymmetric model that we use as a basis for our calculations. We consider a membrane M with thickness d_m and dielectric constant ϵ_m , on which two surface charge densities are present: $\sigma_T(r, t)$ and $\sigma_B(r, t)$ on top and bottom of the membrane, respectively. The membrane's conductivity is characterized by its resistance per square R_s . This conductivity represents the conductive coating on the membrane. We thus assume that charge transport will only occur in the radial direction, along the membrane surface. Axial currents are prohibited by the assumption of perfectly insulating membrane material. The position of the membrane is defined by the circular symmetric deflection $\eta(r, t)$ of the middle of the membrane. At a distance d_0 on both sides of the membrane we have two perfectly conductive plates P_1 and P_2 . Next we define two contours Γ_1 and Γ_2 as depicted in Fig. 4. We assume an electric field $E_T(r, t)$ in the volume

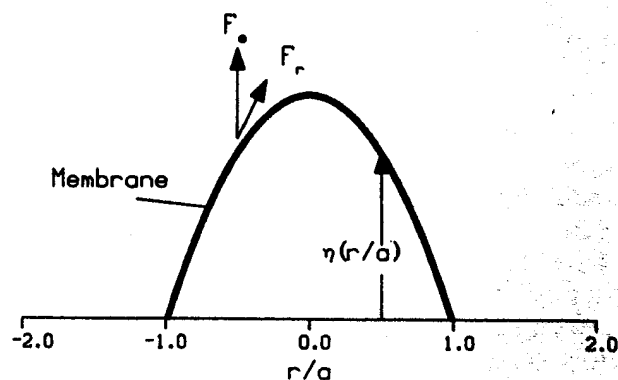


Fig. 2. Creation of in-plane electric force F_r . The nonzero membrane deflection results in an in-plane electric force that causes the membrane charge to move along the membrane surface (axes are on different scale for clarity).

between the membrane and P_1 , and a field $E_B(r, t)$ between the membrane and P_2 . Both these electric fields have components in the z and ρ directions (where ρ is the direction tangent to the membrane) which we denote by $E_{T(z)}(r, t)$, $E_{B(z)}(r, t)$, $E_{T(\rho)}(r, t)$, and $E_{B(\rho)}(r, t)$, respectively. In this situation we can apply Stokes' law to the contours. In the limit for $d_m \rightarrow 0$, $dr \rightarrow 0$, and very small deflections, the application of Stokes' law for Γ_1 yields

$$\begin{aligned} -E_{T(z)}(r, t)[d_0 - \eta(r, t)] + E_{T(z)}(r + dr, t)[d_0 \\ - \eta(r + dr, t)] + E_{T(\rho)}(r, t)dr = 0 \end{aligned} \quad (4)$$

Eq. (4) may be rewritten, using the approximations

$$\begin{aligned} \eta(r + dr, t) &\approx \eta(r, t) + \partial_r\{\eta(r, t)\} dr \\ E_{T(z)}(r + dr, t) &\approx E_{T(z)}(r, t) + \partial_r\{E_{T(z)}(r, t)\} dr \end{aligned} \quad (5)$$

The notation $\partial_r\{\cdot\}$ is short for $\partial\{\cdot\}/\partial r$. Substitution of Eq. (5) into Eq. (4) and linearizing the result then gives

$$\begin{aligned} E_{T(\rho)}(r, t) &= [\eta(r, t) - d_0]\partial_r E_{T(z)}(r, t) \\ &+ E_{T(z)}(r, t)\partial_r \eta(r, t) \end{aligned} \quad (6)$$

The radial field $E_{T(\rho)}(r, t)$ causes a radial surface current density $J_r(r, t)$ defined by

