

## 18.9 Horns and Compression Drivers

Horns play two fundamental roles in acoustics. Firstly, they are directional control devices serving to guide the airborne acoustic energy into particular directions or regions and as such they might well be called waveguides. The familiar plane wave tube may well be considered to be a special case of a horn as it certainly constitutes a waveguide. Secondly, horns act as impedance matching devices similar to the action of an electrical transformer.

Horn surfaces define a bounded region whose cross-sectional area increases from the input to the output in a loudspeaker application. The acoustic power flowing through a cross section of area  $S$ , when the acoustic pressure and particle velocity are in phase, is the product  $puS$ . The product  $uS$  is called the volume velocity and is denoted by  $U$  therefore the acoustic power flow is  $pU$ . At the input end of the horn where  $S$  is small and the acoustic pressure  $p$  is large, the volume velocity for a given acoustic power is small.

At the output end of the horn where  $S$  is large, the volume velocity is large and the acoustic pressure is small for the same acoustic power. This behavior is analogous to an electrical step down transformer that has a large voltage and small current in the primary and a small voltage and large current in the secondary.

Horns were used in acoustics long before their principles of operation were even partially understood. The earliest hearing aid devices were based on horns with sound transmission in the reversed sense in that the object was to convert large  $U$  and small  $p$  into large  $p$  with small  $U$  to accommodate the fact that the ear is a pressure sensitive organ. Some of the earliest acoustic recordings also employed horns operating in reverse with the mouth of the horn collecting energy over a large area and concentrating it into a small area to actuate a small diaphragm mechanically coupled to a recording stylus. This procedure was reversed in the reproduction process wherein the horn was employed in the more conventional sense.

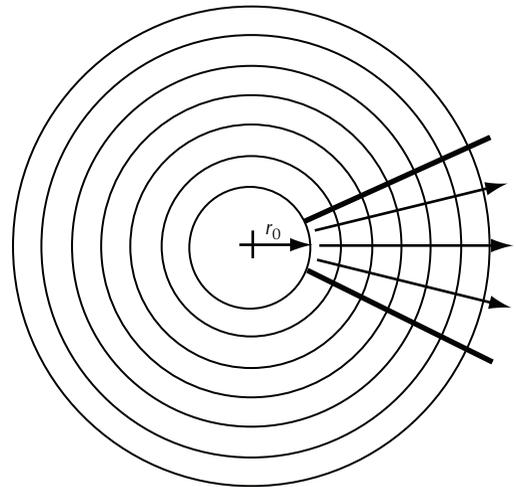
Horns have also been employed as the basis of many musical instruments. The requirements of a horn to be used for sound generation are radically different from that of sound reproduction or reinforcement. In the case of sound generation, resonances in the horn are desirable and in fact, essential. In sound reproduction or reinforcement, however, resonances are undesirable and steps must be taken to minimize their existence. This underscores the necessity for having an underlying theory of horn operation to guide the construction for various applications.

Horn theory stems from the original work of Euler, Lord Rayleigh, and Webster. Webster was the first to introduce the concepts of specific acoustic impedance and analogous acoustic impedance, both of which have proven to be very valuable in acoustic

analysis. What is known as Webster's horn equation is a wave equation that in the strictest sense is correctly applicable to only three waveguide structures. These are the plane wave tube, the conical horn, and the cylindrical horn. Webster's equation employs only a single space variable in the axial direction implying that the acoustic pressure is uniform over an appropriately drawn cross section of the guide or horn structure. This is satisfied exactly in a plane wave tube of limited diameter that is excited at the input by a plane wave.

The equation is also exact for a conical horn that is excited with a spherical wave and for a cylindrical horn that is excited by a cylindrical wave. The application of the equation to other horn structures is only approximately correct and then only when the horn opens up or flares very slowly. It is this last requirement that is often lost sight of in practice.

The simplest horn geometry is that of a truncated cone wherein acoustic energy in the form of a diverging spherical wave is introduced into the small end of the cone and subsequently propagates freely within the cone as an outgoing wave as suggested in Fig. 18-24.



