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Small-signal modelling of self-oscillating switch-mode amplifiers

Oersted Automation DTU Department for Electric Power Engineering

Kgs. Lyngby, September 11, 2006

In response to assignment ♯3 under the Ph.D./Industrial course 31359 on switch-mode audio power amplifiers.
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Contents

Chapter 1

Introduction

This rapport is a hand-in to the assignment ♯3 under the Ph.D./Industrial course 31359 on switch-mode audio power amplifiers. The assignment targets modelling the small signal gain of the only intended nonlinear component in continuous PWM modulators, namely the comparator. It is derived in [1] that the comparator in any pulse-width modulated system, clocked or self-oscillating, can be modelled as a constant gain. This can be exploited to calculate to STF (Signal Transfer Function) and NTF (Noise Transfer Function) of a PWM modulator, without doing time consuming single-tone bode-plot simulations in the time domain.
It is shown [1] that the constant gain is directly related to the slope of the carrier at zero crossing at the input of the comparator together with the amplitude of the supply voltage. This has led to the mathematical proof that the simplest self-oscillating hysteretic amplifiers has an improved NTF compared to it's triangular-clocked counterpart. These result are recreated in this report together with the analysis of phase-shift self-oscillating amplifiers (COM) and more advanced loop configurations that includes an output filter and additional noise shaping. In the case of a phase-shift modulated amplifier, the equivalent small-signal gain of the comparator can be approximated by analysis of the open-loop TF (Transfer Function) alone [2].
Since a PWM modulator is in fact a sampled system, is can be modelled as a discrete time system [1]. This explains aliasing of noise down in the audio band and the existence of image components around the switching frequency and it's harmonics which can also move down into the audio band. In relating to predicting the STF and NTF there is however no need setup a complicated z-domain model.

Chapter 2

PWM model

The basic structure of a PWM modulator with feedback can be seen in Figure 2.1.

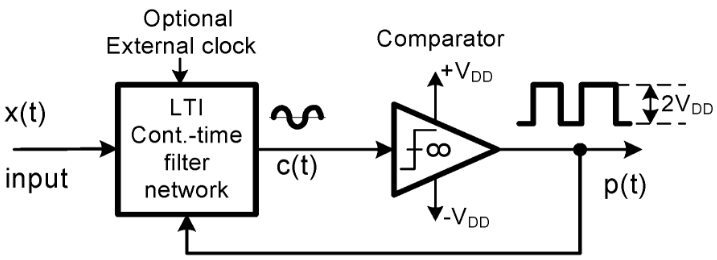

Figure 2.1: *Common PWM model (picture from [1]).*

The system consists of a infinite gain with a saturated output connected to Linear Time Invariant Continuous-time filter network with an optional external clock input. The three interesting signals in the system are the input reference $x(t)$ the carrier $c(t)$ and the PWM output $p(t)$.

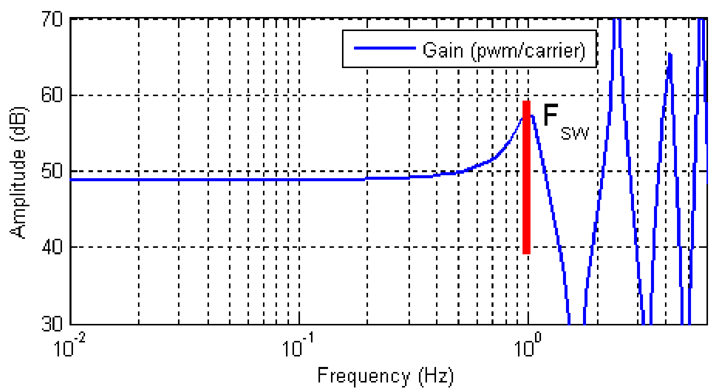

Figure 2.2: *FFT analysis of* 100 *single tome measurement of a COM amplifier with a switching frequency of* $1Hz$*. The ratio between the spectral value of* $c(t)$ *and* $x(t)$ *is constant up to very close to the switching frequency.*

Figure 2.2 shows a series of single tone measurements of the frequency domain gain ratio between $c(t)$ and $x(t)$ for a COM configuration of the LTI network (see more in chapter 4). It appears from the figure that the gain is constant up to very close to the the switching frequency. This result forms the base for the continuous time small-signal model in 2.3 which replaces the switched model in 2.1.

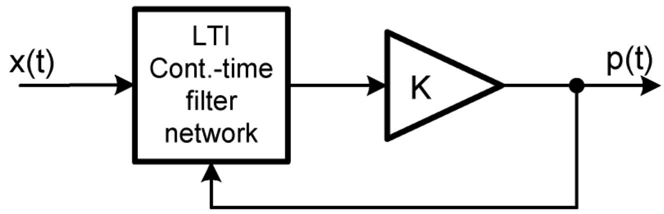

Figure 2.3: *Small-signal model of PWM modulator for s-domain analysis (picture from [1]).*

In the small-signal model the comparator and the optional external clock is replaced by a constant gain block K. This allows the designer to find the s-domain STF and NTF of the system. Especially the NTF is interesting here, since it explains the suppression of distortion components and noise down in the audio band. As it will be shown in the following chapters, this continuous time small-signal model predicts STF and NTF to a very high accuracy under the condition that the correct gain K can be found.

2.1 Discrete time model and noise

The continuous time small signal model has its shortcomings. It does not explain why the system exhibits aliasing. A PWM modulator is in fact a sampled system with sampling rate $2f_{sw}$. For the purpose of examine sampling effects the s-domain model 2.3 should be replaced by the z-domain model in 2.4 according to [1].

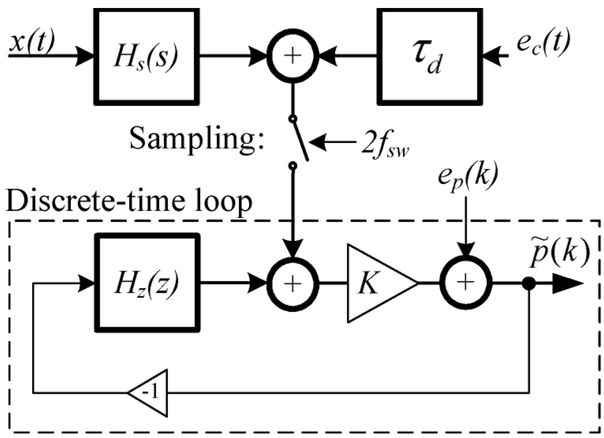

Figure 2.4: *Small-signal model of PWM modulator (picture from [1]).*

The LTI continuous time network is detonated $H(s)$ and $H(z)$ is the equivalent discrete time version. The discrete time model has two new inputs, namely the noise sources $e_c(t)$ (comparator noise) and $e_p(z)$ (power stage errors). Since the sources $x(t)$ and $e_c(t)$ are filtered but then sampled, aliasing of frequency components above the nyquist frequency f_{sw} will occur. $x(t)$ will typically contain quantization errors from a D/A converter as well as random noise. $e_c(t)$ is a noise generated by comparator and the passive components in the filter implementation of $H(s)$. The bandwidth of this noise source is considered infinite [1] but it's limited before the sampling by the equivalent bandwidth of the input stage in the comparator, detonated with the time constant τ_d. Since the bandwidth of the input stage is several orders of a magnitude higher that the nyquist frequency aliasing of the noise down in the audio band will occur. This can decrees the dynamic range of an amplifier or give rise idle tones caused the quantization errors from a D/A converter.

2.2 One model for each purpose

In practical designs with proper pre-filtering of $x(t)$, a low-noise comparator and feedback from the power stage $p(t)$, aliasing of noise down in the audio band is generally not a problem and the signal to noise ratio is large. The dominating problem in PWM modulators is power-stage errors and modulation nonlinearity, which both results in distortion (detonated by the noise source e_p). Concerning this, the NTF explains the suppression of distortion components and is therefore the main performance parameter. For this purpose the s-domain model 2.3 is sufficient and predicts the NTF with a high accuracy if the right gain K is found (See Chapter 3 and Chapter 4).
The NTF also describes the reduction of original open-loop output impedance, which is of a concern in practical amplifiers with an output-LC-filter.