$$J_r(r, t) = \frac{E_{T(\rho)}(r, t)}{R_S} \quad (7)$$

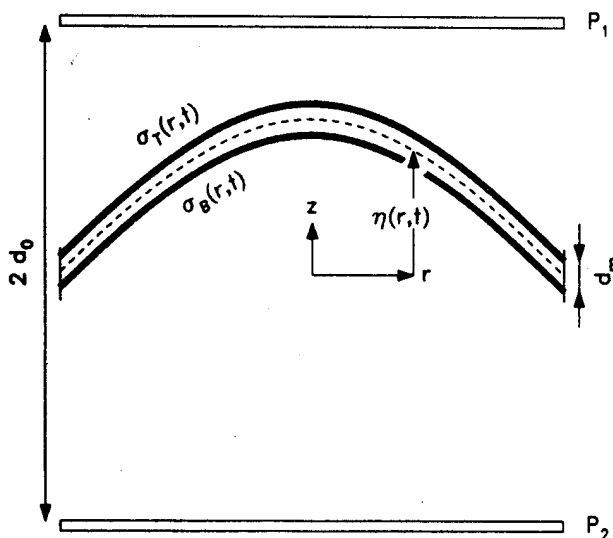


Fig. 3. Definition of calculation model parameters. The axisymmetric edge-clamped membrane with thickness d_m vibrates between two stationary perfectly conductive plates P_1 and P_2 . The membrane surface facing P_1 carries a surface charge density $\sigma_T(r, t)$, the opposite surface carries a charge density $\sigma_B(r, t)$. The membrane surfaces have a resistivity $R_S \Omega/\square$.

Demanding charge continuity, that is, $\nabla J_\rho(r, t) + \partial_t \sigma_T(r, t) = 0$, with $\nabla\{\cdot\} = \partial_r\{\cdot\} + \{\cdot\}/r$, then gives

$$\partial_t \sigma_T(r, t) = -\frac{\nabla E_{T(\rho)}(r, t)}{R_S} \quad (8)$$

Next we combine Eqs. (8) and (6) and arrive at

$$\begin{aligned} \partial_t \sigma_T(r, t) &= -\frac{1}{R_S} \nabla\{[\eta(r, t) - d_0]\partial_r E_{T(z)}(r, t) \\ &+ E_{T(z)}(r, t)\partial_r \eta(r, t)\} \end{aligned} \quad (9)$$

We repeat the analysis for contour Γ_2 , which yields

$$\begin{aligned} \partial_t \sigma_B(r, t) &= -\frac{1}{R_S} \nabla\{[d_0 + \eta(r, t)]\partial_r E_{B(z)}(r, t) \\ &+ E_{B(z)}(r, t)\partial_r \eta(r, t)\} \end{aligned} \quad (10)$$

Since we are interested in the total membrane charge variation, we define the total charge $\sigma_s(r, t)$ according to

$$\sigma_s(r, t) = \sigma_T(r, t) + \sigma_B(r, t) \quad (11)$$

From Eqs. (9)–(11) we see that

$$\begin{aligned} \partial_t \sigma_s(r, t) &= \frac{1}{R_S} \nabla[d_0 \partial_r \{E_{T(z)}(r, t) - E_{B(z)}(r, t)\} \\ &- \eta(r, t) \partial_r \{E_{T(z)}(r, t) + E_{B(z)}(r, t)\} \\ &- \{E_{T(z)}(r, t) \\ &+ E_{B(z)}(r, t)\} \partial_r \eta(r, t)] \end{aligned} \quad (12)$$

The fields $E_{T(z)}(r, t)$ and $E_{B(z)}(r, t)$ may both be con-

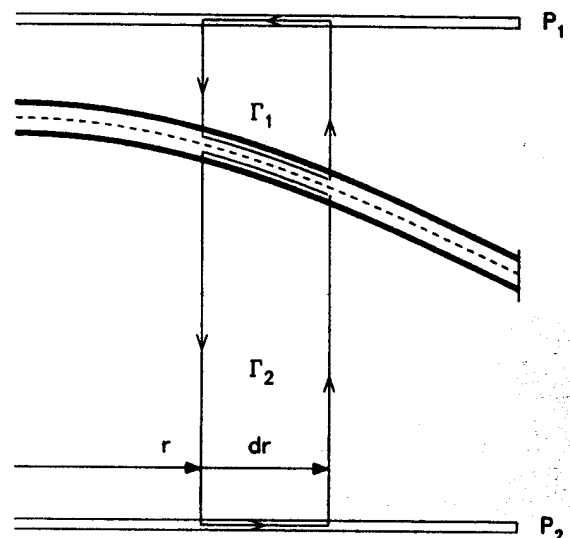


Fig. 4. Definition of integration contours. Contours Γ_1 and Γ_2 define the integration paths along which Stokes' law is applied.