**Figure 18-24.** A spherical wave has a natural coordinate fit in a conical horn.

In order for this horn to work properly, the acoustic wave introduced at its small or throat end must have a radius of curvature equal to  $r_0$ , where  $r_0$  is measured from the virtual apex of the truncated cone. The "natural fit" means that the spherical wavefronts diverging from the apex would everywhere be normal to the bounding surface provided by the cone and all energy flow would be radially directed. The particle velocity in such a wave motion would be tangent to the interior walls of the

horn and there would be no reflections at the interior wall surface. Internal reflections occur when the particle velocity has a component normal to an interior wall surface. Plane waves, as another example, form a natural fit in a straight tube of constant cross section. In both instances the wave motion can be described using only a single spatial coordinate and as such are called single parameter waves. A third geometry that provides a natural fit is that of a horn formed from a sector of a cylinder. Here, however, the wave introduced at the throat must have a cylindrical wavefront. So far as is presently known these three geometries are the only ones that satisfy the natural fit or single parameter conditions exactly.

As a straight tube of constant cross section of course is not a horn in the strictest sense, that leaves only two natural horns. Many other horn shapes have been employed with varying degrees of success, however, but they are only approximately single parameter devices and all suffer from bounding wall reflections to a greater or lesser degree. An analysis by Morse concludes that in order for a horn shape to approximately fall under the conditions necessary to satisfy Webster's horn equation, the rate of change of the square root of the cross-sectional area with respect to the single axial parameter must be much less than one. To what degree some common horns satisfy or fail to satisfy this criterion is a point worthy of examination.

Webster's equations for single parameter horns are

$$\frac{1}{S} \times \frac{\partial \left( S \frac{\partial p}{\partial \chi} \right)}{\partial \chi} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (18-69)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial \chi}$$

In these equations  $\chi$  is the appropriate single space variable and the other symbols have their usual meanings. The area symbol  $S$  must be treated carefully. In a true one parameter horn it is the surface area of the appropriately shaped wavefront as a function of position. In other horns the assumption is made that the wavefronts are approximately plane and  $S$  is the true cross sectional area of the horn as a function of position.

The first of Eq. 18-69 when applied to the conical horn becomes just the spherical wave equation. The second of these equations allows the determination of the specific acoustic impedance at the throat of the horn. The specific acoustic impedance is the ratio of the complex acoustic pressure to the complex particle velocity. The mechanical impedance is thus the product of the specific acoustic

impedance with the wavefront area  $S$ . With a suitable horn driver, these expressions are identical to those of a pulsating sphere of radius  $r_0$  with the exception that all of the energy diverges through the horn rather than through the surface area of a sphere of radius  $r_0$ . The sound intensity on the axis of the horn is thus increased by the ratio of the area of the sphere to the wave entrance area of the horn. This ratio is  $2/(1 - \cos\theta)$  where  $\theta$  is the angle between the horn axis and the interior surface of the horn.

For example, if the coverage angle of the horn is  $40^\circ$ ,  $\theta$  is  $20^\circ$  and this ratio is 33.16. This number is identical to the axial  $Q$  of the horn. This corresponds to a pressure level increase of about 15 dB on the axis of the horn as compared with that produced by a pulsating sphere without the aid of a horn. Thus far the horn has been treated as if there were only an outgoing wave. This would strictly be true if the horn were infinitely long. For any horn of finite length a reflection will occur at the mouth and a portion of the original energy will be directed along the horn back towards the driver. Such reflections can lead to the production of standing waves having resonant frequencies related to the horn length. It has been found in practice that there is only a negligible mouth reflection if the mouth perimeter is about 3 times the free space wavelength at the frequency of operation. Horns intended for use at low frequencies are thus large, unwieldy devices.

Salmon has described a family of horns that can have approximately single parameter behavior when they flare slowly enough. Members of this family are described by

$$S = S_0 \left[ \cosh\left(\frac{\chi}{h}\right) + T \sinh\left(\frac{\chi}{h}\right) \right]^2 \quad (18-70)$$

$S_0$  is the cross-sectional area at the throat where  $\chi$  is 0. A scale factor denoted as  $h$  is indicative of the rapidity of flare with small values of  $h$  corresponding to rapid expansion.  $T$  is a shape factor determining the general properties of the horn near the throat. When  $T = 0$ , Eq. 18-70 generates a catenoidal horn. When  $T = 1$ , an exponential horn results. When  $T = h/\chi_0$  and is allowed to approach  $\infty$  by letting  $h$  become very large, a conical horn results. Each of these horns merits individual attention. The descriptions given in each instance assume the horn is long enough so that mouth reflections can be ignored.