variation of $\sqrt{(R)}$; this represents a signal-to-noise ratio increase with frequency of 1 db/oct due to the fact that the signal voltage is proportional to R and the noise voltage to $\sqrt{(R)}$. The signal-to-noise ratio is, therefore, proportional to $R/\sqrt{(R)} = \sqrt{(R)}$. Figure 12 displays two signal and noise curves with R as parameter which illustrates this relationship. Referring again to Fig. 11, at 450 cps the curve of R intersects a 12 db/oct curve (ω^2) which represents the copper loss due to r. Since Q can be expressed in two ways: $Q = \omega L/r$ or $R/\omega L$, the series resistance r can be transformed to a parallel resistance R using the relationship $R = \omega^2 L^2/r$ where ω^2 represents the 12 db/oct slope. $Q = 1$ occurs at 57 kc where the R curve crosses the ωL curve. Maximum Q is seen to occur in the 500-1000 cps range and is 19 db or 8.9.

According to Section *4.* of the Thermal Noise section (a resistor and a loss-free parallel resonant circuit in parallel, the noise voltage *vs* frequency away from resonance has 6 db/oct slopes. The curve representing the real (noise-producing) part of the head impedance must then have 12 db/oct slope because of the relationship between noise voltage and the value of the noise-generating resistance ($e^2 = 4kTBR$). In the Appendix is given the mathematical demonstration of the 12 db/oct slopes, and Figure 13 shows

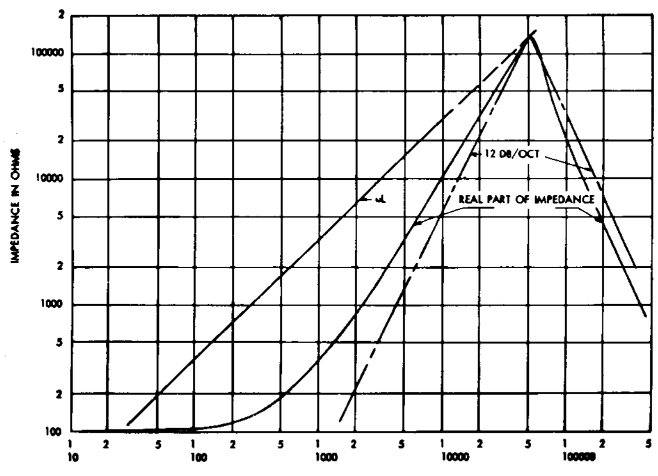

FIG. 13. Measured reactive (ωL) and real impedance of the same head used for the measurements of Fig. 11.

ωL and the real part of the head impedance *vs* frequency as measured on a Dranetz Complex Impedance-Admittance meter, using the same head as used in Fig. 11. (The slopes are not exactly 12 db/oct because of the finite dc winding resistance.) Figure 13 also shows $Q = 1$ ($\omega_0 L = R$) near 57 kc.

The inductance of tape reproducing heads for audio is often in the 0.5 to 2 H range, which has been considered a good compromise between high overall output and high resonant frequency. Most amplifier inputs look like a capacitance and a resistance in parallel. By varying these, the frequency and Q of the head resonance can be adjusted to compensate for reproducing head gap losses.

If higher inductance heads are desirable for higher overall voltage output, it becomes increasingly more difficult to keep the head resonance above the audible range. Figure 14 is a graph of the relative head impedances of three heads with the following inductances: 1.5 H, 15 H and 150 H. All three heads were made from similar laminations and were connected to the grid of a Nuvistor triode. The $1/\omega C$ portions of the resonance curves are seen to be almost identical, indicating that the cable and tube input capacitances in this case are much higher than the distributed capacitance in the head windings. (The 150 H head had 27000 turns of No. 50 wire and had an impedance at resonance of 6.7 Megohm. The dc resistance of the winding was 13.8 kilohm.)