sidered to be a superposition of three fields: the field $E_1(t) = V_1(t)/d_0$, which is caused by the imposed potential difference between the electrodes, and the fields $E_{\sigma T}(r, t)$ and $E_{\sigma B}(r, t)$, which are the result of the two charge layers $\sigma_T(r, t)$ and $\sigma_B(r, t)$ (Fig. 5). In the small-deflection approach we used earlier, the fields $E_{\sigma T}(r, t)$ and $E_{\sigma B}(r, t)$ are approximately parallel to the z axis and may be calculated from Gauss' divergence law, which results in

$$E_{\sigma T}(r, t) \approx \frac{1}{2} \frac{\sigma_T(r, t)}{\epsilon_0} \quad (13)$$

$$E_{\sigma B}(r, t) \approx \frac{1}{2} \frac{\sigma_B(r, t)}{\epsilon_0}$$

(These approximations hold for the regions between membrane and electrodes; inside the membrane we would have to account for ϵ_m .) The superposition of fields may thus be written as

$$E_{T(z)}(r, t) = E_1(t) + \frac{1}{2} \frac{\sigma_T(r, t)}{\epsilon_0} + \frac{1}{2} \frac{\sigma_B(r, t)}{\epsilon_0} \quad (14)$$

$$E_{B(z)}(r, t) = E_1(t) - \frac{1}{2} \frac{\sigma_T(r, t)}{\epsilon_0} - \frac{1}{2} \frac{\sigma_B(r, t)}{\epsilon_0}$$

which in turn may be used in Eq. (12) to yield

$$\partial_t \sigma_s(r, t) = \frac{1}{R_S} \nabla \left[\frac{d_0}{\epsilon_0} \partial_r \sigma_s(r, t) - 2E_1(t) \partial_r \eta(r, t) \right] \quad (15)$$

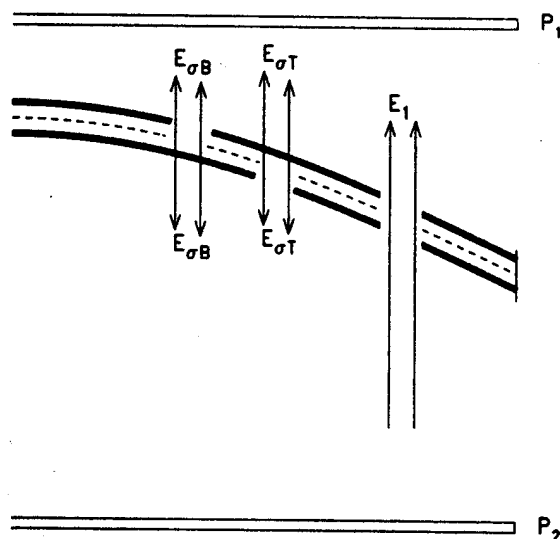


Fig. 5. Composition of electric field between plates. Fields in the gaps between the membrane and the plates are a superposition of field E_1 caused by the potential difference between the plates, and fields $E_{\sigma T}(r, t)$ and $E_{\sigma B}(r, t)$ caused by the charge layers.

where we used $\partial_r E_1(t) = 0$. The total charge $\sigma_s(r, t)$ may be divided into two components,

$$\sigma_s(r, t) = \sigma_0 + \sigma_1(r, t) \quad (16)$$

that is, a static charge σ_0 which is independent of place and time, corresponding to the offset charge in Eq. (1), and a varying charge $\sigma_1(r, t)$ which is the part that we are interested in. Clearly, Eq. (15) still holds if we replace $\sigma_s(r, t)$ by $\sigma_1(r, t)$. When we make this substitution and rearrange terms, we obtain the differential equation that describes the charge variation $\sigma_1(r, t)$,

$$\begin{aligned} \partial_t \sigma_1(r, t) - \frac{d_0}{\epsilon_0 R_S} \nabla^2 \sigma_1(r, t) \\ = \frac{-2}{R_S} E_1(t) \nabla^2 \eta(r, t) \end{aligned} \quad (17)$$

where $\nabla^2\{\cdot\} = \partial_r\{\partial_r\{\cdot\}\} + \partial_r\{\cdot\}/r$. In Eq. (17) we consider the right-hand side as a given function. The membrane deflection $\eta(r, t)$ is the (linear) response of the membrane to the driving field $E_1(t)$. In the remainder of this paper we focus on stationary harmonic excitation, that is,

$$E_1(t) = E_a \cos \omega t = \frac{V_a}{d_0} \cos \omega t \quad (18)$$

$$\eta(r, t) = \eta_c(r) \cos \omega t + \eta_s(r) \sin \omega t$$

If we assume that the membrane coating is perfectly conductive in the region $r > a$ and the dc bias V_0 is connected (i.e., prescribed) at the circumference of the membrane, then the boundary condition for the integration of Eq. (17) is simply

$$\sigma_1(a, t) = 0 \quad (19)$$

The response $\eta(r, t)$ may be calculated using the theory in [3]–[5].

