Dimensioning a single-loop phono preamplifier with subsonic filter

Version 1, Marcel van de Gevel, 24 April 2025

1. Introduction

In the diyAudio thread https://www.diyaudio.com/community/threads/single-stage-active-riaa-correction-with-second-or-third-order-butterworth-high-pass-included.413649/ , I described a way to include a second- or third-order Butterworth high-pass filter in a phono preamplifier with active RIAA correction by using a somewhat more elaborate feedback network than is usually used. An example is shown in Figure 1.

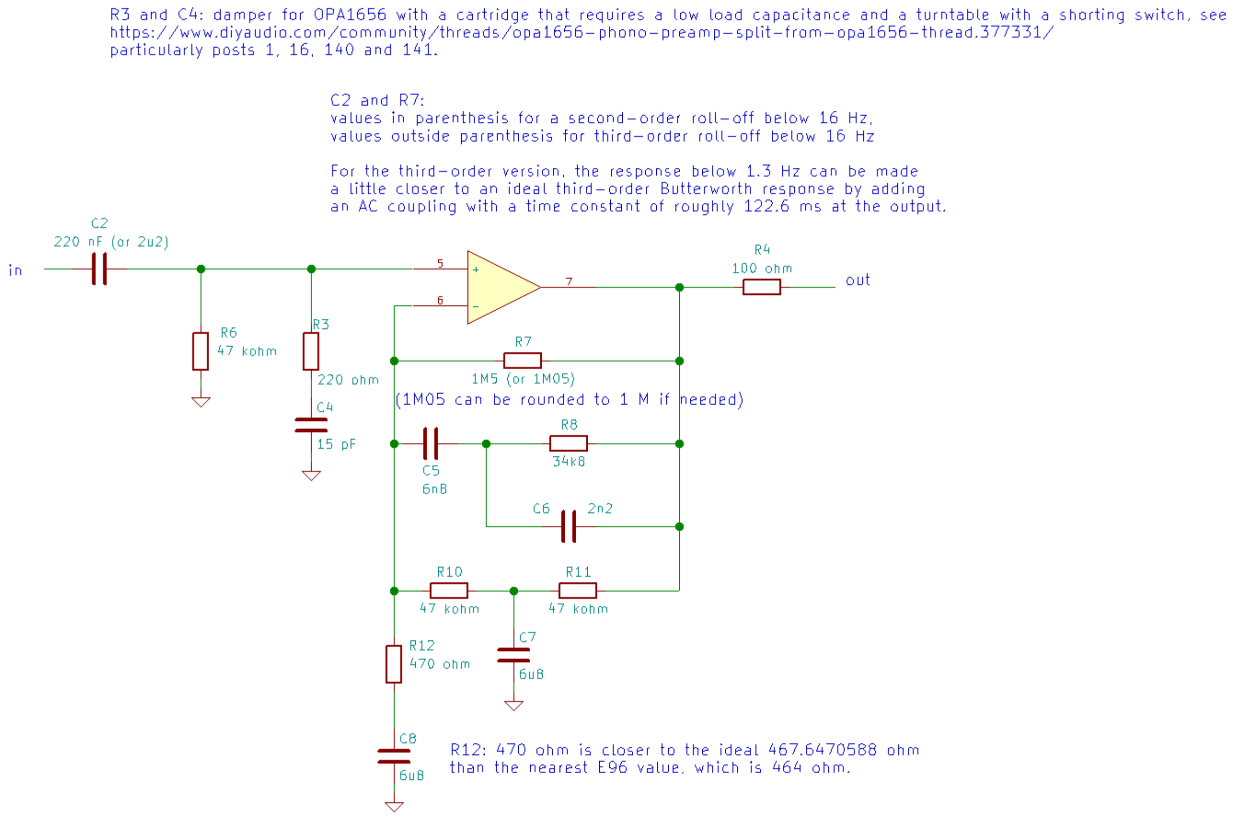

Figure 1: Example of a phono preamplifier circuit as described in the diyAudio thread

Even when the loop gain is assumed to be infinite at all frequencies, due to the high order of the feedback network, exactly calculating the component values is difficult. I therefore resorted to some coarse approximations and tweaking with a pole-zero extraction program.

In section 2 of this document, these coarse approximations are described. In section 3, a more accurate approximation is given, or actually an accurate approximation for placing the RIAA zero and an exact method for placing the RIAA poles and subsonic filter poles once R_8 has been chosen to place the zero. The exact method can easily lead to negative or complex resistor values, though.

In both cases, all components are assumed to be ideal and loop gain is assumed to be infinite at all frequencies. A way to correct for finite loop gain due to finite gain-bandwidth product is given in the attachment of post #100 of the diyAudio thread, https://www.diyaudio.com/community/attachments/riaafinitegbp-pdf.1438365/

The time constant of the first RIAA pole will be called τ_{p1}, the time constant of the second RIAA pole will be called τ_{p2} and the time constant of the RIAA zero will be called τ_{zRIAA}. Their values are in principle τ_{p1} = 3.18 ms, τ_{p2} = 75 μs and τ_{zRIAA} = 318 μs, but the time constants of the poles have to be slightly reduced when you want to precorrect for finite gain-bandwidth product, as explained in the document about that subject.

Although it is not further analysed in this document, Figure 2 shows how the circuit could be combined with an electrically "cold" input resistance for very low noise. This configuration is only applicable to discrete designs.