2.3 Finding the gain

There are several ways to find the small-signal gain of the PWM comparator. The single tone measurement (Figure 2.2 and chapter 4) is the most accurate approach, but it requires time consuming FFT analysis and only provide a numerically value of the gain. The single tone measurement does therefore not provide any insight on what affects gain and is therefore best for verification and not as a design tool.
In [1] and Appendix E it's derived that equivalent small-signal gain K of a PWM comparator can be calculated by looking at the carrier waveform $c(t)$ and the PWM output waveform $p(t)$. Figure 2.5 shows the conceptual waveforms for a COM PWM modulator.

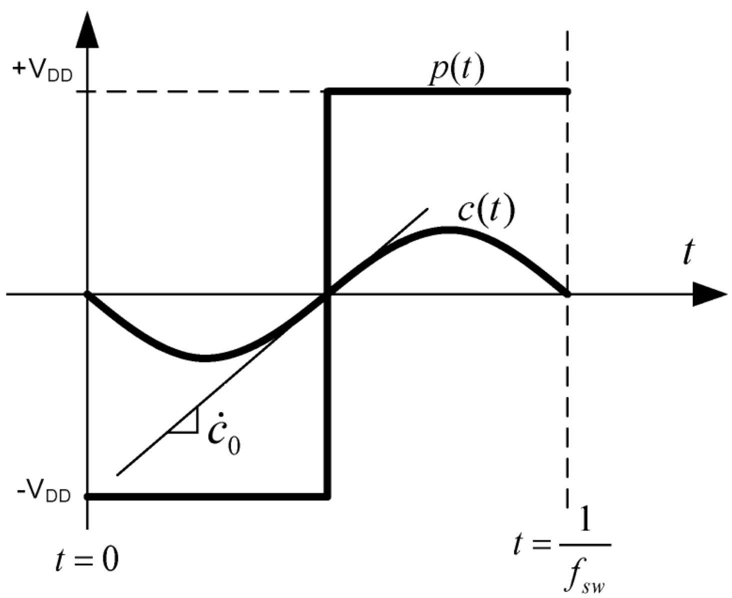

Figure 2.5: *Carrier $c(t)$ and PWM output $p(t)$ of a COM amplifier.*

For a certain switching frequency and supply voltage the gain K is directly proportional to slope of the carrier waveform $\dot{c}_0$ at zero-crossing [1] and is given by:

$$K = 4V_{DD}\frac{f_{sw}}{\dot{c}_0} \tag{2.1}$$

In many cases whether it's a clocked design, a self-oscillating hysteretic modulator or a phase-shift modulator, the three unknowns in (2.1) can often be derived symbolically.
Because the zero-crossing slope approach allows prediction of the gain, it will be used extensively throughout the rest of this report.

Chapter 3

Reference configurations

In Figure 3.1 the generic 1^{st} order PWM model from [1] can bee seen. The model has three configurations, which are listed in Table 3.1, and serves as a reference for what can be achieved with a 1^{st} order clocked or self-oscillating hysteretic design.

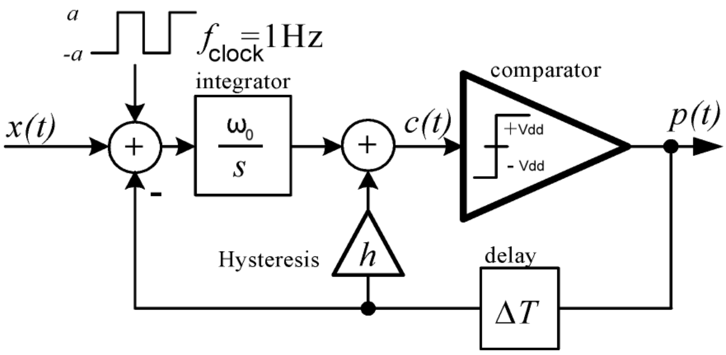

Figure 3.1: *Generic 1^{st} order PWM modulator with five design parameters a, h, ΔT, V_{DD} and ω_0.*

Configuration A is a standart clocked PWM modulator with feedback.
Configuration B is related to A but has a smaller clock amplitude and a stabalizing delay ΔT. This results in a higher small-signal comparator gain K, but makes the configuration ripple unstable with a large input signal $x(t)$.
Configuration C is a non clocked design and represents a self-oscillating hysteretic amplifier.

$V_{DD} = 1$ $\omega_0 = 1$	a	h	ΔT	Comment
A	$2V_{DD}$	0	0	Ripple-stable clocked PWM
B	$1V_{DD}$	0	$\frac{1}{8f_{clock}}$	Zero input marginally ripple-stable clocked PWM
C	0	$\frac{1}{4}$	0	Self-oscillating hysteretic

Table 3.1: *Three special cases of the generic PWM model in Figure 3.1*

The characteristic waveforms of the three configurations can be seen in Figure 3.2.
For case A and B the switching frequency is given by:

$$f_{sw,A|B} = f_{clock} \tag{3.1}$$

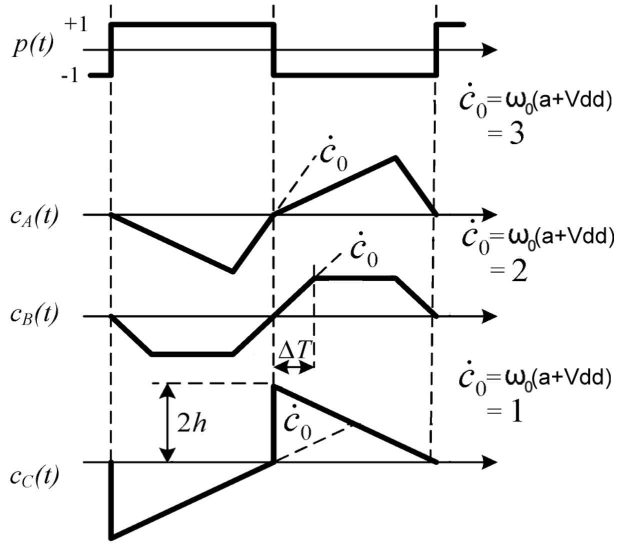

Figure 3.2: *Carrier waveforms of case A, B and C.*

This gives $f_{sw,A|B} = 1Hz$. In the self-oscillating configuration the switching frequency is given by:

$$\begin{aligned} f_{sw,C} &= \frac{1}{4\left(\Delta T + \frac{h}{\omega_0}\right)} \\ &= \frac{\omega_0}{4h} \quad (if\ \Delta T = 0) \end{aligned} \tag{3.2}$$

This equals $f_{sw,C} = 1Hz$ with the parameters in from 3.1.

By looking at the waveforms it's interesting to note that slope of the carrier at zero-crossing in all three cases is given by the same relation:

$$\dot{c_0} = \omega_0 \left(a + V_{DD}\right) \tag{3.3}$$

The slope is not a function of the delay ΔT nor the size hysteretic window h.

In order to maximize the equivalent small-signal comparator gain K from (2.1), the slope $\dot{c_0}$ should be as small as possible. With the parameters from 3.1, A has a slope of 3, B 2 and C 1. By using (2.1) the gain of the three configurations becomes:

$$K_A = 4V_{DD}\frac{f_{clock}}{\omega_0(a+V_{DD})} = \frac{4}{3}\frac{f_{clock}}{\omega_0} \tag{3.4}$$

$$K_B = 4V_{DD}\frac{f_{clock}}{\omega_0(a+V_{DD})} = 2\frac{f_{clock}}{\omega_0} \tag{3.5}$$

$$\begin{aligned} K_C &= 4V_{DD}\frac{1}{4\left(\Delta T + \frac{h}{\omega_0}\right)\omega_0\left(a+V_{DD}\right)} = \frac{1}{\omega_0 \Delta T + h} \\ &= \frac{1}{h} \quad (if\ \Delta T = 0) \end{aligned} \tag{3.6}$$

Case C has a gain of $K_C = 4$ and outperforms B with $K_B = 2$ together with A $K_A = 1.\bar{3}$. The self-oscillating configuration therefore obtains the highest gain at a given switching frequency. In case A and B the gain K can be increased by increasing f_{clock} or by decreasing ω_0 (equation (3.4) and (3.5)). The same property applies to the self-oscillating case C. Decreasing h increases the gain in (3.6) and also the switching frequency (3.2).