The solution to Eqs. (17) and (19) may be found by means of the substitution

$$\sigma_1(r, t) = A(r) + B(r) \cos 2\omega t + C(r) \sin 2\omega t$$

$$A(a) = B(a) = C(a) = 0 \quad (20)$$

and the relations $\cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$ and $\cos \omega t \sin \omega t = \frac{1}{2} \sin 2\omega t$. After substitution into Eq. (17) we obtain three equations from equating non-time-varying terms, terms that vary as $\cos 2\omega t$, and terms that vary as $\sin 2\omega t$, respectively,

$$\nabla^2 A(r) = \frac{\epsilon_0 E_a}{d_0} \nabla^2 \{\eta(r)\} \quad (21)$$

$$C(r) - \frac{d_0}{2\omega\epsilon_0 R_s} \nabla^2 B(r) = \frac{E_a}{2\omega R_s} \nabla^2 \{\eta_c(r)\} \quad (22)$$

$$B(r) + \frac{d_0}{2\omega\epsilon_0 R_s} \nabla^2 C(r) = \frac{E_a}{2\omega R_s} \nabla^2 \{\eta_s(r)\} \quad (23)$$

Clearly, a nonzero value for $A(r)$ is associated with a time-independent change of the membrane charge and may thus lead to a change of sensitivity of the associated loudspeaker. The functions $B(r)$ and $C(r)$ determine the distortion effects. Since these functions are amplitudes of quadrature components, we are basically interested in $\sqrt{B(r)^2 + C(r)^2}$. In the "ideal" situation we would like to have $A(r) = B(r) = C(r) = 0$ simultaneously. In the following sections we see how close we can come to this situation.

2.1 Distortion Effects

At first sight the distortion components are easily tackled by means of an increasing membrane resistivity, that is $R_s \rightarrow \infty$. This, however, may cause very long switch-on times when applied to a loudspeaker in practice. Let us therefore focus on Eqs. (22) and (23), from which the amplitudes $B(r)$ and $C(r)$ of the charge distortion components are determined. Since we want to keep $B(r)$ and $C(r)$ as small as possible, we analyze a worst-case situation which, as can be seen from Eq. (22), occurs for low frequencies and high values of the driving field E_a and membrane curvature $\nabla^2 \{\eta_c(r)\}$ and $\nabla^2 \{\eta_s(r)\}$. For very low frequencies (below the first resonance) we may assume that the membrane deflection is in phase with the driving field, hence $\eta_s(r) \approx 0$. Furthermore, we assume that at these low frequencies the amplitude of the membrane vibration will approximately equal the first eigenmode of the membrane in vacuum, that is,

$$\eta_c(r) \approx d_0 J_0 \left(\frac{j_{01} r}{a} \right) \quad (24)$$

where J_0 is a Bessel function of the first kind and order zero and $j_{01} \approx 2.405$, the smallest positive zero of that function. The corresponding membrane curvature is then found as

$$\nabla^2 \eta_c(r) \approx d_0 \left(\frac{j_{01}}{a} \right)^2 J_0 \left(\frac{j_{01} r}{a} \right) \quad (25)$$

In this situation the membrane will just "touch" the electrodes and is thus at maximum drive level. In a properly designed electrostatic loudspeaker the maximum amplitude of V_a approximately equals V_0 . This is due to the fact that the electrostatic force [see Eq. (2)], which is determined by the product of V_a and V_0 , should be as large as possible, whereas the sum of V_0 and V_a is limited by the breakdown field strength of

air. An optimum is thus found at $V_a = V_0$. By combining this result with Eq. (1), we may write

$$E_a = \frac{V_a}{d_0} \approx \frac{V_0}{d_0} = \frac{1}{2} \frac{\sigma_0}{\epsilon_0} \quad (26)$$

Substitution of Eqs. (25) and (26) into Eq. (22) then yields

$$C(r) - \frac{d_0}{2\omega\epsilon_0 R_s} \nabla^2 B(r) = \frac{-d_0}{2\omega\epsilon_0 R_s} \frac{1}{2} \sigma_0 \left(\frac{j_{01}}{a} \right)^2 J_0 \left(\frac{j_{01} r}{a} \right) \quad (27)$$

Clearly, we can force $B(r)$ and $C(r)$ to take on very small values relative to σ_0 if we demand [knowing that $0 \leq J_0(x) \leq 1$ for $0 \leq x < j_{01}$]

$$\frac{4\omega\epsilon_0 a^2 R_s}{j_{01}^2 d_0} \gg 1 \quad (28)$$

from which we may single out R_s to obtain

$$R_s \gg \frac{j_{01}^2 d_0}{4\omega\epsilon_0 a^2} = \frac{j_{01}^2}{8f} \frac{d_0}{\epsilon_0 \pi a^2} \quad (29)$$