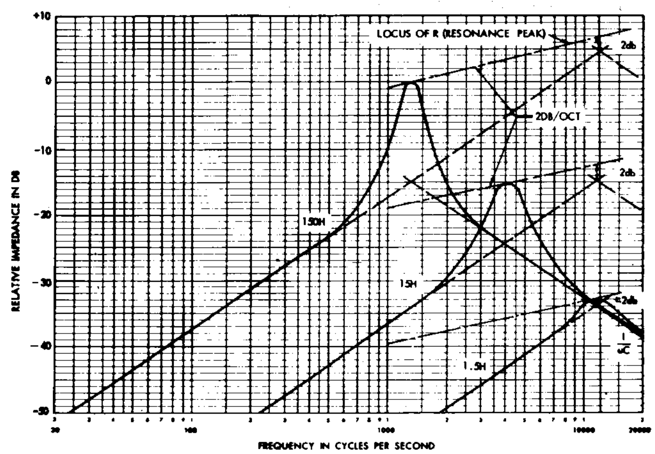

FIG. 14. Measured relative impedance of three different heads made on identical cores but with the inductances 1.5 H, 15 H and 150 H respectively.

Multitrack heads in this inductance range are not easily manufactured since the limited winding space will demand the use of extremely delicate wire. A more practical solution is the use of a low-impedance head in conjunction with step-up transformers. The inherent added noise from a transformer should preferably be kept an order of magnitude lower than the head noise.

By drawing 2 db/oct slope lines through the resonance peaks in Fig. 14, it is evident that all three heads have similar Q's at any given frequency (say 12 kc), indicating identical iron losses. These identical iron losses will produce different absolute values of R due to the difference in the number of turns n on the heads. Since R is proportional to n^2 and the number of turns on the three heads has the ratio $1:\sqrt{(10)}:10$, the respective R's must have the ratio 1:10:100 or 0 db:20 db:40 db as verified in Fig. 14. Since a large R means a high signal-to-noise ratio (see Fig. 12), it is also evident that the amplifier input resistance shall preferably be an order of magnitude higher than R, and R_{eq} (the equivalent noise resistance in series with the amplifier input) as low as possible. For the 150 H head, R_{eq} should preferably be below the dc resistance of the head ($r = 13.8$ kilohm); this will make the thermal head noise the only significant noise contributor up to the frequency where head noise equals amplifier noise. (This frequency is normally above the audible range.)

Figure 14 shows that when the capacitance across the head winding is reduced and the resonance frequency increases, R increases at a 2 db/oct rate. The signal-to-noise ratio, therefore, increases at a 1 db/oct (3.3 db/decade) rate as shown before. Every reduction by a factor of two of the capacitance will then increase the signal-to-noise ratio by 0.5 db. Optimum signal-to-noise ratio is therefore obtained for the following conditions: 1. Minimum head capacitance; 2. Maximum resonant impedance R of the head; 3. Minimum dc resistance r of the head winding; 4. Minimum equivalent input noise resistance R_{eq} of the amplifier; 5. Maximum input resistance of the amplifier.

Shot Noise

Shot noise is found in tubes and transistors. As the dc current through a tube or a transistor is carried by particles with finite velocities (electrons in tubes and electrons and holes in transistors), rather than by a continuous medium, a small ac noise current will be superimposed upon the dc plate and collector current. In the case of a tube with non-interacting electron transits, the noise current at frequencies low compared with the reciprocal of electron transit time is:[7]

$$I_N = \Gamma\sqrt{(2iedf)} \quad \text{or}$$
$$I_{N,rms} = \Gamma\times 1.78\times 10^{-5}\sqrt{(idf)},$$

where Γ is a smoothing factor due to space charge, e is the electron charge 1.59×10^{-19} coulomb, I_N is noise current in μA, i is the dc current in mA and df the bandwidth in cps. The formula points out that for constant bandwidth, lower noise will result from lower plate and collector current and lower Γ (more space charge). The latter can be achieved by lowering the plate voltage at constant cathode temperature.

Shot noise *current* divided by the transconductance of a tube will give the equivalent noise *voltage* at the grid, and since the energy per cps of bandwidth is constant, this noise is similar to thermal noise and can be expressed as the thermal noise of an "equivalent" noise resistor at room temperature, thereby eliminating the bandwidth specification.

At 27°C room temperature and 1000°K cathode temperature, the equivalent noise resistor as given by different authors averages:[8]

$$R_{eq} \sim 3/g_m \quad (R_{eq} \text{ in ohms and } g_m \text{ in mhos}).$$

The lowest equivalent noise resistance is thus obtained with a tube operated at the highest possible transconductance, and a low noise tube is therefore a tube with high transconductance at low plate current.