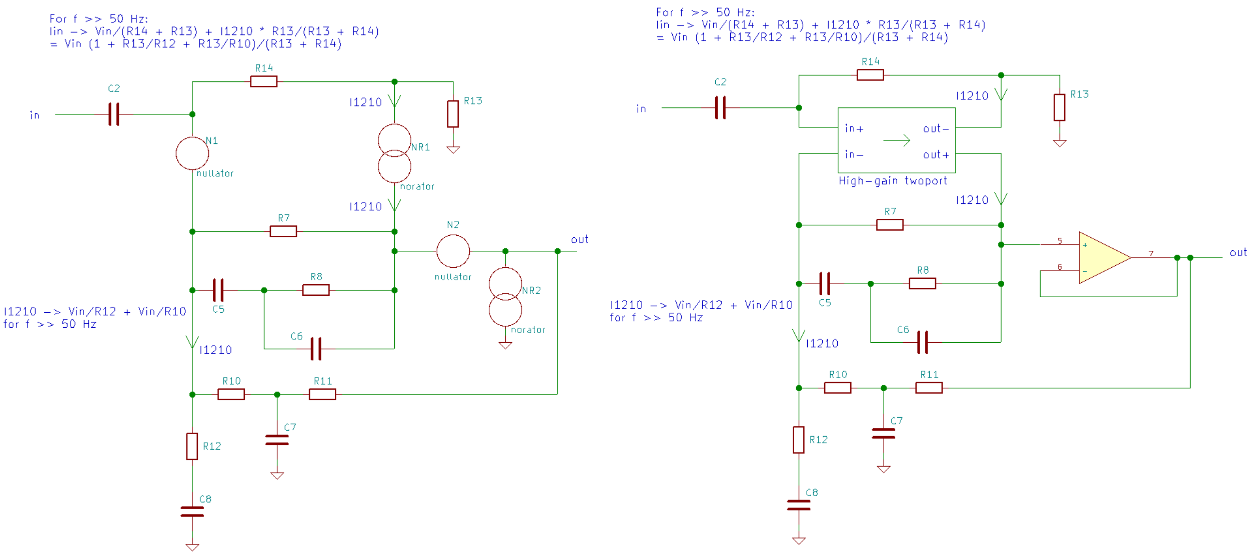

Figure 2: Version with an electrically "cold" input resistance for very low noise

2. Idea behind the circuit and first approximation for finding the component values

When you just look at the topology and ignore the component values, Figure 3 is a rather conventional RIAA amplifier (you could make it even more conventional by connecting R_7 in parallel with C_5, that doesn't matter much for the principle).

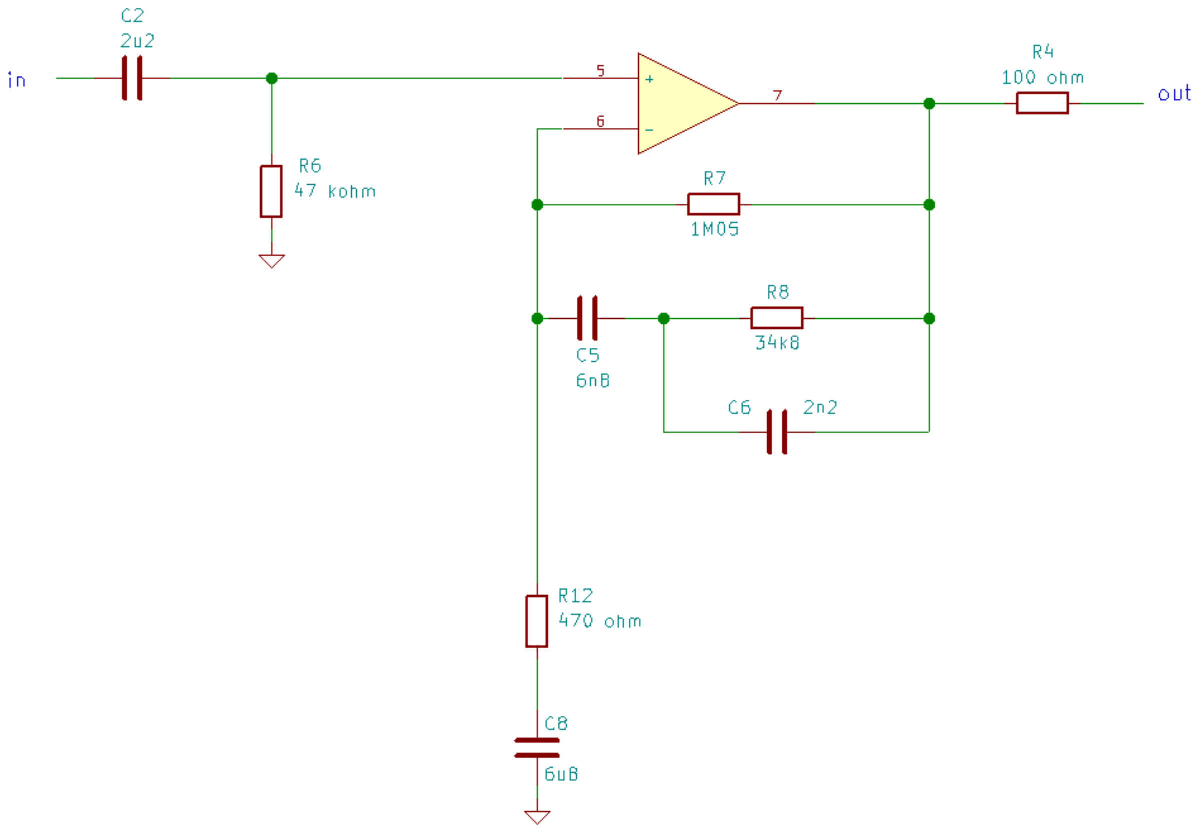

Figure 3: Almost conventional phono preamplifier

Normally, C_8 is used to cause roll-off in the subsonic region and the network R_7...R_8, C_5, C_6 realizes the RIAA poles and zero. In this case, however, I use C_8 to realize the lowest RIAA pole at -1/(3.18 ms) and R_7 to get subsonic roll-off.

Note that C_8/C_5 = 1000, meaning that without the subsonic roll-off, the DC gain would be 1001, a very ordinary value for a moving-magnet amplifier (1 kHz gain roughly 40 dB).

With everything ideal, at the value of the Laplace variable *s* where the impedance of C_8 cancels the impedance of R_{12}, the feedback disappears and the gain goes to infinity. This means that there is a pole at exactly $-1/(R_{12}C_8)$, so if this has to be the first RIAA pole, one needs $R_{12}C_8$ = 3.18 ms. It's actually 3.196 ms in the schematic, pretty close.

The disadvantage of using C_8 for the first RIAA pole is that C_8, which has a relatively large value, needs to be accurate to get an accurate first RIAA pole. (C_8 has practically no effect on the gain at frequencies much greater than 50 Hz, so its tolerance affects deep bass, but not channel balance.) The advantage is that you can include better subsonic filtering in the loop by adding two more resistors and a capacitor.

As an intermediate step, suppose you could add an ideal inductor with a huge value between the output and the negative input of the op-amp, chosen such that it resonates with C_5 at the desired subsonic roll-off frequency, and that you chose R_7 such that it damps the LC circuit to a quality factor of ½ √2. The subsonic response would then be very close to second-order Butterworth. That's because the gain of the RIAA correction amplifier is one plus the ratio of the feedback impedance to the impedance from the negative op-amp input to ground, and that "one" is quite negligible at low frequencies. Mind you, R_8 and R_9 contribute to the damping of the LC circuit, but not by much. You could also choose a quality factor of 1 and design the AC coupling at the input for the same cut-off frequency. The combined response is then third-order Butterworth.