3.1 NTF for case A, B and C

The linear model for the three PWM configurations can be seen in Figure 3.3. The integrator block from the switched model (Figure 3.1) has been split into to, in order to reveal which part of the network that forms the open-loop transfer function $G_o(s)$.

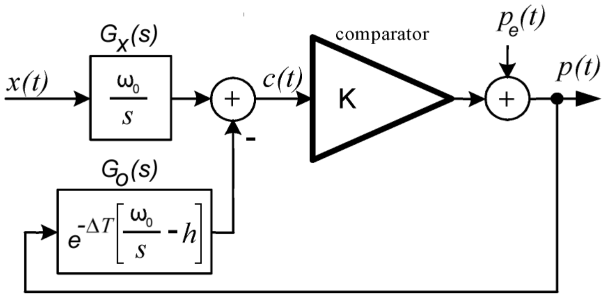

Figure 3.3: *Linear model of the generic* 1^{st} *order switched PWM modulator from 3.1.*

The NTF is given by:

$$\begin{aligned} p(s) &= p_e(s) - KG_o(s)p(s) \Leftrightarrow \\ NTF(s) &= \frac{p(s)}{p_e(s)} = \frac{1}{1 + KG_0(s)} \end{aligned} \tag{3.7}$$

and describes how well powerstage errors, modulation errors and noise, $p_e(s)$, is suppressed.

The STF is given by:

$$\begin{aligned} p(s) &= K\left(x(s)G_x(s) - G(o)p(s)\right) \Leftrightarrow \\ STF(s) &= \frac{p(t)}{x(t)} = \frac{KG_x(s)}{1 + KG_o(s)} \end{aligned} \tag{3.8}$$

and describes the signal gain through the system from the reference $x(s)$ to the output $p(s)$.

We're now able to derive the $NTF(s)$ for the three configurations. In case B a 1^{st} order pade approximation is used to express the delay ΔT:

$$NTF_A(s) = \frac{1}{1 + \frac{\omega_0}{s}\frac{4}{3}\frac{f_{clock}}{\omega_0}} = \frac{3}{4f_{clock}} \frac{s}{\frac{3}{4f_{clock}}s + 1} \tag{3.9}$$

$$NTF_B(s) = \frac{1}{1 + \frac{\omega_0}{s}\frac{-\frac{T}{2}s+1}{\frac{T}{2}s+1}2\frac{f_{clock}}{\omega_0}} = \frac{1}{2f_{clock}\frac{s\left(\frac{T}{2}s+1\right)}{s^2\frac{T}{4f_{clock}}} + s\left(\frac{1}{2f_{clock}} - \frac{T}{2} + 1\right)} \tag{3.10}$$

$$NTF_C(s) = \frac{1}{1 + \left(\frac{\omega_0}{s} - h\right)\frac{1}{h}} = \frac{h}{\omega_0}s = \frac{2}{4f_{sw}} \tag{3.11}$$

The three NTF's is plotted in Figure 3.4.

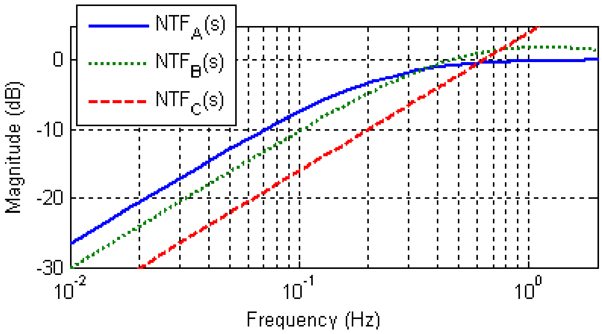

Figure 3.4: *Magnitude plot of the NTF(s) for the three PWM configurations A, B and C.*

It's clearly seen that the self-oscillating configuration C has the best suppression of errors. Although configuration B is an optimized version of A, the NTF proves no reason to use it given it's ripple-instability at large signal inputs.

3.2 NTF bandwidth of the three configuration

The NTF bandwidth of a system con be found by solving:

$$1 = |NTF_{lf}(j2\pi f_{bw})| \tag{3.12}$$

where NTF_{lf} is the low frequency asymptote of the NTF.

For the three configurations the bandwidth is:

$$NTF_{A\_lf}(s) = \frac{3s}{4f_{clock}} \Rightarrow \quad |NTF_{A\_lf}(j2\pi f_{bw})| = 1 = \frac{3 \cdot 2\pi f_{bw}}{4f_{clock}} \Rightarrow f_{A\_bw} = \frac{2}{3\pi} f_{clock} \tag{3.13}$$

$$NTF_{B\_lf}(s) = \frac{s}{2f_{clock}} \Rightarrow \quad |NTF_{B\_lf}(j2\pi f_{bw})| = 1 = \frac{2\pi f_{bw}}{2f_{clock}} \Rightarrow f_{B\_bw} = \frac{1}{\pi} f_{clock} \tag{3.14}$$

$$NTF_{C\_lf}(s) = \frac{s}{4f_{sw}} \Rightarrow \quad |NTF_{C\_lf}(j2\pi f_{bw})| = 1 = \frac{2\pi f_{bw}}{4f_{sw}} \Rightarrow f_{C\_bw} = \frac{2}{\pi} f_{sw} \tag{3.15}$$

Case C has the highest NTF bandwidth of the three configurations. It also follows that to only way to improve the NTF of a 1^{st} order system and increase the NTF bandwidth, is to increase the switching frequency. Changing the proportional gain ω_0 in any of the configurations, leads to no improvement at all. In a normal linear amplifier, an increase in proportional gain would lead to a higher bandwidth and better NTF. This is one of the fundamental differences between switched amplifiers and linear amplifiers.
This concludes that the only way to improve a PWM amplifier, besides increasing the switching frequency, would be to move to a higher order system. This is the subject of Chapter 5.

In Figure (3.5) the three NTF functions are plotted using the script in Appendix A and Appendix B. Each NTF plot has three lines. The 1^{st} line is the calculated NTF(s) based on calculated gain. The 2^{nd} line is the measured NTF based on 100 single tone runs. The 3^{rd} line is the calculated NTF(s) based on a measured gain.

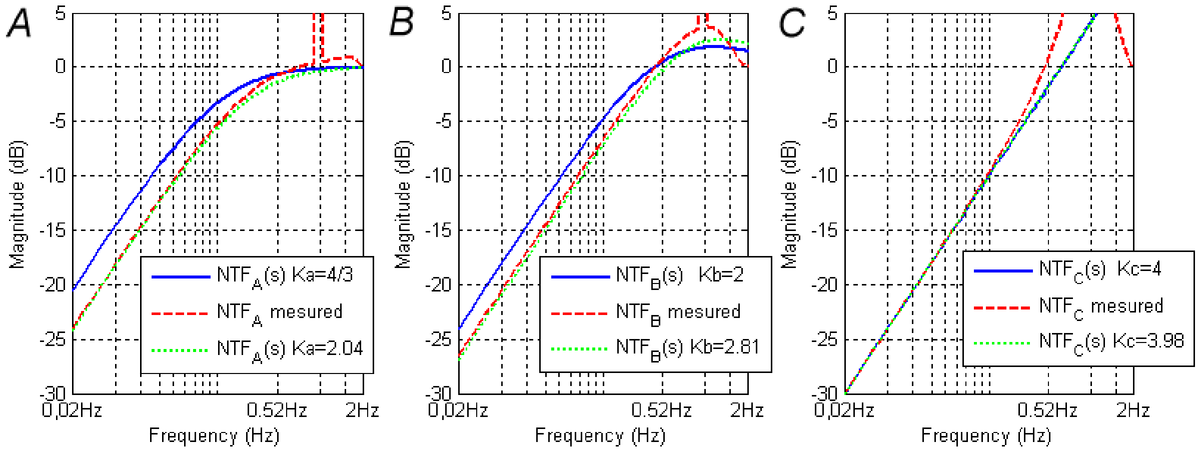

Figure 3.5: *NTF of the three PWM configurations with a switching frequency of* $1Hz$*.* 1^{st} *line: calculated NTF(s) based on calculated gain.* 2^{nd} *line: measured NTF based on* 100 *single tone runs.* 3^{rd} *line: calculated NTF(s) based on single tone measured gain.*

For case A and B the calculated gain does not match the measured gain, which results in approximately $4dB$'s of error in the NTF. With the correct measured gain, the calculated s-domain NTF is consistent with the measured NTF up to the switching frequency. This indicated that the carrier-slope based gain approach [1] is not fully accurate.
Concerning the levels of the three NTFs, case C has $4-6dB$ more error suppression than the clocked configurations. This is not much taking into account that the hysteretic amplifier has a variable switching frequency. On the other hand case A and B rely on a clean clock source, are more complex than the self-running hysteretic configuration and does not modulate directly on power-stage errors. All things considered, the hysteretic modulator is the best configuration.