Since the noise current in a transistor can be expressed as $I_N = \sqrt{(2iedf)}$, it is also called "shot noise." This type of noise is produced when the dc current passes the emitter and collector junctions. Today's transistors, especially the silicon types, are quite competitive with tubes when it comes to thermal, shot and $1/f$ noise, and the price seems to come down at a fairly rapid rate too. Betas of 100 to 1000 are common, making it possible to obtain high input impedance $[\beta \times (R_E + r_e)]$ and high voltage amplification simultaneously; therefore, the coupling capacitors between stages can be reliable inexpensive nonelectrolytic (e.g., paper or mylar) types.

The following example will illustrate typical shot noise frequency distributions for a Nuvistor tube and a silicon transistor.

Example 6: **Measure the noise voltage of a tube** (Nuvistor 7895) and transistor (Fairchild S 3568, a version of 2N 2484) *vs* frequency with the following source impedances: $R_s = 10$ ohm, $R_s = 1$ kilohm and $R_s = 51$ kilohm.

The results are shown in Fig. 15. $R_s = 10$ ohm can be

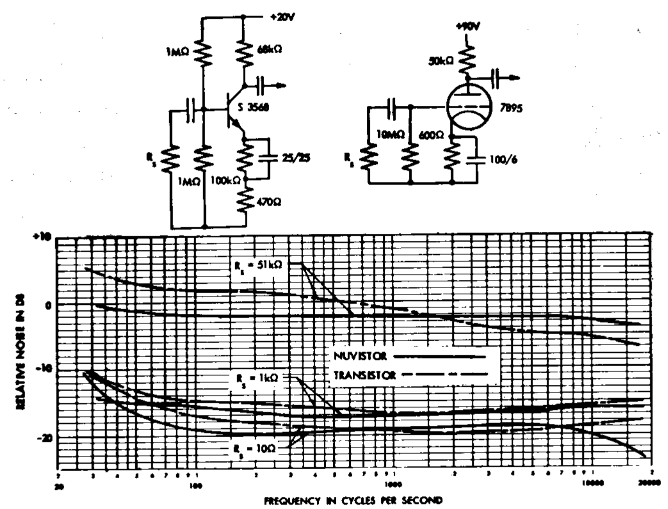

FIG. 15. Measured tube and transistor noise (and noise from the source resistance) *vs* frequency with source resistance as parameter.

considered a short circuit; since the $R_s = 1$ kilohm curves at mid-frequencies are approximately 3 db above the $R_s =$ 10 ohm curves, R_{eq} must also be approximately 1 kilohm for both tube and transistor at these frequencies. Even the low frequency ($1/f$) noise is comparable within a couple of db. The tube appears to be superior at low frequencies and the transistor at high frequencies for $R_s = 51$ kilohm, but the difference is small.

In order to find the maximum signal-to-noise ratio, the output levels which produced 10% second harmonic distortion were also measured: they were 2.2 V_{rms} for the transistor and 33 V_{rms} for the Nuvistor. This considerable difference (24 db) in maximum signal-to-noise ratio is, however, significant only in certain circuit configurations which have to accept a wide range of input signals, such as microphone preamplifiers, or where a passive attenuation is found between stages (volume control, filters, etc.). In a tape-reproducer preamplifier with post-equalization provided by negative feedback, no such attenuation is necessary and tube and transistor amplifiers perform identically.

Since many silicon transistors have a maximum collector to emitter voltage, $V_{CE} = 60$ V (some even 150 V) it is possible to get maximum "undistorted" output levels comparable to those of tubes, if low noise is not the primary design goal.

A semiconductor device which in many respects performs like a tube is the field effect transistor (FET). Its input impedance is inherently in the Megohm range, but the noise factor is low at high source resistances only.

Example 7: Calculate the R_{eq} for a FET with a 1.5 db noise figure when $R_s = 200$ kilohm.

Source resistance noise: $e_s^2 = 4kTBR_s$
Equivalent noise resistance noise: $e_R^2 = 4kTBR_{eq}$
Total noise: $e_{tot}^2 = e_s^2 + e_R^2 = 4kTB\,(R_s + R_{eq})$
Noise factor $(F) = e_{tot}^2/e_s^2 = 1 + R_{eq}/R_s$
Noise figure $(NF) = 10 \log F = 1.5$ db
$F = 1{,}414 = 1 + R_{eq}/R_s$ or
$R_{eq} = 100$ kilohm (compared to 1 kilohm for a tube or regular transistor).