Such an ideal inductor is totally impractical, but it can be approximated with a T network consisting of two resistors with values much smaller than R_7 and a capacitor to ground at the point where they are connected, see Figure 4.

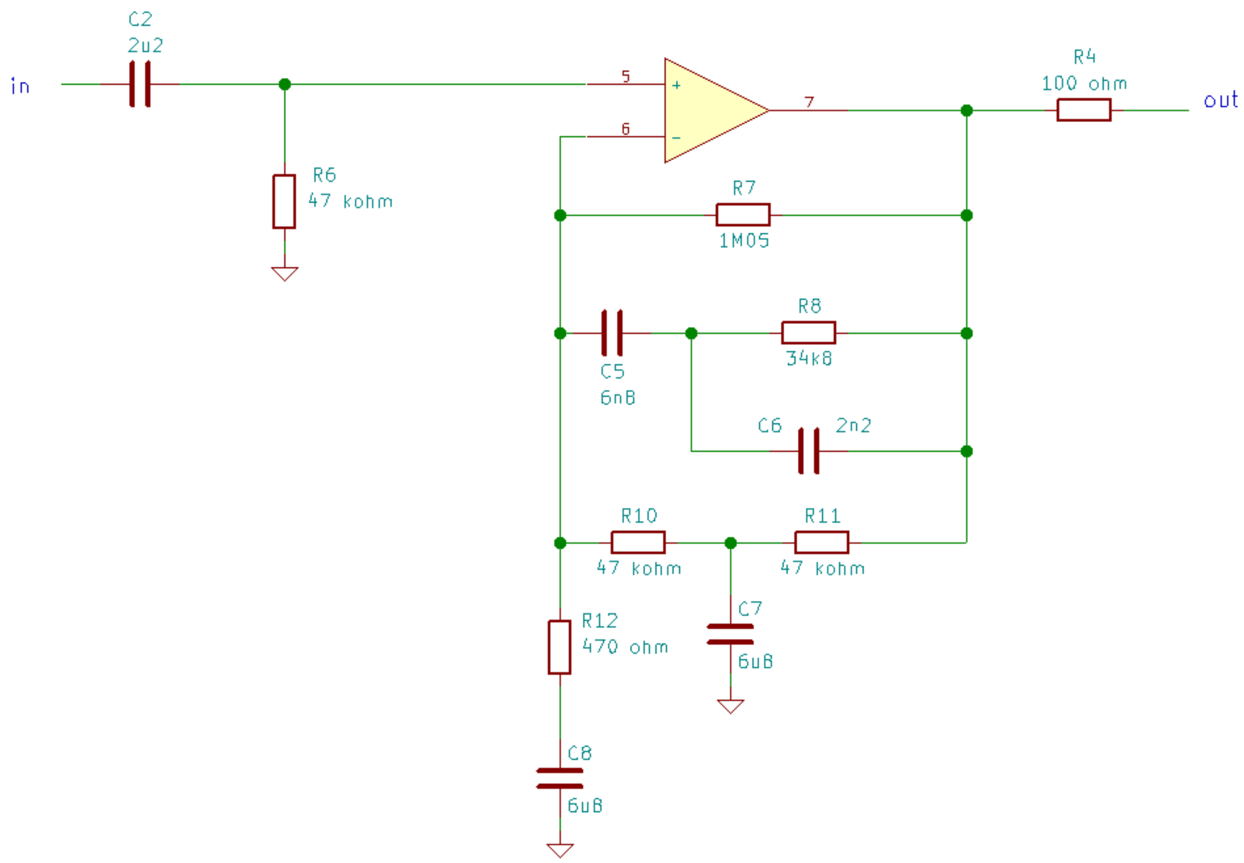

Figure 4: Now with subsonic filter

The transfer from the voltage going into R_{11} to the current coming out of R_{10} rolls off at a first-order rate from some very low frequency onwards, like would be the case with an inductor.

One way to dimension the circuit is to do some rough approximate calculations for the values and to then fine-tune them using a pole-zero extraction program (or a simulator that does AC sweeps, although that is less convenient).

Regarding those approximate calculations:

$$R_{12}C_8 = \tau_{p1}$$

where τ_{p1} is normally 3.18 ms, to get the first RIAA pole at the right spot. This is exact when everything is ideal. I introduce a symbol for the time constant of the first RIAA pole so you can easily reuse the calculations for some other correction or tweak the time constants to precorrect for finite gain-bandwidth product.

The DC gain would be $1 + C_8/C_5$ without subsonic roll-off, so $C_8/C_5 = 1000$ gives you a midband gain of roughly 40 dB, as the midband gain of a RIAA preamplifier is roughly 10 times smaller than the DC gain would be without any subsonic roll-off.

At $s = -1/(R_8(C_5 + C_6))$, the impedance of the network R_8, C_5, C_6 goes to zero and the gain of the circuit becomes 1. As a gain of 1 is pretty close to 0, this must be close to the location of the RIAA zero. That is,

$$R_8(C_5 + C_6) \approx \tau_{zRIAA}$$

where $\tau_{zRIAA} = 318$ µs.

At $s = -1/(R_8C_6)$, the impedance of the parallel connection of C_6 and R_8 goes to infinity. The impedance of the whole feedback network remains finite due to the other branches R_7 and R_{11}, C_7, R_{10}, but it does get pretty large. That means the second RIAA pole must be close, so we get the extra criterion

$$R_8C_6 \approx \tau_{p2}$$

where τ_{p2} is normally 75 µs.

Hence,

$$(C_5 + C_6)/C_6 \approx \tau_{zRIAA}/\tau_{p2}$$

$$C_5 \approx (\tau_{zRIAA}/\tau_{p2} - 1)\ C_6$$

This boils down to $C_5 \approx 3.24\ C_6$ when τ_{zRIAA} and τ_{p2} have the normal values (those with no precorrection for finite gain-bandwidth product). In the pole-zero extraction program, a slightly lower ratio usually works out better due to everything that gets neglected here. 6.8 nF and 2.2 nF (ratio 3.090909...) or 10 nF and 3.3 nF (ratio 3.030303...) tend to work better than 22 nF and 6.8 nF (ratio 3.2352941176...).

