

Noise Limitations in Tape Reproducers*

ERLING P. SKOV

Ampex Audio, Sunnyvale, California

Thermal noise, shot noise, $1/f$ noise, induced grid noise and Barkhausen noise are described and quantitative examples are given. Expressions for thermal noise in mixed resistive and reactive circuits are derived. The relationship between head thermal noise and amplifier shot noise is discussed and measured data are given. The question of tubes *vs* transistors is touched upon with respect to noise and overload capabilities. The relationship between the noise index of a resistor and the crossover frequency between thermal and $1/f$ noise is calculated and displayed as a family of curves, and examples are given. Signal-to-noise measurement data from the Ampex MR-70 master recorder are given along with a brief description of the basic design philosophy with respect to noise reduction.

INTRODUCTION

MAGNETIC recording has been performed since 1899, when Valdemar Poulsen filed patent application in the United States for his telegraphone. His original process has seen considerable improvements through the years: W. L. Carlson and G. W. Carpenter (ac-biasing, 1927), Kurt Stille and Blattner (steel tape), Pfleumer (coated paper and plastic tapes), C. N. Hickman of the Bell Telephone Laboratories (vicalloy tape, 1937), are just some of the contributors.¹

Improvements have continued, and in attempting to obtain the best signal-to-noise ratio in the widest frequency band, the limitations in overload distortion and noise have been investigated more closely, resulting in practical improvements. Lower-noise tubes and transistors have come of age, and the trend in tape development has been towards lower noise, better saturation characteristics at short wavelengths, smoother surfaces for less spacing loss, more homogeneous tapes for fewer dropouts, and better wear characteristics, to mention only some of the advances already obtained. The equipment designer will have to keep pace with tape improvements by providing lower noise head-amplifier designs; better yet, he should keep at least 10 db ahead for possible tape improvements.

Although many noise sources are treated in textbooks, this paper will attempt to pool them all together and to point out some of the basic signal-to-noise limitations in the specific field of audio recording.

NOISE MEASUREMENT TECHNIQUE

Noise was measured as a function of frequency, in small bands of constant width; this measurement may be called the "noise frequency response."²

The noise frequency response was measured with a HP Model 302A wave-analyzer with a 6 cps constant bandwidth. The speed of the meter on this analyzer is much too fast for noise measurements in such a narrow band; its fluctuations made readings very difficult. The averaging time should be several times the reciprocal of the bandwidth in cps:³

$$t = 1/6 \text{ cps} \times 3 = 1/2 \text{ sec.}$$

The addition of a capacitor across the meter movement achieves this result. The meter resistance is 100 ohm; therefore,

$$C = t/R = 1/2 / 100 = 5000 \mu\text{F.}$$

The value 6000 μF was used in all noise measurements in this paper. A more correct method would have been to use an averaging thermocouple meter which would read e^2 instead of e_{eff} , the error, however, is not serious and of course does not appear in relative measurements. The narrow bandwidth of the analyzer makes it possible to measure fairly accurate noise frequency responses of devices with sharply varying frequency responses such as high-Q circuits.

TYPES OF NOISE

Five different types of noise will be considered, along with the devices in which they occur. Among amplifying devices only tubes and transistors will be considered,⁴ since

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they constitute the majority of amplifiers in use. Hum and microphonic effects will not be mentioned since they do not present absolute physical limitations.

The five basic noise sources and the devices in which they are of practical importance are as follows:

- | | |
|------------------------|--|
| 1. Thermal noise: | Resistors, transistors, tape heads, transformers |
| 2. Shot noise: | Tubes, transistors |
| 3. 1/f noise: | Resistors, tubes, transistors |
| 4. Induced grid noise: | Tubes |
| 5. Barkhausen noise: | Tape heads, transformers |

Thermal Noise

Thermal or Johnson noise is a measure of the kinetic energy of the conduction electrons. This noise, if measured with a constant bandwidth wave analyzer, will exhibit a flat frequency response (white noise). This type of noise is found in resistors, in the base area of transistors,⁵ and in tape playback heads and transformers, where the combined copper resistance and effective resistance from eddy current losses will produce thermal noise. (Thermal noise in vacuum tubes appears to be negligible at audio frequencies.) Wire-wound resistors have thermal noise only; that is, noise will not increase if dc current is sent through them, unless 1. the dissipated power increases the temperature of the resistor or 2. it has faulty contacts. In low-level input circuits, however, the heat rise is negligible.