Chapter 4

The COM modulator

In Section 3.1 we saw that the self-oscillating hysteretic amplifier outperformed the clocked PWM configurations. These types of free running amplifiers is therefore the subject of further analysis in this chapter. Related to the hysteretic modulator is the phase-shift modulator or the COM[1] (Controlled Oscillation Modulator). It works be enforcing a -180^o phase-shift at the switching frequency and have similar performance as the hysteretic modulator.

The COM is derived from the hysteretic modulator by adding two poles in the loop at the desired switching frequency and removing the hysteretic window. This is shown in Figure 4.1.

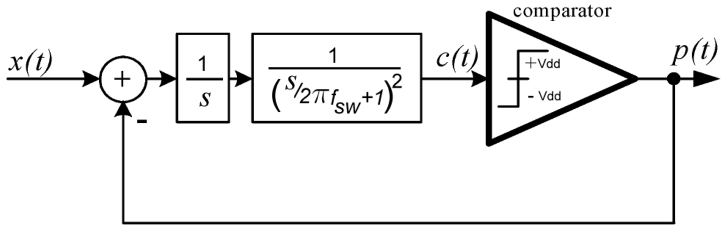

Figure 4.1: *COM modulator.*

Based on the carrier-slop principle [1] the small-signal gain of the comparator can be found, as it was the case in Chapter 3. This will allow a performance comparison with the hysteretic modulator.

4.1 The small-signal comparator gain of a COM

Since the COM loop is effectively a third order system at the switching frequency and above, the carrier $c(t)$ is approximately an attenuated version of the fundamental of the PWM output $p(t)$. This can be exploited to derive the slope of the carrier $\dot{c_0}$ by easy means [2]. The gain is found in the following equation and is derived in Appendix F:

$$K_{com} = \frac{1}{2\,|G_o(j2\pi f_{sw})|} \tag{4.1}$$

[1]COM is Bang and Olufsen Icepowers name for a phase-shift oscillating amplifier that self-oscillates on the PWM signal. If the output filter is included it's covered by the UCD (Universal class D amplifier) patent by Philips.

The open-loop TF for the 1^{st} order COM in Figure 4.1 is:

$$G_{o\_com1^{st}}(s) = \frac{1}{s\left(\frac{s}{2\pi f_{sw}}+1\right)^2} \tag{4.2}$$

By inserting the open-loop TF (4.2) into (4.1) the equivalent small-signal gain of the COM comparator becomes:

$$K_{com1^{st}} = \frac{1}{2\left|G_{o\_com1^{st}}(2\pi f_{sw})\right|} = 2\pi f_{sw} \tag{4.3}$$

The resulting gain of the COM is $\pi/2$ times larger than the hysteretic modulator, so better performance can be expected.

Using $K_{com1^{st}}$ (4.3) and (3.7) the NTF for the COM becomes:

$$NTF_{com1}(s) = \frac{1}{1+2\pi f_{sw}\frac{1}{s\left(\frac{s}{2\pi f_{sw}}+1\right)^2}} = \frac{1}{2\pi f_{sw}}\,\frac{s\left(\frac{s}{2\pi f_{sw}}+1\right)^2}{\frac{s^3}{(2\pi f_{sw})^2}+\frac{s^2}{2(\pi f_{sw})^2}+\frac{s}{2\pi f_{sw}}+1} \tag{4.4}$$

The $NTF_{com1}(s)$ is plotted in figure 4.2 together with a 200 run single tone measurement (Appendix C) on the switched model 4.1.

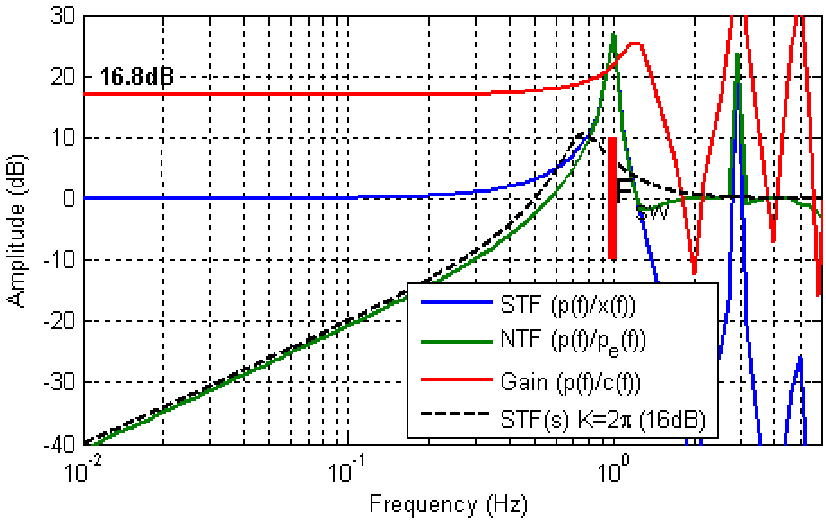

Figure 4.2: *Measured gain, STF and NTF of COM modulator and calculated $NTF_{com1}(s)$.*

The calculated and the measured gain only differ by $1dB$, as a result the calculated and the measured NTF align. By looking at the level of the NTF, the COM appears to outperform the hysteretic modulator in Figure 3.5 by approximately $4dB$.

As it was done in the previous chapter, the NTF bandwidth of the COM can be found by analyzing the $0dB$ crossing of the low frequency asymptote of the NTF:

$$NTF_{com1\_lf}(s) = \frac{s}{2\pi f_{sw}} \Rightarrow \quad \left|NTF_{com1\_lf}(j2\pi f_{bw})\right| = 1 = \frac{2\pi f_{bw}}{2\pi f_{sw}} \Rightarrow f_{com1\_bw} = f_{sw} \tag{4.5}$$

(4.6)

The bandwidth of the COM equals the switching frequency. This is slightly higher that the hysteretic case.
From an overall point of view the COM seams to outperform the hysteretic modulator by a small amount. But what we don't know anything about is the distortion mechanisms of the two modulators. There's no benefit from having a good NTF to suppress distortion components, if the modulation is very nonlinear. This problem is however best analyzed by simulation.

Chapter 5

High order phase shift modulator

In many practical applications a LC-output filter is placed after the output stage of an amplifier to suppress switching residuals. Such a filter has a second order transfer function which is given by:

$$H_{LC}(s) = \frac{1}{s^2LC + s\frac{L}{R} + 1} \tag{5.1}$$

with a quality factor and cutoff frequency of:

$$Q = R\sqrt{\frac{C}{L}} \tag{5.2}$$

$$f_0 = \frac{1}{2*\pi\sqrt{LC}} \tag{5.3}$$

Typically the cutoff frequency lies in between $30kHz$ and $100kHz$ with a quality factor of $1-3$ (with a load of $4-8\Omega$). If a low cutoff frequency is chosen, high residual attenuation is obtained, but it also opens the possibility to include the filter in the modulator loop and use it as a noise shaper, i.e. improve the NTF. This will be the main topic for analysis in this chapter.

5.1 Output filter as a noise shaper

A switching frequency of $400kHz$ is chosen. The output filter is going to be designed around $C = 1\mu F$, $L = 22\mu H$ and a load between 4Ω and $1k\Omega$ (no load condition). This equals a cutoff frequency of $34kHz$ and a quality factor between 0.85 and 213.
The Q is very high and there are three ways to deal with it inside the loop to avoid instability:

- Add a zobel-network to the output filter and lower the Q: Dissipative and costly solution.
- Detect when there's no load attached and shut down the amplifier: Complex solution since it requires load detection.
- Use saturable integrators: Normal opamps does saturate at the supply voltage but diodes can also be used although they are nonlinear.