A full-range electrostatic loudspeaker typically has an electrical capacity $C_s = \epsilon_0 \pi a^2 / d_0$ in the range of 1.5 nF, and a lowest operating frequency of approximately $f = 50$ Hz. In this situation the value of R_s has to satisfy $R_s \gg 10$ M Ω , so practical values of 1000 M Ω and higher are typical for full-range loudspeakers designed to operate at extreme drive levels (i.e., close to the breakdown field strength of air). Notice that the time constant $R_s C_s$, which is a measure for the switch-on time of the loudspeaker, is in the range of a second, which is fully acceptable in practice.

2.2 DC Shift of Charge and Level-Dependent Characteristics

In the previous section we have seen that harmonic distortion may be suppressed by means of a proper choice of the surface resistivity of the membrane. This means in fact that harmonic distortion caused by charge variations is of minor interest for (properly designed) electrostatic push-pull loudspeakers. In contrast, we will see that the dc shift of charge will always occur in steady-state operation, so it is worth examining this effect in more detail.

We know that the dc part $A(r)$ of the charge variation is found from Eq. (21) and the boundary condition $A(a) = 0$. The straightforward solution for $A(r)$ is thus

$$A(r) = \epsilon_0 E_a \left\{ \frac{\eta_c(r)}{d_0} \right\} \quad (30)$$

(The homogeneous solution $A(r) = A_0 + A_1 \log(r/a)$,

with A_0 and A_1 arbitrary constants, is 0 due to the restrictions $A(a) = 0$ and $A(0)$ is finite.) From Eq. (30) we may draw a number of conclusions. First we see that the level of the dc shift cannot be influenced by the choice of the resistivity of the membrane. (The choice of R_S does affect the time to reach the steady state however.) The second observation is that since in the small-signal approach $\eta(r)$ varies linearly with E_z , the dc shift will vary as E_z . We may therefore expect that in the case of a large membrane deflection the (static) distribution of charge along the membrane surface will not be the same as the distribution in the case of a small membrane deflection. An important consequence of this effect is that since the radiation characteristics of the associated electrostatic loudspeaker will depend on the membrane charge distribution, these characteristics must be level dependent. This effect may be illustrated with the following experiment. We measure the on-axis frequency response of an electrostatic loudspeaker in an anechoic room at two different driving levels, a low level and a high level. Since the charge distribution will be different in both cases, we may expect a different response, especially in the low-frequency region where the largest membrane deflections will occur. In Fig. 6 we see the results of a measurement made with a commercially available electrostatic full-range loudspeaker, driven at a low level V_L and a high level $V_H = 30V_L$, respectively, and at a sweep rate of approximately 1 decade per minute. The measurements are normalized with respect to the driving level, which means that in the case of a purely linear behavior the characteristics should be exactly the same; this is clearly not true here. We see that at the high level the sensitivity of the loudspeaker increases below the first resonance frequency, whereas it decreases above the first resonance frequency. The cause of this anti-symmetrical effect around the resonance frequency is found in the change of phase near resonance. Below

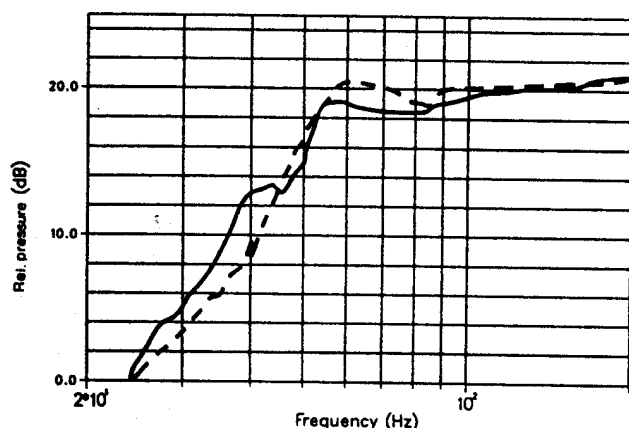


Fig. 6. On-axis response of commercial electrostatic loudspeaker. Measured response at 2 m demonstrates the effect of a dc shift of charge. A high-level (V_H , solid curve) input causes a change of sensitivity compared with a low-level ($V_L = V_H/30$, dashed curve) input. Measured data are normalized with respect to driving level.

resonance the membrane response will roughly be in phase with the electric driving field, which causes a current to flow toward the center of the membrane (see Fig. 2), and consequently an increase in charge and a higher sensitivity. Above resonance the membrane response will roughly be in opposite phase with the field, causing an outwardly flowing current, a charge decrease, and thus lower sensitivity.