Example 1: How many db will the thermal noise increase in a wire-wound resistor if its temperature is raised 30°C above room temperature?

Noise voltage, $e_n = \sqrt{4kTRB}$, where k = Boltzmann's constant (1.374×10^{-23} Joules/°K), T = absolute temperature in °K, R = resistance in ohm, B = bandwidth in cps.

Noise voltage increase for 27°C room temperature (300°K):

$$20 \log \sqrt{(4kT_1RB/4kT_2RB)} = 20 \log \sqrt{[(300+30)/300]} = 0.4 \text{ db.}$$

Film resistors exhibit thermal noise and often also some semiconductor (1/f) noise. Films of thicknesses less than the mean free path of the conduction electrons will show higher resistivity than the bulk material,⁶ due to the scattering of the electrons when they meet the surfaces of the film.

The base resistance of transistors will only contribute to noise in transistors with low shot noise.

Thermal noise is present in any device with a resistive component, when the temperature is sufficiently above 0°K. Normal amplifier circuits operate near or above room temperature so their resistive components do produce thermal noise. The reactive circuit components do not produce noise but serve only as energy storage devices. They will modify the frequency distribution of the thermal noise produced by the resistive components.

The noise frequency response of five basic resistive-reactive combinations will now be investigated: 1) Two resistors in parallel; 2) A resistor and a capacitor in paral-

lel; 3) A resistor and an inductor in parallel; 4) A resistor and a loss free parallel resonant circuit in parallel; 5) A resistor and a lossy parallel resonant circuit in parallel.

Sample Calculations

1. Derive the noise frequency response of two resistors in parallel (see Fig. 1):

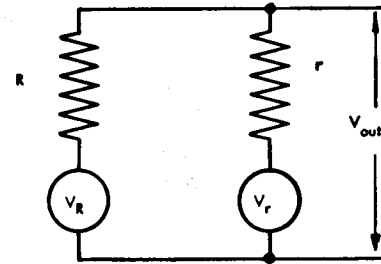


FIG. 1. Circuit for calculating the noise voltage from two resistors in parallel.

$$V_r \text{ contribution to } V_{out}: [(R/r+R)]V_r$$

$$V_R \text{ contribution to } V_{out}: [(r/r+R)]V_R$$

$$\text{Total noise: } \sqrt{([V_r \cdot (R/r+R)]^2 + [V_R \cdot (r/r+R)]^2)}$$

$$\text{Since } V_r^2 = 4kTBr \text{ and } V_R^2 = 4kTBR;$$

$$\text{Total noise:}$$

$$\sqrt{[4kTBr(R/r+R)^2 + 4kTBR(r/r+R)^2]} = \sqrt{(4kTB) \times (rR/r+R)}$$

which is the thermal noise of a resistor composed of r and R in parallel.

2. Derive the noise frequency response of a resistor and a capacitor in parallel (see Fig. 2):

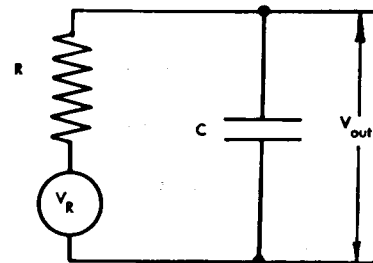


FIG. 2. Circuit for calculating the noise voltage from a resistor and a capacitor in parallel.

$$V_R \text{ contribution to } V_{out}:$$

$$V_R \frac{1/j\omega C}{R + (1/j\omega C)} = V_R \frac{1}{j\omega CR + 1}$$

$$\text{Since } V_R^2 = 4kTBR,$$

$$V_{out} = \sqrt{(4kTBR)/(j\omega CR + 1)}.$$

$$\text{If } j\omega CR \ll 1: V_{out} = \sqrt{(4kTB) \times (R)}$$

(low frequencies)

$$\text{If } j\omega CR \gg 1: V_{out} = \sqrt{(4kTB)/[j\omega C \times (R)]}$$

(high frequencies)

At low frequencies where the capacitor has no effect, the noise is pure thermal noise from R ; therefore, the noise voltage increases 3 db with a doubling of R ($V_{out} \propto \sqrt{R}$).