The last option is optimal and will be chosen.

The proposed model for inclusion of the output LC-filter in the loop can be seen in Figure 5.1. The integrator and the output-filter effectively works as a 3^{rd} order system above the LC cutoff frequency. To compensate for this a double lead-compensator has been placed in the feedback

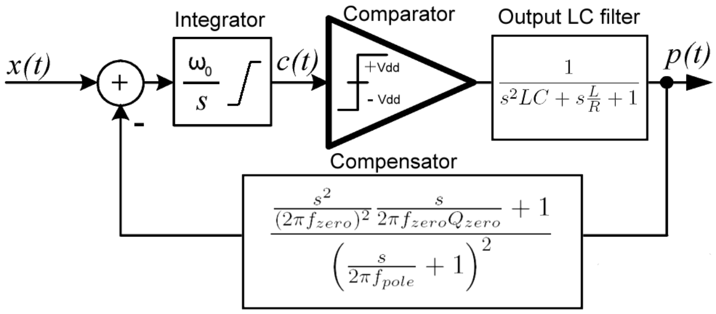

Figure 5.1: *High order phase-shift modulator.*

path. As a special feature the lead-compensator has complex zeros, which allows a steeper phase slope[1].

The open-loop transfer function for the system is:

$$G_{o\_hi}(s) = \frac{\omega_0}{s} \frac{1}{s^2LC + s\frac{L}{R} + 1} \frac{\frac{s^2}{(2\pi f_{zero})^2} \frac{s}{2\pi f_{zero} Q_{zero}} + 1}{\left(\frac{s}{2\pi f_{pole}} + 1\right)^2} \tag{5.4}$$

5.2 Fitting the parameters

Finding the right parameters for the model in Figure 5.1 is best done by trail and error. The Matlab script "comhi.m" in appendix D is used for this purpose. The chosen parameters are:

$$\omega_0 = 20k \quad f_{zero} = 100kHz \quad f_{pole} = 450kHz \quad Q_{zero} = 2 \tag{5.5}$$

The resulting open-loop bode-pole with $R = 1k$ can be seen in Figure 5.2 together with a time domain simulation.
The amplitude characteristic of the bode-plot cleanly shows that open-loop TF has a 3^{rd}order slope between $34kHz$ and $100kHz$, as it was intended.
Because of the high Q of the LC-filter, the phase characteristic crosses -180^o at both $34kHz$ and at $400kHz$. According to nyquist stability criterium, a -180^o phase shift does not result in instability if the gain differ from 1. In [2] and appendix F it's derived that the equivalent gain of the comparator adjusts scutch that $|K_{comp}G_{o\_hi}(j\omega_{f_{sw}})|$ equals $-6dB$. Since the system chosen to oscillate at the -180^o crossing with the highest frequency, the gain of the whole loop including the comparator is $-6dB$ at $400kHz$ and much more at $34kHz$. Therefore the system does not become unstable at $34kHz$.

The time-domain simulation in Figure 5.2b verifies that the system is stable, even when the output saturates in the supply rails $(+/-1V)$. However it was necessary to saturate the integrator and therefore also the carrier amplitude[2] at $1mV$.

[1] There are various ways to implement zomplex zeros. A passive (2 resistor, 1 capacitor) LC-filter feed-forward solution has been reported in [3].

[2] A typical amplifier implementation operates with supply voltages in excess of $40V$ and with a larger ω_0. The carrier amplitude in such a system will often be in the order of $100mV_{p2p}$ to $2V_{p2p}$. With such carrier levels it becomes possible to build an analog saturable integrator.

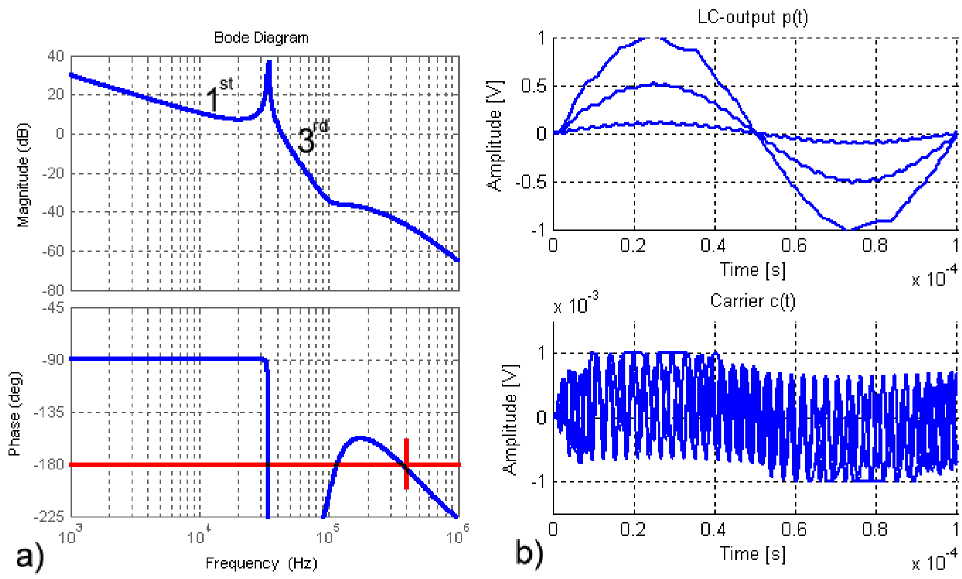

Figure 5.2: *a) Open-loop bode-plot of high order phase-shift modulator. b) Time-domain simulation with various amplitude sinusoidals at* $10kHz$.

At 4Ω the switching frequency nor the transfer characteristic differ much from what is seen in the figure.

5.3 NTF of the high order phase-shift modulator

To verify the performance of the high order phase-shift modulator, we have to evaluate its NTF.

Using (4.1) the gain of the comparator becomes:

$$K_{com1^{st}} = \frac{1}{2\left|G_{o\_hi}(2\pi f_{sw})\right|} = \frac{1}{2 \cdot 10^{\frac{-47.4dB}{20dB}}} = 117 \tag{5.6}$$

By inserting the gain and the open-loop TF into (3.7), the NTF can be found. Because of the rather large expression, this has been done in Matlab (see Appendix D) and will not be shown here. The bode-plot of the NTF is shown in Figure 5.3.

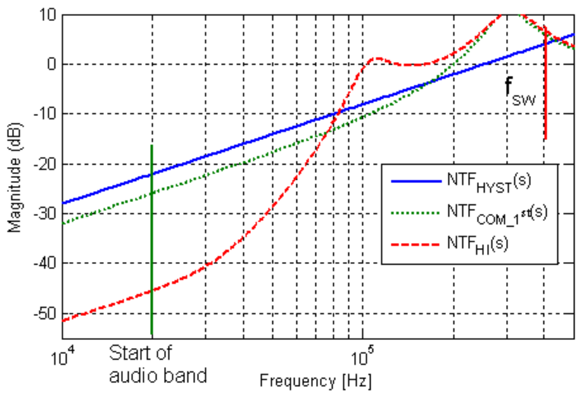

Figure 5.3: *NTF of the high order phase-shift modulator compared with the simple COM and the hysteretic amplifier.*

The high order phase-shift modulator has a $19dB$ better NTF down in the audio band than the simple COM and $23dB$ better that the hysteretic amplifier. This is achieved only by exploiting the noise-shaping property of the output-LC filter. Further improvement can be achieved by adding active poles in the loop. A stable high order COM configuration with a NTF suppression of $101dB$ at $1kHz$ has been reported in [3], by exploiting the LC-filter and adding to active orders two the loop.

Chapter 6

Conclusion

The NTF (Noise Transfer Function) of an amplifier it's the main performance parameter, since it expresses it's ability to suppress errors such as noise and distortion. The NTF of two clocked PWM configurations has been compared with their hysteretic self-oscillating counterpart. By modelling the PWM comparator as a constant gain, the NTF has been computed and verified by simulations. The clocked designs have a constant switching frequency, but they are outperformed by the hysteretic modulator, in terms of NTF, by $4 - 6dB$.