Since $A(r)$ is the charge distribution along the radius of the vibrating membrane, we may define the value of

$$Q_S = 2\pi \int_0^a A(r)r dr \quad (31)$$

to be the amount of charge that flows to or from the vibrating membrane during the time that is needed to reach the steady state. Clearly, Q_S varies as V_d^2 . The real amount of charge Q_{SR} that flows to or from the vibrating membrane in practice is determined from a current measurement.

As an example we consider the case of a circular electrostatic push-pull loudspeaker with the following parameters:

Dc charging voltage V_0	1.5 kV
Ac signal voltage amplitude V_a	500 V
Membrane radius a	0.125 m
Membrane tension T	100 N/m
Membrane density ρ_m	0.02 kg/m ²
Distance membrane-electrode d_0	2.3 mm
Additional acoustic damping R_a	40 Rayl

where the additional acoustic damping is used to suppress resonance effects in order to obtain a realistic loudspeaker model. In Fig. 7(a) we see the calculated value of the total membrane charge Q_S and the normalized value of the charge distribution $A(r)$ for the frequency range of 30 to 100 Hz. The value of $A(r)$ is depicted as a contour plot; positive $A(r)$ values are indicated by solid contours, negative values by dotted contours. In Fig. 7(b) we see the results for the frequency range of 100 Hz to 1 kHz. We see that the value of Q_S (i.e., the change of total membrane charge) indeed predicts a change of low-frequency loudspeaker sensitivity according to the results in Fig. 6. Below the first resonance frequency (some 70 Hz in this case) there is an increase in total charge; above the resonance frequency there is a decrease in total charge [Fig. 7(a)]. Notice that above resonance frequency, the value of Q_S is not necessarily a good measure for the change of sensitivity, since the value of $A(r)$ shows one or more changes of sign (for fixed frequency). Since the central part of the membrane will account for the largest contribution in the total volume velocity, we have to give relatively more weight to the on-axis value of $A(r)$, which does not show up in the value of Q_S . The value of Q_S predicts a "steady-state ripple" in the loudspeaker's sensitivity [Fig. 7(b)], the periodicity of which is determined by the successive membrane resonance frequencies. The variation of both total and local charge

reduces considerably with increasing frequency, however, which means that in this case the frequency region above some 300 Hz (i.e., above the second resonance frequency) appears to be of minor importance.

The $A(r)$ contours predict the distribution of charge. From these contours we see that with varying frequency the charge variations at most membrane positions have sign reversals. The most "stable" position in this respect is the region near the membrane edge (near $r = a$). Furthermore we see that the contour pattern builds up defined regions of charge depletion and enhancement, which may have an important influence on the high-frequency sensitivity when only part of the membrane is driven at higher frequencies. (In order to prevent high-frequency focusing effects, see [5].) Consider, for example, a situation where the loudspeaker, according to Eqs. (32), is driven with a high-level low-frequency tone (steady state). According to the charge distribution pattern in Fig. 7(a), we may expect relatively large charge variations of the membrane area near the center. Thus if only the central part of the loudspeaker is used for high frequencies, the high-frequency sensitivity will vary with the frequency of the steady-state signal. Therefore in a case like this it may be worth moving the high-frequency region toward a position of less charge variation, for example, near the edge of the membrane. This region shows relatively

stable charge distribution values (and not much different from zero) as compared with other regions. Clearly, the center of the membrane is the worst place of all in this respect. Analogously, if a large part of the membrane is driven with two signals, a fixed high-level low-frequency signal and a signal of higher frequency, the high-level low-frequency tone will cause the sensitivity for the higher frequencies to follow the Q_s function. (For an experiment, see Baxandall [2, p. 132].)

We may compare theoretical and measured results from a "real" case, where the following input parameters were measured:

Dc charging voltage V_0	1.5 kV
Ac signal voltage amplitude V^a	750 V
Membrane radius a	0.125 m
Membrane tension T	100 N/m
Membrane density ρ_m	0.02 kg/m ²
Distance membrane-electrode d_0	2.3 mm
No additional acoustic damping.	