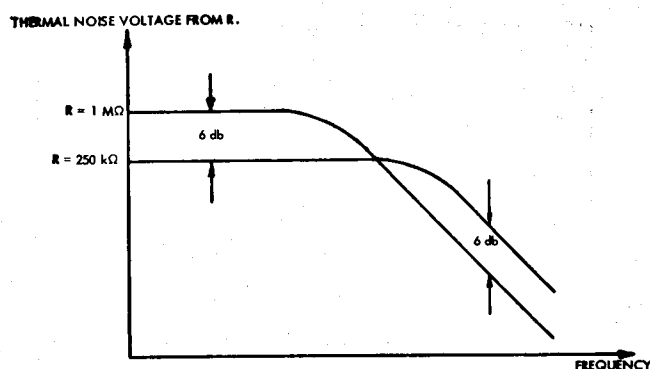


FIG. 3. Noise voltage frequency response of a resistor R and a capacitor in parallel, with R as parameter and $C = 300$ pF.

At high frequencies the noise voltage decreases 6 db/oct with frequency ($V_{out} \propto 1/j\omega C$), and 3 db with a doubling of R ($V_{out} \propto 1/\sqrt{(R)}$).

Example 2: Plot the noise curves for $C = 300$ pF in parallel with $R = 1$ Megohm and $R = 250$ kilohm respectively (see Fig. 3). This shows that greater resistance produces more noise at low frequencies, but less noise at high frequencies.

3. Derive the noise frequency response of a resistor and an inductor in parallel (see Fig. 4):

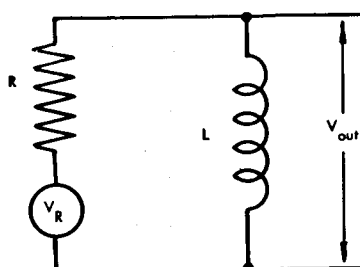


FIG. 4. Circuit for calculating the noise voltage from a resistor and an inductor in parallel.

V_R contribution to V_{out} :

$$V_R \frac{j\omega L}{j\omega L + R} = V_R \frac{1}{1 + (R/j\omega L)}$$

Since $V_R = \sqrt{(4kTBR)}$,

$$V_{out} = \sqrt{(4kTBR)} / [1 + (R/j\omega L)]$$

If $R \ll j\omega L$: $V_{out} = \sqrt{(4kTB)} \times \sqrt{(R)}$ (high frequencies)

If $R \gg j\omega L$: $V_{out} = [\sqrt{(4kTBR)}] j\omega L / R$
 $= j\omega L \sqrt{(4kTB)} / \sqrt{(R)}$ (low frequencies)

At high frequencies where the inductor has no effect, the noise is pure thermal noise from R ; therefore, the noise voltage increases 3 db with a doubling of R ($V_{out} \propto \sqrt{(R)}$). At low frequencies the noise voltage increases 6 db/oct with frequency ($V_{out} \propto j\omega L$) and decreases 3 db with a doubling of R ($V_{out} \propto 1/\sqrt{(R)}$).

Example 3: Plot the noise curves for $L = 100$ H in parallel with 1 Megohm and 250 kilohm respectively (see Fig. 5). This shows that the greater resistance produces more noise at high frequencies, but less noise at low frequencies.

4. Derive the noise frequency response of a resistor and a loss-free parallel resonant circuit in parallel (Fig. 6):

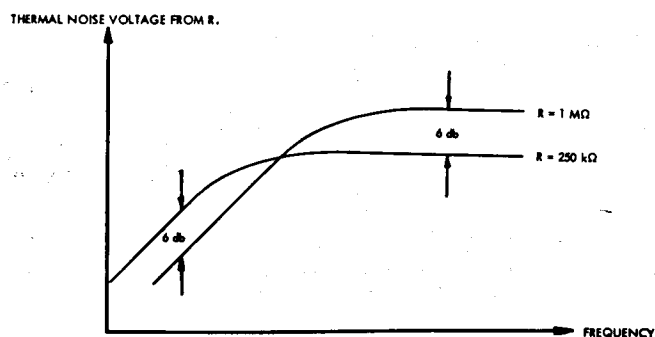


FIG. 5. Noise voltage frequency response of a resistor R and an inductor in parallel, with R as parameter and $L = 100$ H.