1/f Noise

The term "$1/f$ noise" means that the rms noise power in a band of constant absolute width is inversely proportional to frequency. The noise voltage is then proportional to $\sqrt{(1/f)}$, i.e., it has a 3 db/oct slope on this basis. Such $1/f$ noise is found in resistors, tubes and transistors. There are two different types of $1/f$ noise: one is "flicker" noise from the fluctuation of thermionic emission from oxide coated cathode in tubes, which is reduced by space charge; the other may be termed "semiconductor noise" since it can be interpreted[9] as fluctuations of conductivity in a semiconductor (fluctuations in the number of carriers in the conduction band). "Excess noise" (over thermal) and "current noise" are other expressions for $1/f$ noise. This noise is found in a variety of materials:[10] carbon granules, carbon filaments, graphite powder, pyrolytic carbon films, germanium filaments, thin metal films, lead sulphide films, all types of rectifying barriers in germanium, silicon and metallic oxides, single crystal cuprous oxide and the interface layer (barium orthosilicate) in tubes between the nickel cathode tube and its oxide coating.

1/f Noise in Tubes

It is especially important to have the orthosilicate interface layer in tubes under control, as it may change with time. A tube with low initial $1/f$ noise can grow a considerable "interface" layer with time, unless the purity of the nickel is controlled when the tube is manufactured. The interface layer is produced by the impurity (silicon) in the nickel cathode which, by diffusion, combines with the oxide coating and forms orthosilicate. Since this interface layer exhibits a parallel connection of a capacitance and a resistance in series with the cathode lead, it is important that it be eliminated if the tube is to be used at rf frequencies. Thus, this type of improvement in tubes for rf and computer use will also benefit their use at audio frequencies. Tubes like 6SN7 and 12AU7 have been tested for interface $1/f$ noise, and a 40 db increase in equivalent noise at 30 cps was measured after several hundreds hours of use.[11] It was distinguished from flicker $1/f$ noise by its closer dependence upon the square of the plate current, whereas flicker noise is proportional initially to the square of the emission current. A test of aged tubes at different cathode temperatures gave the following results:[11]

Filament voltage	6 V	16 V	18 V
Interface resistance	5000 ohm	40 ohm	0.5 ohm
Noise resistance (30 cps)	7×10^{10} ohm	6.3×10^{6} ohm	No value given

Another illustration of the difference between tube types with respect to $1/f$ noise is displayed in Fig. 16 which shows that approximately 16 db lower noise can be obtained at 30 cps with a tube with controlled interface, such as 6922 or 7586, as compared to a standard audio tube such as 12AX7. Rf tubes with controlled interface are not necessarily good audio tubes, however. Some are very microphonic at frequencies which are difficult to damp, and since rf tubes have relatively small mechanical structures in order to keep the inter-electrode capacitance down the cathode temperature must be rather high for a given maximum plate current, with reduced life as a result. The RCA Nuvistors have controlled interface and low cathode temperature; because of the small size, the microphonics are at a high frequency (3-5 kc) which is fairly easy to damp; so far, the Nuvistors have been found to be an excellent type of low noise tube.

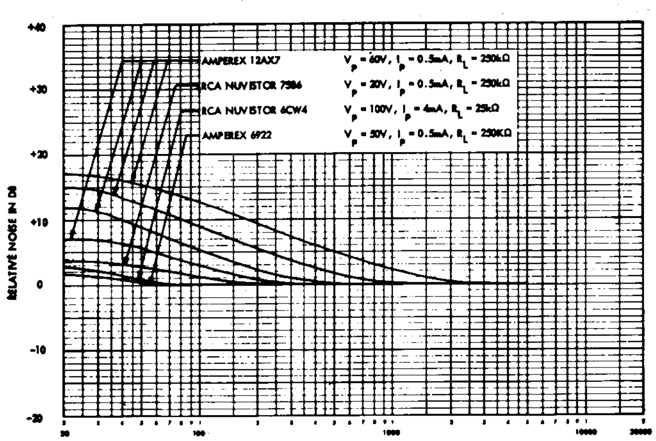

FIG. 16. Noise frequency response of different tubes showing different amounts of $1/f$ noise.