The (theoretical) inductance L is chosen to resonate with C_5 at the required subsonic roll-off frequency and R_7 is chosen to get the desired quality factor. R_{10} and R_{11} get convenient values much smaller than R_7 with R_{10} also much greater than R_{12}. We then have

$$C_7 = L/(R_{10}R_{11})$$

The RCR T-network that approximates an inductor actually approximates an inductor with inductance $L = R_{10}\ R_{11}\ C_7$ and a series resistance of $R_{10} + R_{11}$. At low frequencies, it stops behaving inductively, it just turns into the series connection of the two resistors. As a result, one of the zeros of the high-pass filter that are supposed to lie at $s = 0$ actually lies somewhere around $s = -(R_{10} + R_{11})/(R_{10}\ R_{11}\ C_7)$.

For the second-order cases, I have used the first-order high-pass (AC coupling) at the input to cover this zero by making the input *RC* time constant approximately equal to $R_{10} R_{11} C_7/(R_{10} + R_{11})$, or actually to a more accurate value for the displaced zero found by the LINDA pole-zero extraction program. With a FET op-amp and split supply, one can also decide not to correct for the zero and leave out the input AC coupling altogether.

For the third-order case, I have used the input AC coupling to make the real pole of the third-order Butterworth response, so I couldn't use it to cover the displaced zero. I used the output AC coupling in that case, or simply did not cover the zero. The effect of the zero not being in the origin is typically only seen below 1.something Hz anyway.

There is another zero not exactly in the origin, this is related to the + 1 term in the gain expression of a non-inverting op-amp amplifier. It is so close to 0 that I decided not to bother correcting for it.

A minor improvement was implicitly suggested by hbtaudio, see Figure 5. When you split R_6 into a resistor R_6 after the input coupling capacitor and a resistor R_0 before the input coupling capacitor, the time constant of the input RC coupling network can be set accurately without needing awkward values for C_2. You will then need awkward values for the resistors, but those are available in E96 values, while capacitors are at best available in E24 values and more often than not only in E6 values.

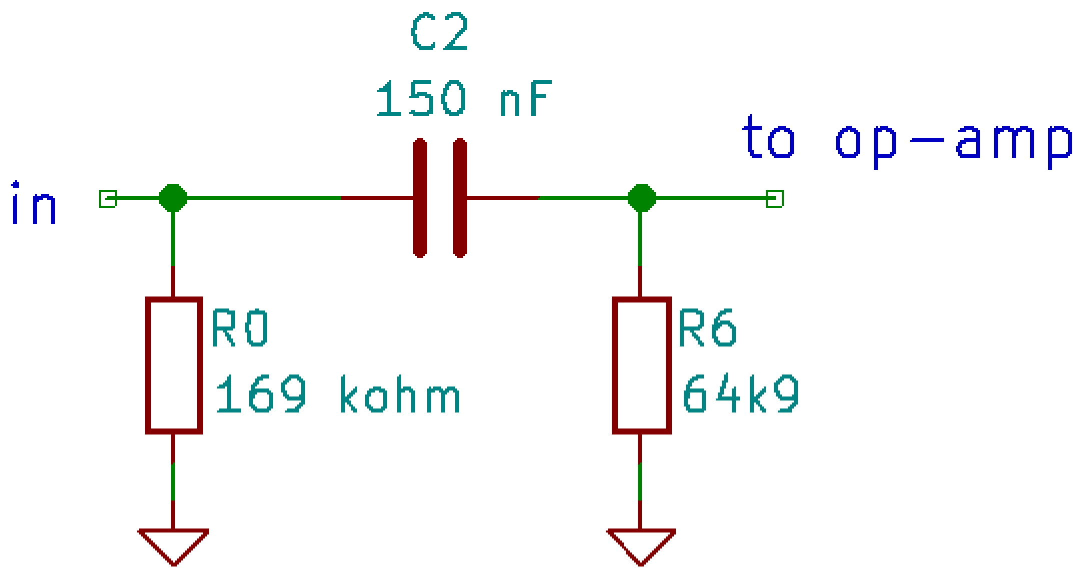

Figure 5: Example of splitting R_6. Assuming 1 kΩ *source resistance, the cut-off frequency is within* 1 % *from* 16 Hz *while only standard values are used.*

2.1. Step-by-step procedure

A. Look at the document https://www.diyaudio.com/community/attachments/riaafinitegbp-pdf.1438365/ to decide if you want to correct for finite gain-bandwidth product. Correcting is useful when you use a relatively slow op-amp to make a phono preamplifier with a high gain. If not, τ_{p1} = 3.18 ms, τ_{p2} = 75 µs and τ_{zRIAA} = 318 µs.

B. Choose convenient values for C_5 and C_6 with a ratio just above 3, such as 6.8 nF and 2.2 nF (ratio 3.090909...) or 10 nF and 3.3 nF (ratio 3.030303...). These tend to work better than 22 nF and

6.8 nF, even though their ratio (3.2352941176...) is closer to the theoretical 3.24.

C. Choose a convenient value for C_8 that is about 10 times the desired gain at 1 kHz times C_5. For example, C_8 = 6.8 µF when C_5 = 6.8 nF for a gain at 1 kHz of about 100 times (40 dB).

D. Calculate R_{12} using $R_{12} = \tau_{p1}/C_8$. Assuming a moving-magnet phono preamplifier, one would normally want R_{12} to be somewhere in the range from 100 Ω to 1 kΩ to keep the load on the op-amp reasonable and to ensure that the feedback network won't dominate the noise. Scale the capacitors and start over again when the calculated R_{12} is impractical.

E. Calculate R_8 twice, once using $R_8 \approx \tau_{p2}/C_6$ and once using $R_8 \approx \tau_{zRIAA}/(C_5 + C_6)$. Pick a value somewhere in the middle.

F. When the desired subsonic cut-off frequency is f_{sub}, calculate $L = 1/(4\pi^2 f_{sub}^2 C_5)$.

G. If a second-order Butterworth response (so $Q = ½\sqrt{2}$) is desired, choose $R_7 = ½\sqrt{2}\,\sqrt{(L/C_5)}$. For a third-order Butterworth response (pole pair Q = 1) is desired, choose $R_7 = \sqrt{(L/C_5)}$.

H. Pick values for R_{10} and R_{11} that are much greater than R_{12}, yet much smaller than ½ R_7.

I. Calculate $C_7 = L/(R_{10}\, R_{11})$. You can tweak R_{10} and/or R_{11} a bit to get a more convenient value for C_7. For example, round the calculated C_7 to the nearest standard value and then change R_{11} into $R_{11} = L/(R_{10}\, C_7)$.