Related to the hysteretic modulator is controlled phase-shift modulators. By using the comparator linearizing method, it's shown that self-oscillating phase-shift modulators has a better NTF than hysteretic modulators in the order of $4dB$.

By adding an output LC-filter to a phase-shift modulator and including it in the loop, the filter can be used to suppress PWM switching residuals as well as performing additional noise shaping. The last option has been exploited and the NTF has been improved by $19dB$ compared to the simple phase-shift modulator.

Bibliography

[1] Lars Risbo. Discrete-time modeling of continuous-time pulse width modulator loops. *AES 27^{th} International Conference*, September 2005. Copenhagen, Denmark.

[2] Bruno Putzeys. Simple self-oscillating class d amplifier with full output filter control. *Audio Engineering Society*, 118th Convention, May 2005. Barcelona, Spain.

[3] Kaspar Sinding Meyer. Minimizing distortion in self-oscillating switching amplifiers. *Oersted DTU Department for Electric Power Engineering*, July 2006. Kgs. Lyngby, Denmark.

Appendix A

Matlab code for NTF of A B and C

This script ('abc.m') computes the NTF of the three linearized PWM configurations A, B and C.

```
clear; load abcA; load abcB; load abcC;
fc=1; fsw=fc; T=1/8;
NTFa=3/(fc*4)*tf([1 0],[3/(fc*4) 1]);
NTFb=2/(4*fc)*tf(conv([1 0],[T/2 1]),[T/(4*fc) (1/(2*fc)-T/2) 1]);
NTFc=1/(4*fsw)*tf([1 0],[1]);
[ma pa wa]=bode2(NTFa);
[mb pb wb]=bode2(NTFb);
[mc pc wc]=bode2(NTFc);
clf; figure(1);
subplot(2,1,1);
semilogx(wa./(2*pi),20*log10(ma),wb./(2*pi),20*log10(mb),':',wc./(2*pi),20*log10(mc),'--','linewidth',2); grid on;
ylabel('Magnitude (dB)'); axis([1e-2 2 -30 5]);
subplot(2,1,2);
semilogx(wa./(2*pi),pa,wb./(2*pi),pb,':',wc./(2*pi),pc,'--','linewidth',2); grid on;
set(gca,'YTick',[-180 -125 -90 -45 0 45 90 125])
xlabel('Frequency (Hz)'); ylabel('Phase (deg)');
axis([1e-2 2 0 95]);
legend('NTF_A(s)','NTF_B(s)','NTF_C(s)')

Ka=2.04; Kb=2.81; Kc=3.98;
NTFa2=1/Ka*tf([1 0],[1/Ka 1]); [ma2 pa2 wa2]=bode2(NTFa2);
NTFb2=1/Kb*tf(conv([1 0],[T/2 1]),[T/(2*Kb) (1/(Kb)-T/2) 1]); [mb2 pb2 wb2]=bode2(NTFb2);
NTFc2=1/Kc*tf([1 0],[(1-0.25*Kc)/Kc 1]); [mc2 pc2 wc2]=bode2(NTFc2);

figure(2);
subplot(1,3,1);
semilogx(wa./(2*pi),20*log10(ma),mfA,20*log10(mNTFa),'r--',wa2./(2*pi),20*log10(ma2),'g:','linewidth',2); grid on;
xlabel('Frequency (Hz)'); ylabel('Magnitude (dB)'); axis([2*1e-2 2 -30 5]); legend('NTF_A(s) Ka=4/3','NTF_A mesured','NTF_A(s) Ka=2.04')
subplot(1,3,2);
semilogx(wb./(2*pi),20*log10(mb),mfB,20*log10(mNTFb),'r--',wb2./(2*pi),20*log10(mb2),'g:','linewidth',2); grid on;
xlabel('Frequency (Hz)'); ylabel('Magnitude (dB)'); axis([2*1e-2 2 -30 5]); legend('NTF_B(s) Kb=2','NTF_B mesured','NTF_B(s) Kb=2.81')
subplot(1,3,3);
semilogx(wc./(2*pi),20*log10(mc),mfC,20*log10(mNTFc),'r--',wc2./(2*pi),20*log10(mc2),'g:','linewidth',2); grid on;
```

```
xlabel('Frequency (Hz)'); ylabel('Magnitude (dB)'); axis([2*1e-2 2 -30 5]);
    legend('NTF_C(s) Kc=4','NTF_C mesured','NTF_C(s) Kc=3.98')
```

Appendix B

Matlab code for mesured NTF of A B and C

This script ('ana_abc.m') uses the simulink model "abcmodel.mdl" to measure the NTF of case A B and C by single tone measurement.

```
%init
bw=6; Ain=0.01;
sample_freq=6*bw;
[AAnum,AAden]=butter(6,bw*2*pi,'s');
num=[1]; den=conv([1],conv([1 2*pi],[1 2*pi]));
simtime=2^8;
M=0; %Modulation index

%Init transfer function
simu=1; EvalIt=1;
f_in=logspace(log10(0.005),log10(10),3);


%Correct f_in to match the FFT blocks
freq=0.1; noise=0;
opt=simset('SignalLogging','off');
sim('abcmodel',simtime,opt);
[U,F]=spec(pwm,1,sample_freq);
f_inc=f_in;
for i=1:length(f_in)
    f_inc([1,2],i)=[F(find(F>f_in(i),1));find(F>f_in(i),1)];
end
f_inc=unique(f_inc','rows')';
%Find the switching frequency and plot it
[val,where]=max(U);
f_sw=[F(where);where];
line([f_sw(1) f_sw(1)],[-10 10],'LineWidth',4,'Color','r');
text(f_sw(1)*1.02,0,'F_{sw}','FontSize',14);


%Find the transfer function
if simu==1
    for noise=0:1
        clear pwmV carrierV errorV inputV
        for i=1:length(f_inc(1,:))
            fprintf('\nProgress %f%%',50*noise+50*i/length(f_inc(1,:)));
            freq=f_inc(1,i);
            sim('abcmodel',simtime,opt);
            [fpwm,F]=spec(pwm,1,sample_freq);
```

```
            pwmV(i)=fpwm(f_inc(2,i));
            [fcarrier,F]=spec(carrier,0,sample_freq);
            carrierV(i)=fcarrier(f_inc(2,i));
            [finput,F]=spec(input,0,sample_freq);
            inputV(i)=finput(f_inc(2,i));
            line([f_inc(1,i) f_inc(1,i)],[-10 10],'LineWidth',4,'Color','r');
            text(f_inc(1,i)*1.02,0,'f_{in}','FontSize',14);
            axis([min(F) max(F) -120 20]);
        end
        switch noise
            case 0
                gain=pwmV./carrierV;
                STF=pwmV./Ain;
            case 1
                NTF=pwmV./Ain;
        end
    end
    save run1_abc gain STF NTF f_inc
end


if EvalIt==1
    figure(3)
    semilogx(f_inc(1,:),20*log10(STF),f_inc(1,:),20*log10(NTF),f_inc(1,:),20*
        log10(gain),'LineWidth',2)
    grid on;
    ylabel('Amplitude (dB)');
    xlabel('Frequency (Hz)');
    axis([1e-2 bw -40 50])
    legend('STF (output/input)','NTF (output/noise)','Gain (pwm/carrier)');
    line([f_sw(1) f_sw(1)],[-10 10],'LineWidth',4,'Color','r');
    text(f_sw(1)*1.02,0,'F_{sw}','FontSize',14);
end

if EvalIt==3
    figure(3)
    semilogx(f_inc(1,:)./1.2,20*log10(gain),'LineWidth',2)
    grid on;
    ylabel('Amplitude (dB)');
    xlabel('Frequency (Hz)');
    axis([1e-2 bw 30 70])
    legend('Gain (pwm/carrier)');
    line([f_sw(1) f_sw(1)],[20*log10(gain(ceil(end/2)))-10 20*log10(gain(ceil(
        end/2)))+10],'LineWidth',4,'Color','r');
    text(f_sw(1)*1.02,20*log10(gain(ceil(end/2))),'F_{sw}','FontSize',14);
end

if EvalIt==2
    figure(3)
    semilogx(f_inc(1,:),20*log10(STF),f_inc(1,:),20*log10(NTF),f_inc(1,:),20*
        log10(gain),f,20*log10(STFm),'--',f,20*log10(NTFm),'--','LineWidth',2)
    grid on;
    ylabel('Amplitude (dB)');
    xlabel('Frequency (Hz)');
    axis([1e-2 bw -40 50])
    legend('STF (output/input)','NTF (output/noise)','Gain (pwm/carrier)','STFm
        (model)','NTF (model');
    line([f_sw(1) f_sw(1)],[-10 10],'LineWidth',4,'Color','r');
    text(f_sw(1)*1.02,0,'F_{sw}','FontSize',14);
end
```