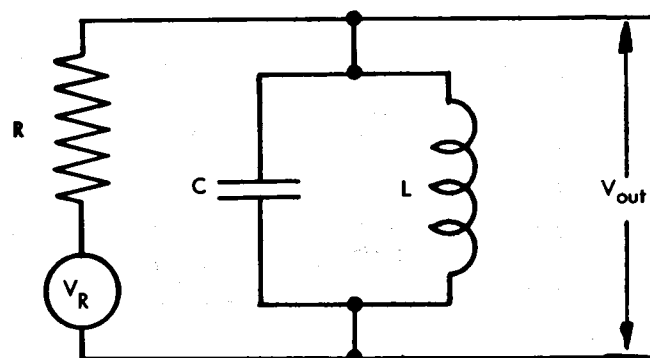


FIG. 6. Circuit for calculating the noise voltage from a resistor and a loss-free parallel resonant circuit in parallel.

V_R contribution to $V_{out} =$

$$= V_R \frac{[j\omega L(1/j\omega C)] / [j\omega L + (1/j\omega C)]}{R + \{[j\omega L(1/j\omega C)] / [j\omega L + (1/j\omega C)]\}}$$

$$= V_R \left\{ 1 / \left[\frac{R[1 - (\omega/\omega_0)^2]}{j\omega L} + 1 \right] \right\}$$

where $\omega_0^2 = 1/LC$.

At resonance ($\omega = \omega_0$):

$$V_{out} = V_R = \sqrt{(4kTB)} \times \sqrt{(R)}$$

Below resonance ($\omega \ll \omega_0$):

$$V_{out} = V_R \{1 / [(R/j\omega L) + 1]\}$$

which is identical to 3. above.

Above resonance ($\omega \gg \omega_0$):

$$V_{out} = V_R [1 / (j\omega CR + 1)]$$

which is identical to 2. above.

At resonance where the parallel circuit has no effect (infinite impedance) the noise is pure thermal noise from R . At low frequencies (below resonance) the noise voltage increases 6 db/oct with frequency ($V_{out} \propto j\omega L$) and decreases 3 db with a doubling of R ($V_{out} \propto 1/\sqrt{(R)}$). At high frequencies

(above resonance) the noise voltage decreases 6 db/oct with frequency ($V_{out} \propto 1/j\omega C$) and 3 db with a doubling of R ($V_{out} \propto 1/\sqrt{R}$).

Example 4: Plot the noise curves for $L=100$ H, $C=300$ pF, ($f_0=900$ cps) and $R=1$ Megohm and 250 kilohm, respectively. This may be done by any one of three approaches:

a) Exactly, by direct calculation from the equation

$$V_{out} = V_R \frac{1}{\{R[1 - (\omega/\omega_0)^2] / j\omega L\} + 1}$$

b) Approximately, by first locating the voltage at resonance [$(V_{out} \text{ at } \omega_0) = \sqrt{(4kTB) \times \sqrt{R}}$]; this is the voltage due to R alone, since the L - C circuit is assumed to have an infinite impedance at resonance. The 6 db/octave asymptotes may then be located, using the knowledge that the resonant Q may be determined from the equation $Q = R/\omega_0 L$, and that the asymptotes meet at the point where $Q = 1$. In this example, when $R=1$ Megohm, $Q=1.77$; this ratio is 5 db. Therefore the intersection of the asymptotes must be 5 db below the resonant impedance, as is shown in Fig. 7. Likewise, when $R=250$ kilohm, $Q=0.44$;