1/f Noise in Transistors and Resistors

Bell[12] defines metallic conductors and semiconductors as follows: "In a metallic conductor the number of conduction electrons remains constant and therefore temperature variation of conductivity is a function only of their mobility. In a semiconductor the charge carriers (electrons and holes) are not permanently in the conduction band, but are raised to it from their ground states by thermal or other excitation."

More practically measurable properties of semiconductors are negative temperature coefficient and $1/f$ noise.

The $1/f$ noise in transistors is due[13] to leakage current and contaminated or imperfect semiconductor surfaces and can be reduced by proper manufacturing until the crossover frequency with shot noise is below 100 cps even at low collector currents. (See Fig. 15.)

The $1/f$ noise in resistors will occur when current passes through them. The National Bureau of Standards has recommended as a noise index that the unit μV noise per V dc in a frequency decade in db be used as a measure of $1/f$ noise in a resistor.

A resistor has a 0 db noise index when 1 μV noise is measured in a frequency decade with 1 V dc applied across it. The resistor value is immaterial. If 2 V dc is applied, 2 μV is produced in a decade if the noise index is 0 db. A noise index of -6 db results in ½ μV noise in a decade for 1 V dc applied voltage.

Different resistor types have different noise indexes. Resistor noise measurements using the Quan-Tech Laboratories, Inc., Model 315 Resistor Noise Test Set gave the following results: -30 to -45 db for good metal film resistors and $+3$ to -30 db for carbon film and composition resistors. All three types showed highest noise index for the highest value resistors. The low value resistors are made of a material with properties close to those of the bulk material, whereas the high value resistors, for example film resistors, rely on the resistance of extremely thin conducting layers with properties several orders of magnitude different from those of the bulk material. One brand of metal film resistors had the following distribution:

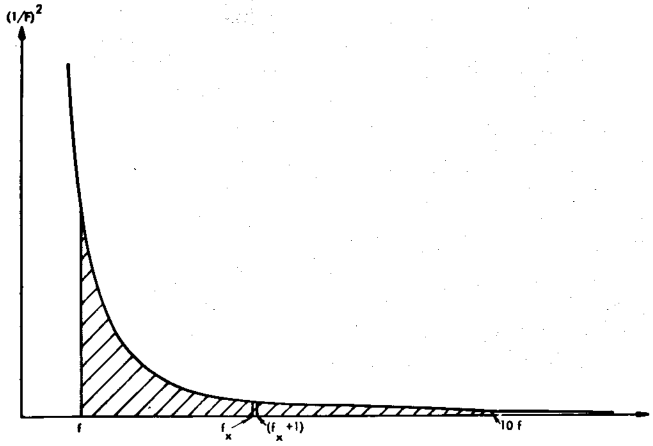

FIG. 17. The curve shows the square of $1/f$ noise voltage *vs* frequency, using linear scales. The crosshatched area represents the total $(1/f \text{ noise})^2$ in a decade. The area from f_x cps to f_{x+1} cps represents the $(1/f \text{ noise})^2$ in a one cps wide band.

47 kilohm	–45 db
100 kilohm	–33 db
825 kilohm	–30 db

Ten resistors of each value were measured.

Relationship Between Noise Index of a Resistor and the Crossover Frequency Between Thermal Noise and $1/f$ Noise

In this section will be calculated the total $1/f$ noise in a decade produced by a resistor R, the average 1 cps bandwidth noise in a decade and the frequency at which it occurs. The frequency f_x where it equals the 1 cps bandwidth thermal noise will be found and an expression for the relationship between resistance value R, crossover frequency f_x, noise index and applied dc voltage will be derived.

Figure 17 shows the square of noise voltage *vs* frequency of $1/f$ noise using linear scales. Since noise must be added rms, $(e_{noise})^2$ in a decade (f to $10f$) is:

$$\int_f^{10f} C\times 1/f\times df = C(\ln 10f - \ln f) = C\times \ln(10f/f) = C\ln 10.$$

The average (*noise voltage*)^2 per cps is thus:

$$C\ln 10/(10f-f) = C\ln 10/9f,$$

where f is the lowest frequency in the decade measured, and C is a constant.