Appendix C

Matlab code for mesured NTF of the simple COM

This script ('ana_com_1.m') uses the simulink model "com_1.mdl" to measure the NTF of the simple com by single tone measurement.

```
%init
bw=6; Ain=0.01;
sample_freq=6*bw;
[AAnum,AAden]=butter(6,bw*2*pi,'s');
num=[1]; den=conv([1],conv([1 2*pi],[1 2*pi]));
simtime=2^8;
M=0; %Modulation index

%Init transfer function
simu=1; EvalIt=1;
f_in=logspace(log10(0.005),log10(10),3);


%Correct f_in to match the FFT blocks
freq=0.1; noise=0;
opt=simset('SignalLogging','off');
sim('abcmodel',simtime,opt);
[U,F]=spec(pwm,1,sample_freq);
f_inc=f_in;
for i=1:length(f_in)
    f_inc([1,2],i)=[F(find(F>f_in(i),1));find(F>f_in(i),1)];
end
f_inc=unique(f_inc','rows')';
%Find the switching frequency and plot it
[val,where]=max(U);
f_sw=[F(where);where];
line([f_sw(1) f_sw(1)],[-10 10],'LineWidth',4,'Color','r');
text(f_sw(1)*1.02,0,'F_{sw}','FontSize',14);


%Find the transfer function
if simu==1
    for noise=0:1
        clear pwmV carrierV errorV inputV
        for i=1:length(f_inc(1,:))
            fprintf('\nProgress %f%%',50*noise+50*i/length(f_inc(1,:)));
            freq=f_inc(1,i);
            sim('abcmodel',simtime,opt);
            [fpwm,F]=spec(pwm,1,sample_freq);
```

```
            pwmV(i)=fpwm(f_inc(2,i));
            [fcarrier,F]=spec(carrier,0,sample_freq);
            carrierV(i)=fcarrier(f_inc(2,i));
            [finput,F]=spec(input,0,sample_freq);
            inputV(i)=finput(f_inc(2,i));
            line([f_inc(1,i) f_inc(1,i)],[-10 10],'LineWidth',4,'Color','r');
            text(f_inc(1,i)*1.02,0,'f_{in}','FontSize',14);
            axis([min(F) max(F) -120 20]);
        end
        switch noise
            case 0
                gain=pwmV./carrierV;
                STF=pwmV./Ain;
            case 1
                NTF=pwmV./Ain;
        end
    end
    save run1_abc gain STF NTF f_inc
end


if EvalIt==1
    figure(3)
    semilogx(f_inc(1,:),20*log10(STF),f_inc(1,:),20*log10(NTF),f_inc(1,:),20*
        log10(gain),'LineWidth',2)
    grid on;
    ylabel('Amplitude (dB)');
    xlabel('Frequency (Hz)');
    axis([1e-2 bw -40 50])
    legend('STF (output/input)','NTF (output/noise)','Gain (pwm/carrier)');
    line([f_sw(1) f_sw(1)],[-10 10],'LineWidth',4,'Color','r');
    text(f_sw(1)*1.02,0,'F_{sw}','FontSize',14);
end

if EvalIt==3
    figure(3)
    semilogx(f_inc(1,:)./1.2,20*log10(gain),'LineWidth',2)
    grid on;
    ylabel('Amplitude (dB)');
    xlabel('Frequency (Hz)');
    axis([1e-2 bw 30 70])
    legend('Gain (pwm/carrier)');
    line([f_sw(1) f_sw(1)],[20*log10(gain(ceil(end/2)))-10 20*log10(gain(ceil(
        end/2)))+10],'LineWidth',4,'Color','r');
    text(f_sw(1)*1.02,20*log10(gain(ceil(end/2))),'F_{sw}','FontSize',14);
end

if EvalIt==2
    figure(3)
    semilogx(f_inc(1,:),20*log10(STF),f_inc(1,:),20*log10(NTF),f_inc(1,:),20*
        log10(gain),f,20*log10(STFm),'--',f,20*log10(NTFm),'--','LineWidth',2)
    grid on;
    ylabel('Amplitude (dB)');
    xlabel('Frequency (Hz)');
    axis([1e-2 bw -40 50])
    legend('STF (output/input)','NTF (output/noise)','Gain (pwm/carrier)','STFm
        (model)','NTF (model');
    line([f_sw(1) f_sw(1)],[-10 10],'LineWidth',4,'Color','r');
    text(f_sw(1)*1.02,0,'F_{sw}','FontSize',14);
end
```

Appendix D

Matlab code for higher order phase-shift modulator

This scripts ("comhi.m") computes the open-loop bode-plot for the high order phase-shift modulator. The model "hicomp.mdl" is also needed and serves to make a time-plot of the switched modulator, to verify that the system works in the time domain.

```
clear; clf;
%High order com
w=2*pi*logspace(log10(1e3),log10(1e6),500);
f_sw=400*1e3;

%The LC filter
R=1e3; C=1e-6; L=22*1e-6; %F0=34kHz
R=4;
num_LC=1; den_LC=[L*C L/R +1]
GLC=tf(num_LC,den_LC);

%The dual lead compensator
f_zero=100*1e3; Q_zero=2; f_pole=425*1e3; sat=1*1e-3;
num_comp=[1/(2*pi*f_zero)^2 1/(2*pi*f_zero*Q_zero) 1];
den_comp=(conv([1/(2*pi*f_pole) 1],[1/(2*pi*f_pole) 1]));
Gcomp=tf(num_comp,den_comp);

%The integrator
Gi=20*1e4*tf(1,[1 0]);

figure(1); subplot(2,2,[1 3]);
bode(GLC*Gcomp*Gi,w);
line([f_sw f_sw],[-200 -160],'LineWidth',1,'Color','r');
line([min(w/(2*pi)) max(w/(2*pi))],[-180 -180],'LineWidth',1,'Color','r');
axis([1e3 1e6 -225 -45])

%init simulink
bw=6*f_sw;  sample_freq=6*bw;
[AAnum,AAden]=butter(6,bw*2*pi,'s');
noise=0; freq=1e4; simtime=1/freq; M=0;
AinV=[0.1 0.5 1]; i=1:2;
for i=1:length(AinV)
    Ain=AinV(i);
    sim('hicomp');
    figure(1); subplot(2,2,2); hold on;
    plot((1:length(pwm))./sample_freq,pwm,'LineWidth',2); axis([0 length(pwm)/
        sample_freq  -1 1])
```

```
    hold off; grid on; title('LC-output p(t)'); xlabel('Time [s]'); ylabel('Amplitude [V]')

    subplot(2,2,4); hold on;
    plot((1:length(carrier))./sample_freq,carrier,'LineWidth',2); axis([0 length(carrier)/sample_freq  -sat*1.5 sat*1.5])
    hold off; grid on;title('Carrier c(t)'); xlabel('Time [s]'); ylabel('Amplitude [V]')
end

figure(4)
%Hi COM NTF
NTFhi=feedback(1,117.2*GLC*Gcomp*Gi);
[mhi phi whi]=bode2(NTFhi);

%1st COM
f_sw=400*1e3;
num=conv([1 0],conv([1/(2*pi*f_sw) 1],[1/(2*pi*f_sw) 1]));
den=[1/(2*pi*f_sw)^3 1/(2*pi^2*f_sw^2) 1/(2*pi*f_sw) 1]
NTFcom=1/(2*pi*f_sw)*tf(num,den);
[mcom pcom wcom]=bode2(NTFcom);

%Hyst
NTFc=1/(4*f_sw)*tf([1 0],[1]);
[mc pc wc]=bode2(NTFc);

clf; figure(3);
semilogx(wc./(2*pi),20*log10(mc),wcom./(2*pi),20*log10(mcom),':',whi./(2*pi),20*log10(mhi),'--','linewidth',2); grid on;
ylabel('Magnitude (dB)'); %axis([1e-2 2 -30]);
legend('NTF_{HYST}(s)','NTF_{COM\_1^{st}}(s)','NTF_{COM\_{3^{rd}}}(s)')

STFhi=feedback(117.2*GLC*Gi,Gcomp);
```

Appendix E

Modelling of the ideal comparator

Note: This derivation of the small-signal gain for any PWM modulator is directly copied from the paper by Lars Risbo [1].