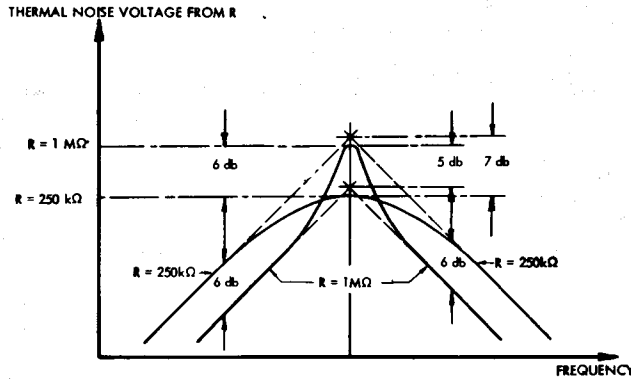


FIG. 7. Noise voltage frequency response of a resistor R and a loss-free parallel resonant circuit in parallel, with R as parameter, $L=100$ H and $C=300$ pF.

this ratio is -7 db. Therefore this intersection of asymptotes must be 7 db above the resonant impedance, as also shown.

c) Approximately, by locating the voltage at resonance [$(V_{out} \text{ at } \omega_0) = \sqrt{(4kTB) \times \sqrt{R}}$], and by using the curves of Examples 2 and 3 at the frequencies well above and well below resonance to determine the asymptotes of response.

5. Derive the noise frequency response of a resistor and a lossy parallel resonant circuit in parallel (see Fig. 8):

V_R contribution to $V_{out} =$

$$\begin{aligned} &= V_R \frac{[(j\omega L + r)(1/j\omega C)] / [(j\omega L + r) + (1/j\omega C)]}{R + [(j\omega L + r)(1/j\omega C)] / [(j\omega L + r) + (1/j\omega C)]} \\ &= V_R \frac{j\omega L + r}{R[1 - (\omega/\omega_0)^2] + j\omega C R r + j\omega L + r} \end{aligned}$$

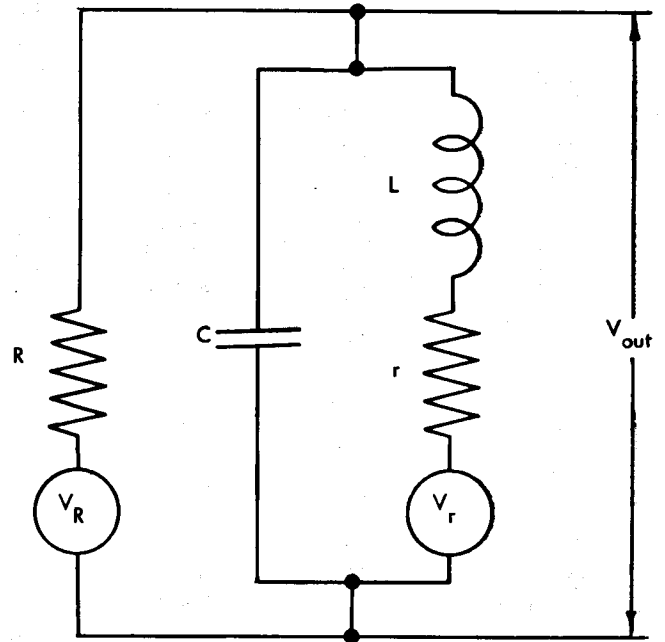


FIG. 8. Circuit for calculating the noise voltage from a resistor and a lossy parallel resonant circuit in parallel.

V_r contribution to $V_{out} =$

$$\begin{aligned} &= V_r \frac{[R(1/j\omega C)] / [R + (1/j\omega C)]}{r + j\omega L + \{R(1/j\omega C) / [R + (1/j\omega C)]\}} \\ &= V_r \frac{R}{R[1 - (\omega/\omega_0)^2] + j\omega C R r + j\omega L + r} \end{aligned}$$

Total noise voltage $V_{out} =$

$$\begin{aligned} &= \sqrt{\frac{V_R^2 (j\omega L + r)^2 + V_r^2 R^2}{R[1 - (\omega/\omega_0)^2] + j\omega C R r + j\omega L + r}} \\ &= \sqrt{(4kTB) \times \frac{\sqrt{[R(j\omega L + r)^2 + r^2 R^2]}}{R[1 - (\omega/\omega_0)^2] + j\omega C R r + j\omega L + r}} \end{aligned}$$

At very low frequencies ($\omega \sim 0$),

$$\begin{aligned} V_{out} &= \sqrt{(4kTB) [\sqrt{(Rr^2 + r^2 R^2)} / (R + r)]} \\ &= \sqrt{(4kTB) \sqrt{[Rr / (R + r)]}}, \end{aligned}$$

which is the thermal noise of r and R in parallel.