J. For the second-order case, decide whether you want to use an input coupling capacitor. If so, choose it such that $(R_6 + R_{DC,cartridge})\, C_2 \approx (R_{10}\, R_{11}\, C_7)/(R_{10} + R_{11})$, or if you use a resistor R_0 to ground before the coupling capacitor, $(R_6 + R_{DC,cartridge}\, R_0/(R_{DC,cartridge} + R_0))\, C_2 \approx (R_{10}\, R_{11}\, C_7)/(R_{10} + R_{11})$. Obviously, $R_6\, R_0/(R_6 + R_0)$ needs to be equal to the parallel load resistance that the cartridge requires, usually 47 kΩ for a moving-magnet cartridge.

For the third-order case, you need an input coupling capacitor, and it needs to be chosen such that $(R_6 + R_{DC,cartridge}\, R_0/(R_{DC,cartridge} + R_0))\, C_2 = 1/(2\pi f_{sub})$.

K. Either fine-tune the values using a pole-zero extraction program or an ordinary simulator or measuring equipment, or use the method of section 3 of this document to find more accurate values.

L. If the gain at 1 kHz is far below 40 dB, it may be necessary to add a first-order low-pass filter at the output to correct for the ultrasonic zero. Its cut-off frequency has to be approximately 2122 Hz times the voltage gain at 1 kHz.

3. More accurate approximation

Without the tweaking step K at the end of the procedure of section 2, the RIAA correction values can be off by a few percent, that is, by an amount that is not negligible compared to the tolerances of accurate capacitors and resistors. It would be nice to find an exact solution or a better approximation.

I have only found a better approximation for placing the zero. Once R_8 is chosen based on the desired zero location, I have an exact procedure for placing the RIAA poles and the complex pole pair of the subsonic filter. It consists of these steps:

1. Choose and calculate values for C_5, C_6, C_8 and R_{12} using steps A...D from section 2.

2. Calculate the R_8 needed to get the zero at the right place while approximately taking into account the effect of R_{12}, as will be explained in section 3.1.

3. Use some fairly complicated exact expressions for the rest, see section 3.2.

4. Execute steps J and L from section 2 to dimension the input coupling capacitor and output first-order filter (if any).

I'm still not sure whether the results of the fairly complicated expressions are of any practical use.

3.1. Recalculating R_8

According to the approximation of section 2, the RIAA zero must be close to the value of *s* that makes the impedance of the network C_5-R_8-C_6 equal to zero. It is not exactly at that value of *s*, though, because the gain actually becomes 1 rather than 0 when the impedance of the network C_5-R_8-C_6 is zero.

The gain would become zero if the impedance of the network C_5-R_8-C_6 became the opposite of the parallel impedance of everything else that is connected to the negative op-amp input. This follows directly from the gain equation for a non-inverting op-amp stage: $1 + Z_A/Z_B$, where Z_A is the feedback impedance and Z_B the impedance to ground. When $Z_A = -Z_B$, the sum is zero.

At frequencies around 500 Hz (that is, around the desired corner frequency of the RIAA zero), the parallel impedance of everything else that is connected to the negative op-amp input is dominated by R_{12}. One can therefore equate the impedance of the network C_5-R_8-C_6 to $-R_{12}$ to get a better approximation of the location of the zero.

Writing out the equation for the impedance of the network C_5-R_8-C_6 results in $\frac{s R_8(C_5+C_6)+1}{s C_5(s R_8 C_6+1)}$.

Equating this to $-R_{12}$ results in

$$\frac{s R_8(C_5+C_6)+1}{s C_5(s R_8 C_6+1)}=-R_{12}$$

$$s R_8(C_5+C_6)+1=-s R_{12} C_5(s R_8 C_6+1)$$

$$s^2 R_{12} C_5 R_8 C_6+s\left(R_8(C_5+C_6)+R_{12} C_5\right)+1=0$$

$$s=\frac{-R_8(C_5+C_6)-R_{12} C_5 \pm \sqrt{(R_8(C_5+C_6)+R_{12} C_5)^2-4 R_{12} C_5 R_8 C_6}}{2 R_{12} C_5 R_8 C_6}$$

This equation gives two zero locations. The plus solution is the zero closest to the origin. This must be (the approximation to) the desired RIAA correction zero, the other must be (an approximation to) the ultrasonic zero that non-inverting RIAA preamplifiers always have because the gain goes to 1 rather than 0 at high frequencies.

Hence,

$$z_{\text{RIAA}}=-\frac{1}{\tau_{\text{zRIAA}}}=\frac{-R_8(C_5+C_6)-R_{12} C_5+\sqrt{(R_8(C_5+C_6)+R_{12} C_5)^2-4 R_{12} C_5 R_8 C_6}}{2 R_{12} C_5 R_8 C_6}$$

$$2R_{12}C_5R_8C_6z_{\mathrm{RIAA}}+R_8(C_5+C_6)+R_{12}C_5=\sqrt{(R_8(C_5+C_6)+R_{12}C_5)^2-4R_{12}C_5R_8C_6}$$

Squaring both sides,

$$4R_{12}^2C_5^2R_8^2C_6^2z_{\mathrm{RIAA}}^2+4R_{12}C_5R_8C_6z_{\mathrm{RIAA}}(R_8(C_5+C_6)+R_{12}C_5)+(R_8(C_5+C_6)+R_{12}C_5)^2=(R_8(C_5+C_6)+R_{12}C_5)^2-4R_{12}C_5R_8C_6$$

$$4R_{12}^2C_5^2R_8^2C_6^2z_{\mathrm{RIAA}}^2+4R_{12}C_5R_8C_6z_{\mathrm{RIAA}}(R_8(C_5+C_6)+R_{12}C_5)=-4R_{12}C_5R_8C_6$$

$$R_{12}C_5R_8C_6z_{\mathrm{RIAA}}^2+z_{\mathrm{RIAA}}(R_8(C_5+C_6)+R_{12}C_5)=-1$$

$$R_8\left(R_{12}C_5C_6z_{\mathrm{RIAA}}^2+z_{\mathrm{RIAA}}(C_5+C_6)\right)=-1-z_{\mathrm{RIAA}}R_{12}C_5$$

$$R_8=\frac{-1-z_{\mathrm{RIAA}}R_{12}C_5}{R_{12}C_5C_6z_{\mathrm{RIAA}}^2+z_{\mathrm{RIAA}}(C_5+C_6)}=\frac{-\frac{1}{z_{\mathrm{RIAA}}}-R_{12}C_5}{R_{12}C_5C_6z_{\mathrm{RIAA}}+C_5+C_6}$$