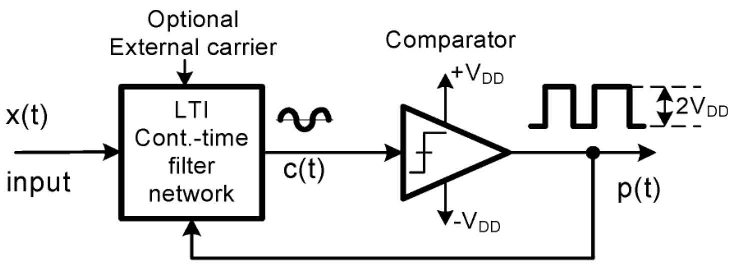

Figure E.1: *Generic PWM loop*

A small-signal model reflects a systems response deviation from a steady state condition (i.e., DC bias point) when being perturbed by a small amplitude input. However, for a comparator operating as a pulse width modulator, we will model perturbations to the periodic steady state condition, where the system receives a zero input and the comparator hence produces a 50% duty cycle signal. Conceptually, this can be done as shown in Figure E.2 by operating two identical systems; one reference system being un-perturbed and the other being perturbed by a small amplitude input. The small-signal response is then the difference between the perturbed system and the reference system. The comparator and power stage will initially be modeled as a high gain G followed by saturation to the supply rails as shown in Figure E.1. The ideal comparator is approximated by letting G go to infinity. Both comparators receive a periodic carrier signal $c(t)$ with frequency f_{sw} having two uniformly spaced zero crossings per period. The perturbed system receives a superimposed small-amplitude perturbation signal $y(t)$.
At each zero-crossing of $c(t)$, the comparator will for a short time Δt be in its non-saturated mode acting as a linear gain G:

$$\Delta t \approx \frac{2V_{DD}}{|\dot{c_0}|G} \tag{E.1}$$

, where $\dot{c_0}$ is the derivative (slope) of the carrier signal at the zero-crossing. Outside these time intervals of duration Ät the comparator is saturated and does not respond to the perturbation signal, i.e., here the gain is zero. The small-signal PWM output $\tilde{p(t)}$ is effectively the product between $y(t)$ and a pulse train $g(t)$ of frequency $2f_{sw}$, where each pulse has duration Δt and amplitude G as shown in Figure E.3.

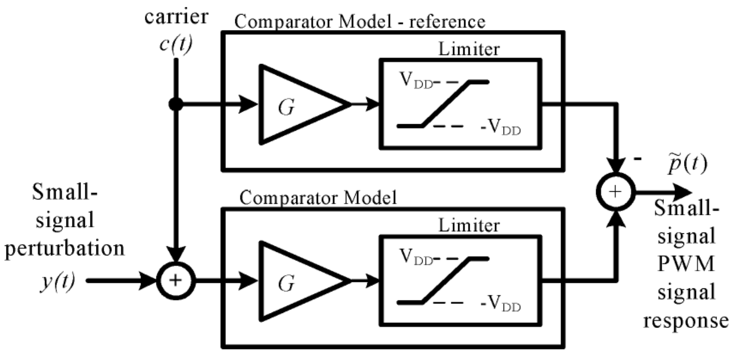

Figure E.2: *Conceptual small-signal model*

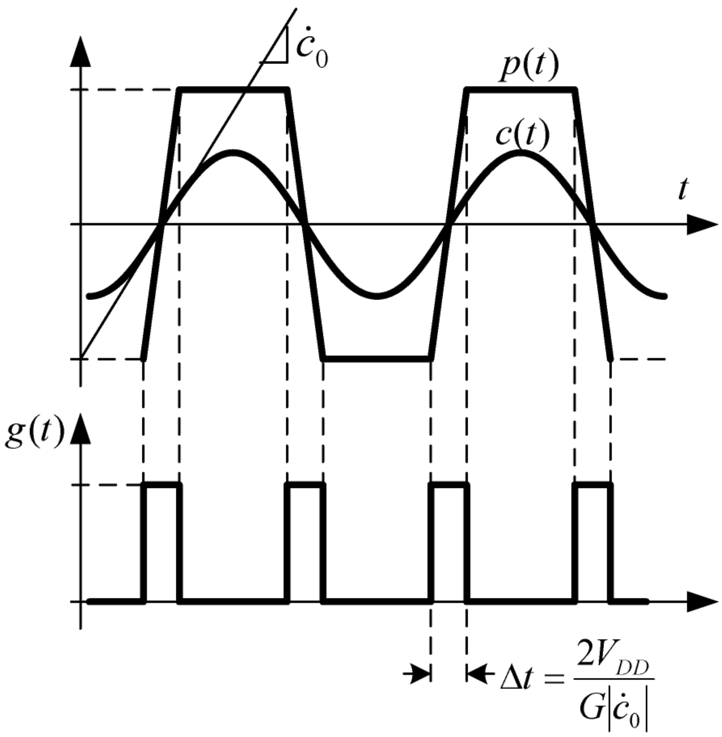

Figure E.3: *Waveforms for the finite-gain comparator*

Each pulse of $g(t)$ has an area of:

$$A = G\Delta t \approx \frac{2VDD}{|\dot{C}_0|} \tag{E.2}$$

I.e., the area is independent of the gain G and the duration of each pulse goes to zero for infinite G. Consequently, for infinite gain G we can approximate the pulse train $g(t)$ as a periodic repetition of delta impulses. Consequently, multiplication with $g(t)$ corresponds to a sampling operation with frequency $2f_{sw}$ followed by a multiplication by an effective comparator gain K which is the mean value of $g(t)$:

$$K = mean[g(t)] = 2f_{sw}A = 4V_{DD}\frac{f_{sw}}{|\dot{c}_0|} \tag{E.3}$$

Appendix F

Linearized DC gain of the comparator and the power stage for a phase-shift modulator

Note: This derivation of the small-signal gain of the PWM comparator in phase-oscillating modulation mode, is to a large extent copied directly from the UCD paper [2].

In the following analysis, amplitude is always meant to be peak amplitude.
In class D amplifiers employing a triangle wave or saw tooth oscillator to compare the control signal to, DC gain of the combined comparator and power stage is the amplitude of the square wave before the output filter (equals the supply voltage) divided by the amplitude of the triangle wave:

$$A_{DC} = \frac{V_{sq}}{V_{tri}} \tag{F.1}$$

In the present circuit, the reference waveform is the signal found at the comparator inputs as a result of the self-oscillation. Of the square wave produced by the power stage, little more than an attenuated fundamental is left.
When the reference waveform is not a triangle or sawtooth, the modulation becomes nonlinear. For small signal use, the gain is approximated based on the slope of the waveform. For a sinusoidal reference waveform of amplitude V_c, small-signal gain is identical to that found with a triangle wave that has the same slope at the zero crossings (i.e. is tangential). This is a triangle wave with an amplitude $\pi/2$ that of the sine wave.

$$A_{DC} = \frac{V_{sq}}{\frac{\pi}{2} V_c} \tag{F.2}$$

The fundamental of a square wave with amplitude V_{sq} has an amplitude of:

$$V_{fund.} = \frac{4}{\pi} V_s q \tag{F.3}$$

The amplitude at the comparator input becomes

$$V_c = \frac{4}{\pi} V_s q \left| H(s) \right| \tag{F.4}$$

Following from (F.4) and (F.2), DC gain becomes

$$A_{DC} = \frac{1}{2 \left| H(j 2 \pi f_s w) \right|} \tag{F.5}$$

A result worth remembering. The linearized DC gain of the comparator and the power stage in a self-oscillating system with 180-degree oscillation condition equals one half divided by the gain of the feedback network. If the feedback network has eq. 40dB of loss, the linearized gain of the comparator is 34dB.