This setup differs from Eqs. (32) only in the value of the driving force (i.e., V_a) and the acoustic damping, the reason simply being that we want to emphasize the dc shift in order to simplify the measurement. We focus on the frequency region near the lowest resonance frequency since the investigated effect will be most pronounced there. In Fig. 8 we see the results obtained

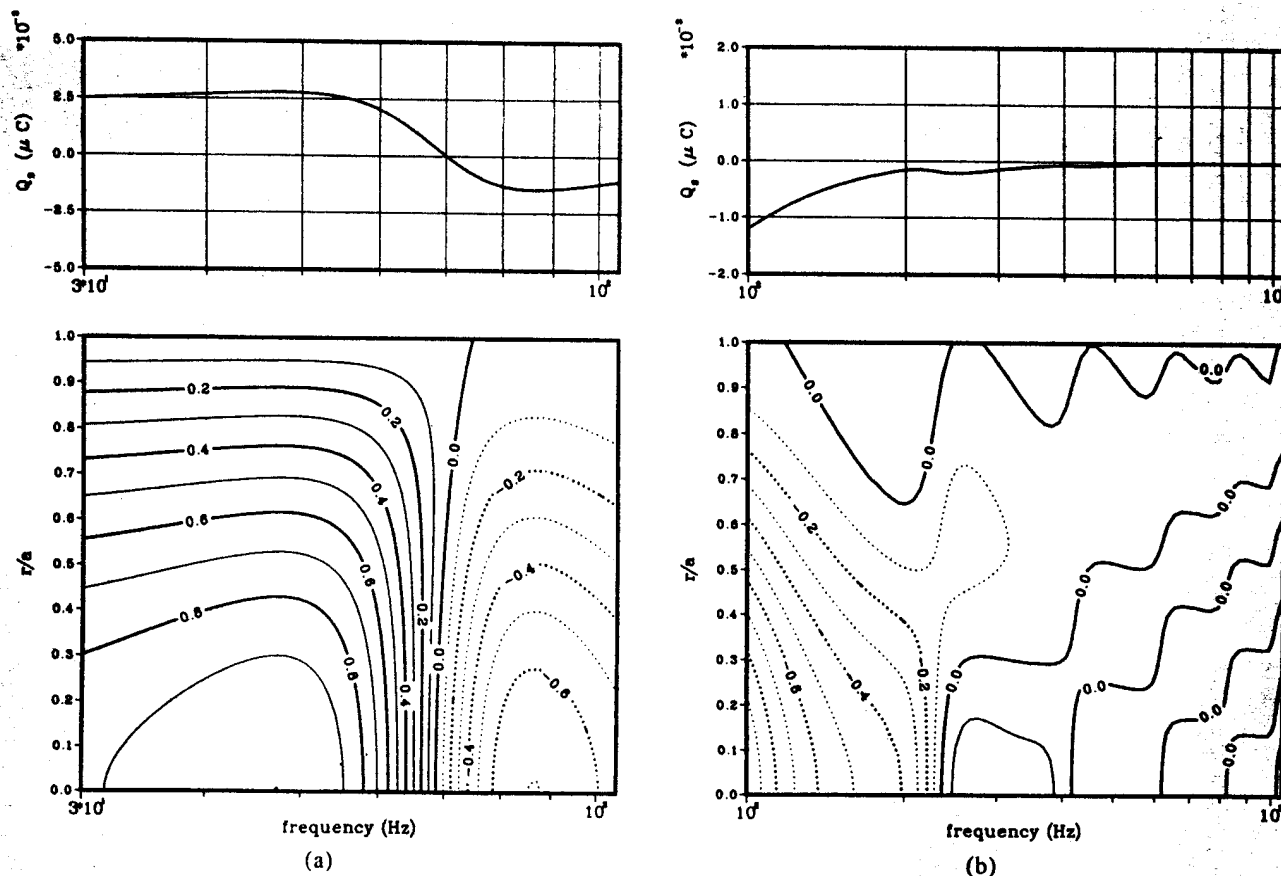


Fig. 7. Calculated charge variations in circular electrostatic loudspeaker. (a) Frequency range 30 to 100 Hz. (b) Frequency range 100 Hz to 1 kHz. The integrated charge variation Q_s is given in top graph. The calculated steady-state charge distribution $A(r)$ (bottom graphs) is denoted by contour lines; dotted lines represent negative values, solid lines represent positive values. The contour labels denote the fraction of the maximum $|A(r)|$ value. Loudspeaker parameters are given in Eqs. (32).

from calculation and measurement. Clearly there is a close agreement between measured Q_{SR} and calculated Q_S functions.