Using

$$z_{\mathrm{RIAA}}=-\frac{1}{\tau_{\mathrm{zRIAA}}} \quad ,$$

$$R_8=\frac{\tau_{\mathrm{zRIAA}}-R_{12}C_5}{C_5+C_6-\frac{R_{12}C_5C_6}{\tau_{\mathrm{zRIAA}}}}=\frac{\tau_{\mathrm{zRIAA}}}{C_5+C_6}\left(\frac{1-\frac{R_{12}C_5}{\tau_{\mathrm{zRIAA}}}}{1-\frac{R_{12}\frac{C_5C_6}{C_5+C_6}}{\tau_{\mathrm{zRIAA}}}}\right)$$

The first factor of the right-hand side of the equation equals the rough approximation from section 2, the second factor corrects for the effect of R_{12}.

3.2. Calculating the rest given C_5, C_6, C_8, R_{12}, R_8 and the desired pole positions

Not counting the input (and output, if applicable) AC coupling networks, the poles of the phono preamplifier are the zeros of its feedback network: around the values of *s* that result in no feedback, the gain tends to infinity. To get the RIAA correction poles and the pair of complex poles for subsonic filtering at their right places, it therefore suffices to calculate the zeros of the feedback network, see Figure 6.

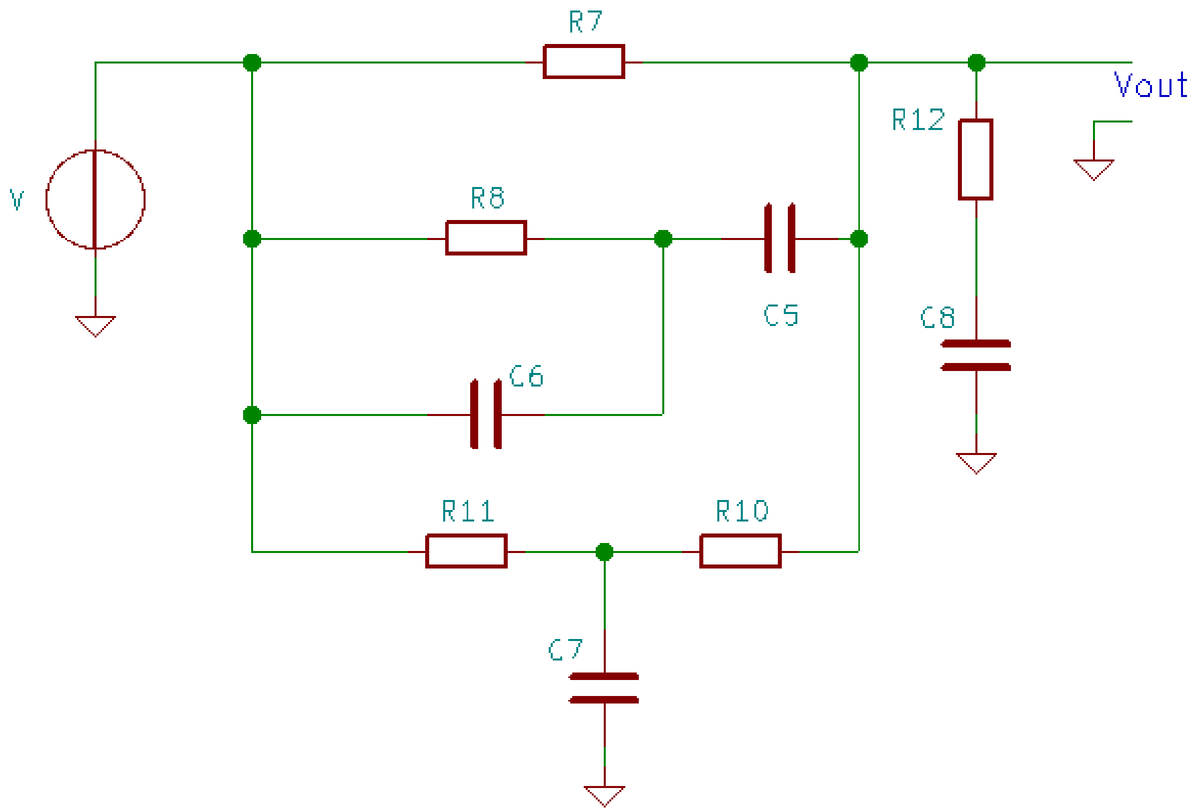

Figure 6: Feedback network

The transfer from feedback network input voltage to feedback network output voltage is zero when $s=-1/(R_{12}C_8)$, because the impedance of the branch R_{12}-C_8 becomes zero at this value of *s*. There is therefore a zero of the feedback network and a pole of the whole amplifier at $s=-1/(R_{12}C_8)$, as was already known.

For other values of *s*, the feedback network transfer can only be 0 when there is no current flowing through the branch R_{12}-C_8. The values of *s* that make the current cancel can be calculated using the simplified network of Figure 7.