The (*noise voltage*)^2 in a one cps frequency band from f_x to (f_x+1) cps is

$$C\int_{f_x}^{f_x+1}(1/f)df = C\times\ln[(f_x+1)/f_x].$$

To find the frequency f_x at which this one cps bandwidth (*noise voltage*)^2 equals the average (*noise voltage*)^2 in a decade:

$$(C\ln 10)/9f = C\ln[(f_x+1)/f_x]$$

or, multiplying with the factor $\log N/\ln N = 1/2.3026$ and eliminating C:

$$\log 10/9f = 1/9f = \log[(f_x+1)/f_x] = \log[1+(1/f_x)]. \quad (1)$$

If the audio range only is considered, f will vary between 20 cps and 2 kc (20 kc will be the highest frequency in the decade). In order to solve this equation with known accuracy in the audio frequency spectrum, the two limiting frequencies $f = 20$ cps and 2 kc are inserted in Eq (1):

$$\log(1+1/f_x) = 1/9f = 1/180 = 0.00555$$
$$1+1/f_x = 1.012874 \text{ or } f_x = 77.68 \text{ cps} = 77.68/20 = 3.88f$$
$$\log(1+1/f_x) = 1/9f = 1/18000 = 0.0000555$$
$$1+1/f_x = 1.0001278 \text{ or } f_x = 7825 \text{ cps or } 7825/2000 = 3.91f.$$

The difference is seen to be less than 1%.

The average one cps bandwidth $1/f$ (*noise voltage*)^2 is:

$$(C\ln 10)/9f = (C\ln 10)/(9f_x/3.9) = (C/f_x)(2.3026\times 3.9/9) = .998\times(C/f_x)\sim C/f_x.$$

The average one cps bandwidth thermal (*noise voltage*)^2 is $4kTR$. The average one cps bandwidth thermal and $1/f$ (*noise voltage*)^2 are equal when

$$4kTR = C/f_x \text{ or } Rf_x = C/4kT.$$

The value of C is found from

$$e_1/f_{\text{decade}}{}^2 = C\ln 10 = (1\ \mu\text{V/Vdc})^2 \qquad (0 \text{ db noise index})$$

$$C = (1/10^{12}\ln 10)[V^2/(Vdc)^2]$$

$$Rf_x = 10^{23}/(4\times1.374\times300\times10^{12}\times\ln 10) \qquad (2)$$
$$= 26.4\times10^6. \qquad \text{(0 db noise index, 1 V dc)}$$

The thermal noise voltage across resistors from 1 ohm to 100 Megohm is calculated from $e=\sqrt{(4kTBR)}$. The bandwidth is chosen to be 1 cps and the voltage is expressed in db below 1 V. Since thermal noise has a flat frequency response, the resistor thermal noises are plotted as horizontal lines, as shown in Fig. 18. $R=1$ Megohm inserted in

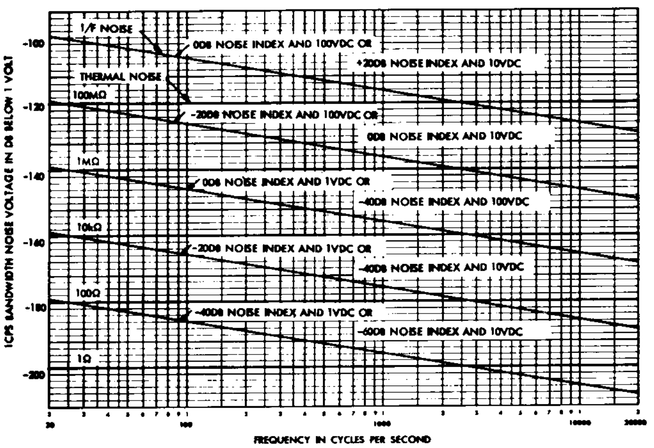

FIG. 18. **Relationship** between thermal noise and $1/f$ noise in a resistor at audio frequency, with noise index and applied dc voltage as parameters.