The maximum value of the measured charge equals approximately $0.06 \mu\text{C}$, which is about 10% of the total charge of the membrane at rest. Since the loudspeaker sensitivity is linearly related to the membrane charge (in the small-signal approach), we may expect that sensitivity will increase by approximately 0.8 dB compared to the constant-charge model. Bearing in mind that we are dealing here with a relatively small loudspeaker (full range would need some 0.5 m^2) and relatively low voltages (about 5 kV would be maximum in this case), it is not unlikely that level-dependent sensitivity changes well above 1 dB are found in full-range electrostatic loudspeakers.

Another demonstration of the effect of a moving charge is seen from the burst response (Fig. 9) of the electrostatic push-pull loudspeaker according to Eqs. (33). The surface resistivity of the membrane of this loudspeaker is such that harmonic distortion plays no significant role. The loudspeaker is driven just below its fundamental resonance frequency and thus the dc shift (a better term would be "quasi dc shift" here, since we observe the transition from one steady state to another) causes an extra amount of charge to flow toward the membrane. The gradually increasing amount of charge then causes a gradually increasing acoustic output, until eventually a steady state is reached.

3 CONCLUSIONS

The theoretical model, developed to describe the membrane charge movement in circular electrostatic push-pull loudspeakers, predicts two important effects, the first being harmonic distortion and the second, a dc shift of the charge with subsequent change of sensitivity in a steady-state situation. The generation of harmonic distortion may be suppressed by means of a large membrane resistance, typically in the range of thousands of megohms per square for full-range applications. The dc shift will always occur in steady-state operation (assuming no special measures have been taken) and may cause a change of sensitivity well above 1 dB at high drive levels. The effect is most pronounced for large membrane deflections (low fre-

quencies). If linear operation of the loudspeaker is desired, then additional measures have to be taken to ensure that the amount of charge on the membrane remains constant, especially in the case of large membrane deflections.

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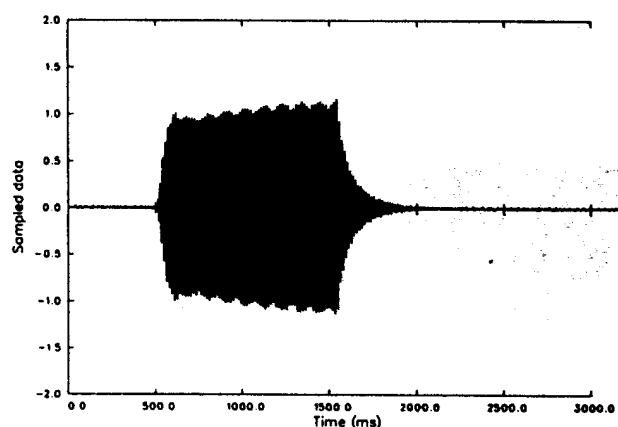


Fig. 9. Influence of charge variation on burst response. The loudspeaker with parameters according to Eqs. (33) is driven by a repetitive burst. Repetition frequency 0.5 Hz; burst frequency 65 Hz; burst duration 1 s. Samples are obtained from pressure response at 0.5 m on axis.

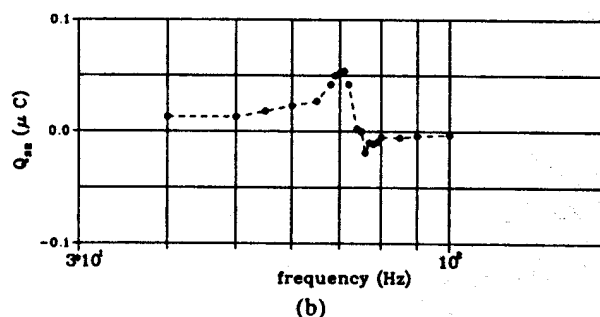
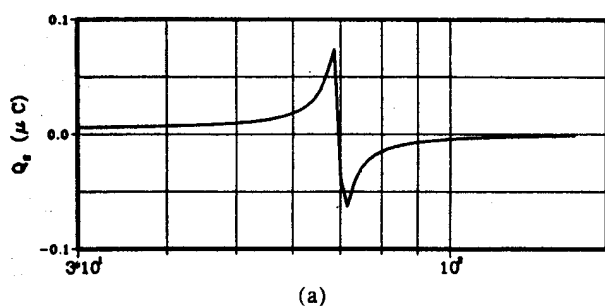


Fig. 8. (a) Measured and (b) calculated charge variations for loudspeaker parameters according to Eqs. (33).

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