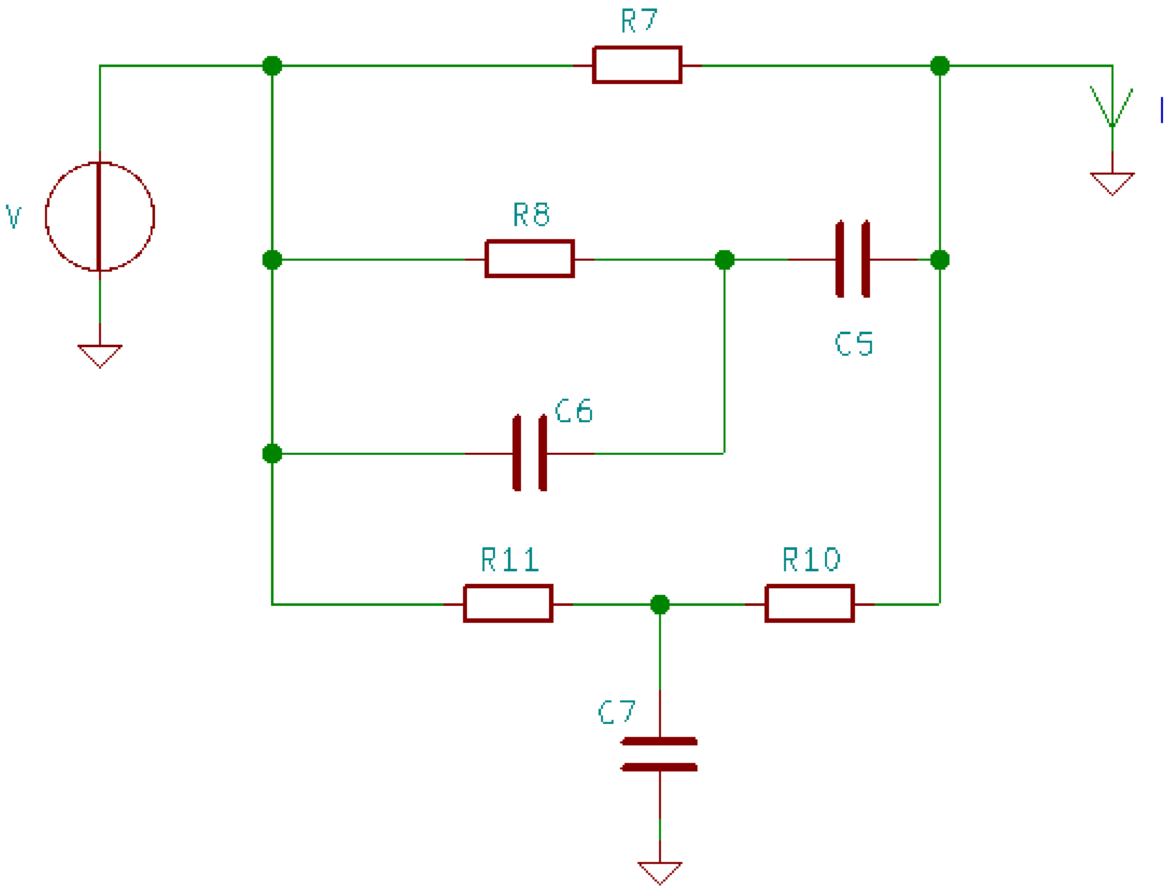

Figure 7: Simplified feedback network for calculating all feedback network zeros (which are phono amplifier poles) but s = -1/(R_{12} C_8)

Clearly, using $L = R_{10}\, R_{11}\, C_7$,

$$\frac{I}{V}=\frac{1}{R_7}+\frac{1}{sL+R_{10}+R_{11}}+\frac{1}{\dfrac{1}{sC_5}+\dfrac{R_8}{sR_8C_6+1}}$$

Hence,

$$\frac{I}{V}=\frac{1}{R_7}+\frac{1}{sL+R_{10}+R_{11}}+\frac{sC_5(sR_8C_6+1)}{sR_8(C_5+C_6)+1}$$

$$\frac{I}{V}=\frac{(sL+R_{10}+R_{11})(sR_8(C_5+C_6)+1)+R_7(sR_8(C_5+C_6)+1)+sC_5(sR_8C_6+1)R_7(sL+R_{10}+R_{11})}{R_7(sL+R_{10}+R_{11})(sR_8(C_5+C_6)+1)}$$

Rewriting the numerator and dividing it by R_{10} + R_{11} + R_7 to make the zeroth-order term 1 results in

$$a_3s^3+a_2s^2+a_1s+1$$

with

$$a_3 = \frac{L R_7 R_8 C_5 C_6}{R_{10} + R_{11} + R_7}$$

$$a_2 = \frac{L R_8 (C_5 + C_6) + R_7 C_5 L + R_7 (R_{10} + R_{11}) R_8 C_5 C_6}{R_{10} + R_{11} + R_7}$$

$$a_1 = \frac{L}{R_{10} + R_{11} + R_7} + R_8 (C_5 + C_6) + \frac{R_7 (R_{10} + R_{11})}{R_7 + R_{10} + R_{11}} C_5$$

The required values of a_3, a_2 and a_1 follow from the desired RIAA and subsonic filter pole positions. That is, as the first RIAA pole is realized with the network R_{12}-C_8 that is excluded from Figure 7, the second RIAA pole and the complex pole pair of the subsonic filter determine a_3, a_2 and a_1. The second RIAA pole contributes a factor $s\tau_{p2} + 1$ to the denominator of the transfer function of the phono preamplifier, while the complex pole pair of the subsonic filter contributes a factor $\frac{1}{\omega_n^2} s^2 + \frac{1}{\omega_n Q} s + 1$, where for a Butterworth filter, $\omega_n = 2\pi f_{sub}$ and $Q = ½\sqrt{2}$ for the second-order case and $Q = 1$ for the third-order case. Multiplying these factors,

$$(s\tau_{p2} + 1)\left(\frac{1}{\omega_n^2} s^2 + \frac{1}{\omega_n Q} s + 1\right) = \frac{\tau_{p2}}{\omega_n^2} s^3 + \left(\frac{1}{\omega_n^2} + \frac{\tau_{p2}}{\omega_n Q}\right) s^2 + \left(\tau_{p2} + \frac{1}{\omega_n Q}\right) s + 1$$

so the required values are

$$a_3 = \frac{\tau_{p2}}{\omega_n^2}$$

$$a_2 = \frac{1}{\omega_n^2} + \frac{\tau_{p2}}{\omega_n Q}$$

$$a_1 = \tau_{p2} + \frac{1}{\omega_n Q}$$

The a_i are therefore known, as are C_5, C_6 and R_8 (although the value we have for R_8 only puts the RIAA zero at approximately the right place).