Eq. (2) yields a crossover frequency f_x between thermal and $1/f$ noise of 26.4 cps. The $1/f$ frequency response for 0 db noise index and 1 V applied dc is then obtained by drawing a 3 db/oct sloping line through the crossover point. The $1/f$ noise curves for different noise indexes and applied dc voltages are obtained by drawing lines parallel to the 3 db/oct sloping line. If 10 times the dc voltage is applied, the noise voltage increases 10 times or 20 db; if the noise index is 20 db lower the noise curve will move down 20 db. The data given by Stansbury[14] also prove the validity of Fig. 18. The *1/f noise*, unlike thermal noise, is independent of resistor value. This is due to the fact that the noise is proportoinal to the dc *current* through a resistor (current noise).

The following two examples may illustrate the magnitude of $1/f$ noise in resistors:

Example 8: What is the noise voltage increase near 30 cps of a 1 W 10 kilohm carbon film resistor with a noise index of 0 db when 1 W dc power is dissipated in it?

$V=\sqrt{(RW)}=\sqrt{(10^4\times1)}=100$ V. Figure 18 shows a $1/f$ noise at 30 cps for 0 db noise index and 100 V dc of -101 db. The thermal noise of a 10 kilohm resistor is -158 db (Fig. 18) so the increase is 57 db!

Example 9: How much dc can be applied across a 100 ohm metal film resistor with a -40 db noise index if $1/f$ noise shall be negligible in the audible frequency range (30-15000 cps)?

The $1/f$ noise is negligible when it is below the thermal noise. Figure 18 shows the thermal noise of a 100 ohm resistor to be -178 db. The -40 db noise index, 1 V dc curve crosses -178 db at 26.4 cps, so the dc voltage must be kept below 1 V.

The curves for thermal noise and *1/f noise* in Fig. 18 are plotted for 1 cps bandwidth for reasons of simplicity. Other bandwidths, for example, $B=10$ cps, 5 cps or 1/10 cps, could just as well be used; in this case the relationship between thermal and $1/f$ noise would not change, but the whole system of curves would move up $20\log\sqrt{(B)}$ db.

When a pure ac signal is applied across a non-wirewound resistor, $1/f$ noise will be produced proportionally to the amplitude of the signal and a process similar to modulation noise can be observed.

Induced Grid Noise

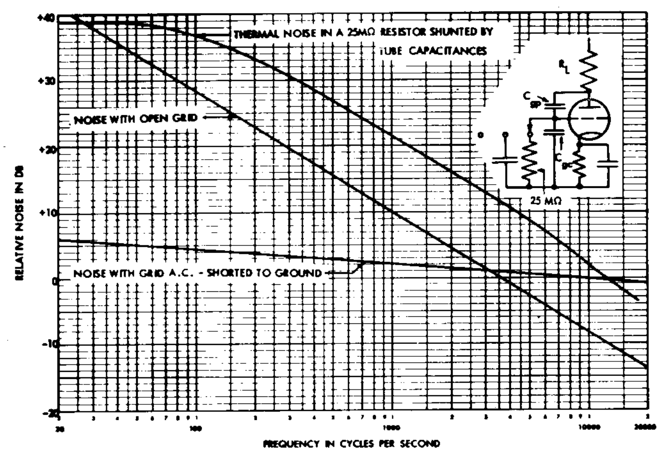

FIG. 19. **Measured** noise frequency responses for the determination of induced grid noise.

Induced grid noise is found in tubes. When the electrons emitted from the cathode pass through the grid wires on their way to the plate, they create small current pulses as they approach and move away from the grid wires. If the grid is shorted to ground, no grid noise voltage will develop. If there is a finite impedance between grid and ground, the noise voltage will be the noise current times this impedance. Maximum noise is obtained with the highest impedance, which means that a tube with an open grid would exhibit maximum induced grid noise; in this case, the noise is attenuated only by the tube capacitances.

Since the same electrons which pass the grid will eventually hit the plate, a certain correlation between induced grid noise and shot noise seems to exist. At rf frequencies the electron transit time becomes significant. Van der Ziel and others have found a noise minimum at these frequencies by detuning a resonant circuit slightly so as to obtain a certain phase relationship between the grid and the plate. At audio frequencies, however, little use can be made of this transit time phenomenon. The induced grid noise is approximately equal to the thermal noise arising from the grid-cathode conductance at a temperature $1\frac{1}{4}$ times that of the cathode, according to Bakker[15] and others, and can therefore, at audio frequencies, be considered as having the same noise spectrum as the thermal noise in a resistor at room temperature.