As the factor $\frac{L}{R_{10} + R_{11} + R_7}$ occurs all over the place, I will call this τ_L to simplify the equations. I will also introduce R_{par}, which denotes the parallel value of R_7 and the series connection of R_{10} and R_{11}. That is,

$$\tau_L \stackrel{\text{def}}{=} \frac{L}{R_{10} + R_{11} + R_7}$$

$$R_{par} \stackrel{\text{def}}{=} \frac{R_7 (R_{10} + R_{11})}{R_7 + R_{10} + R_{11}}$$

The equations for the polynomial coefficients can now be simplified to:

$$\frac{a_3}{R_8 C_5 C_6} = \tau_L R_7$$

$$a_2 = \tau_L R_8 (C_5 + C_6) + \tau_L R_7 C_5 + R_{par} R_8 C_5 C_6$$

$$a_1 - R_8 (C_5 + C_6) = \tau_L + R_{par} C_5 \Leftrightarrow \tau_L = a_1 - R_8 (C_5 + C_6) - R_{par} C_5$$

Substituting the first of these three equations into the second:

$$a_2 = \tau_L R_8 (C_5 + C_6) + \frac{a_3}{R_8 C_6} + R_{par} R_8 C_5 C_6$$

which eliminates R_7. Using the third equation to also eliminate τ_L,

$$a_2 - \frac{a_3}{R_8 C_6} = (a_1 - R_8 (C_5 + C_6) - R_{par} C_5) R_8 (C_5 + C_6) + R_{par} R_8 C_5 C_6$$

$$a_2 - \frac{a_3}{R_8 C_6} = a_1 R_8 (C_5 + C_6) - R_8^2 (C_5 + C_6)^2 - R_{par} C_5 R_8 (C_5 + C_6) + R_{par} R_8 C_5 C_6 =$$
$$a_1 R_8 (C_5 + C_6) - R_8^2 (C_5 + C_6)^2 - R_{par} C_5^2 R_8$$

$$R_{par} = \frac{a_1 R_8 (C_5 + C_6) - R_8^2 (C_5 + C_6)^2 - a_2 + \frac{a_3}{R_8 C_6}}{R_8 C_5^2}$$

The formerly unknown R_{par} is now expressed in known quantities.

As $\tau_L = a_1 - R_8 (C_5 + C_6) - R_{par} C_5$, τ_L is now also solved.

$$R_7 = \frac{a_3}{R_8 C_5 C_6 \tau_L}$$

$$R_{10} + R_{11} = \frac{1}{\frac{1}{R_{par}} - \frac{1}{R_7}}$$

$$L = \tau_L (R_7 + R_{10} + R_{11})$$

$$C_7 = \frac{L}{R_{10} R_{11}}$$

R_{10} and R_{11} are not uniquely found, because only their series value and the value of $L = R_{10} R_{11} C_7$ matter for the operation of the circuit of Figure 7. That is, there is some freedom to choose a standard value for C_7 and adjust the resistors.

Defining $R_{1011} \stackrel{\text{def}}{=} R_{10} + R_{11}$, the values for R_{10} and R_{11} for a given C_7 can be calculated like this:

$$R_{1011} = \frac{1}{\frac{1}{R_{par}} - \frac{1}{R_7}}$$

$$R_{10} + R_{11} = R_{1011}$$

$$R_{11}=\frac{L}{R_{10}C_7}$$

$$R_{10}+\frac{L}{R_{10}C_7}=R_{1011}$$

$$R_{10}^2-R_{1011}R_{10}+\frac{L}{C_7}=0$$

$$R_{10}=\frac{R_{1011}\pm\sqrt{R_{1011}^2-4\frac{L}{C_7}}}{2}$$

$$R_{11}=\frac{R_{1011}\mp\sqrt{R_{1011}^2-4\frac{L}{C_7}}}{2}$$

That is, either the plus solution has to be chosen for R_{10} and the minus solution for R_{11} or the other way around. I'm in favour of using the plus solution for R_{10}, as this makes the approximation used for R_8 slightly more accurate.

In order to get real values out of these equations, the following inequality has to be satisfied:

$$R_{1011}^2-4\frac{L}{C_7}\geq 0$$

or

$$C_7\geq\frac{4L}{R_{1011}^2}$$

A practical approach is to choose the smallest standard value greater than or equal to $4L/R_{1011}^2$ for C_7 and then to calculate R_{10} and R_{11}.

When you attempt to use this method, you will find that R_7, the minimum allowable C_7, R_{10} and R_{11} are all very sensitive to the ratio C_5/C_6. This is logical: in first approximation, the ratio of the RIAA correction second pole to the RIAA correction zero depends only on C_5/C_6. We now use the small dependence on R_7, C_7, R_{10} and R_{11} that was neglected before to correct for a slightly wrong ratio C_5/C_6, but as there hardly is a dependence on R_7, C_7, R_{10} and R_{11}, they have to change a lot to correct a small change in C_5/C_6. An example is shown in Figure 8.

C5	**6.80E-009**	**6.70E-009**	**6.90E-009**
C6	**2.20E-009**	**2.20E-009**	**2.20E-009**
C8	**6.80E-006**	**6.80E-006**	**6.80E-006**
tau_p1	0.00318	0.00318	0.00318
tau_p2	0.000075	0.000075	0.000075
tau_zRIAA	0.000318	0.000318	0.000318
fsub	16	16	16
Q	0.707106781	0.707106781	0.707106781
R12	**467.6470588**	**467.6470588**	**467.6470588**
R8	**35065.7162**	**35464.66443**	**34675.52917**
omega_n	100.5309649	100.5309649	100.5309649
a3	7.4210E-009	7.4210E-009	7.4210E-009
a2	0.000100002	0.000100002	0.000100002
a1	0.014142442	0.014142442	0.014142442
Rpar	344090.5738	-328896.6786	993257.4434
tau_L	0.011487035	0.016030415	0.006973419
R7	**1231513.867**	**885571.1631**	**2021719.715**
R1011	477508.6662	-239826.3702	1952515.139
L	19631.60181	10351.5568	27714.00391
C7,min	3.4439E-007	7.1990E-007	2.9078E-008
C7	**4.70E-007**	**4.70E-007**	**4.70E-007**
R10	**362181.5019**	**Fout:502**	**1921832.99**
R11	**115327.1643**	**Fout:502**	**30682.1488**

Figure 8: Spreadsheet screenshot illustrating the large sensitivity to C_5/C_6. Fout:502 is an error message due to real variables getting complex values. When the value for C_7 in the second column is increased to 1 μF, *R_{10} and R_{11} get real, but go negative.*

Putting the values of the first column into a pole-zero extraction program shows that the poles end up where they should within the numerical precision of the program. The RIAA zero is off, but only by about 0.077 %, which seems to show that at least for this example, the approximation used for calculating R_8 is good enough for most practical purposes.

It should be noted that the approximation for the RIAA zero was made assuming that

$R_{12} \ll \frac{R_7 R_{10}}{R_7 + R_{10}}$. Solutions that actually meet this criterion are therefore to be preferred.

4. Conclusion

A simple way to incorporate a second- or third-order Butterworth high-pass filter in a single-stage active-correction phono preamplifier has been presented. Calculating the component values can either be done with a couple of rough approximations and some tweaking, or with a whole bunch of relatively complicated expressions that can lead to negative or complex values for two resistors when the ratio between two capacitances is not quite what it should be.