At resonance ($\omega = \omega_0$),

$$V_{out} = \sqrt{(4kTB) \frac{\sqrt{[R(j\omega_0 L + r)^2 + r^2 R^2]}}{j\omega_0 C R r + j\omega_0 L + r}}$$

Example 5: Find V_{out} at resonance when $R=1$ Megohm ($f_0=900$ cps), $L=100$ H, $C=300$ pF and $r=10$ kilohm. Plot the noise curves for $R=1$ Megohm and 250 kilohm respectively (see Fig. 9).

$V_{out} =$

$$\begin{aligned} &= \sqrt{\{4kTB [10^6 (j2\pi \times 900 \times 100 + 10^4)^2 + 10^4 \times 10^{12}]\}} \\ &= \sqrt{j2\pi 900 \times 300 \times 10^{-12} \times 10^6 \times 10^4 + j2\pi 900 \times 100 + 10^4} \end{aligned}$$

$$V_{out} = \sqrt{(4kTB)} \times \sqrt{(0.886 \text{ Megohm} + j0.016 \text{ ohm})}$$

The impedance at resonance is thus reduced from 1 Megohm to 0.886 Megohm because r prevents the resonant circuit impedance from reaching infinity.

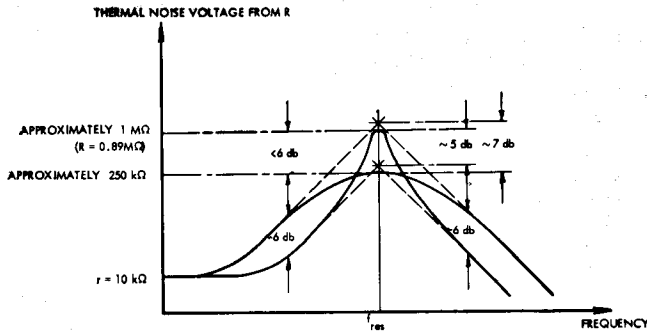


FIG. 9. Noise voltage frequency response of a resistor R and a lossy parallel resonant circuit in parallel, with R as parameter, $L = 100 \text{ H}$, $C = 300 \text{ pF}$ and $r = 10 \text{ kilohm}$.

Thermal Noise in a Tape Reproducing Head

The circuit configuration in Section 5. in the Thermal Noise section (Fig. 8) above is typical of a tape reproducing head. The dc resistance of the copper winding is constant $= r$; the parallel resistance R due to eddy current losses in the laminations varies with frequency as a function of the mechanical, electrical and magnetic properties of the laminations. A lamination can be considered a "shorted" turn around the flux lines through it, (Fig. 10), the turn having

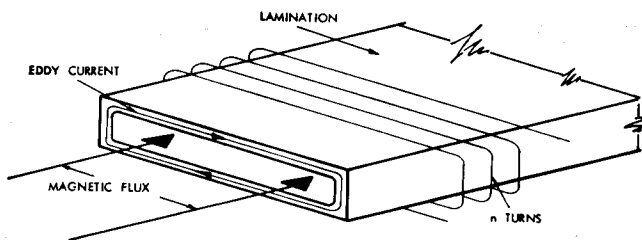


FIG. 10. Cross-sectional view of a lamination, indicating the paths of the eddy currents and the magnetic flux.

a finite resistance similar to a shorted turn on a regular transformer. The resistance of this single turn, R' , times n^2 , where n is the total number of turns on the head, is the transformed parallel resistance $R = n^2 R'$ of one lamination as measured across the head winding. Since eddy currents will resist the penetration of magnetic fields into the central parts of a lamination, the induction decreases from the surface towards the center when a magnetic field is applied from the outside. If the magnetic field is developed inside the lamination by the thermal noise in the conducting lamination material, the outer layers will both produce thermal noise and act as shields for the magnetic field produced by the thermal noise near the center of the lamination, so that R cannot be calculated directly from the cross-sectional area of the lamination. For the complete head all the laminations must be considered and the noise voltages added

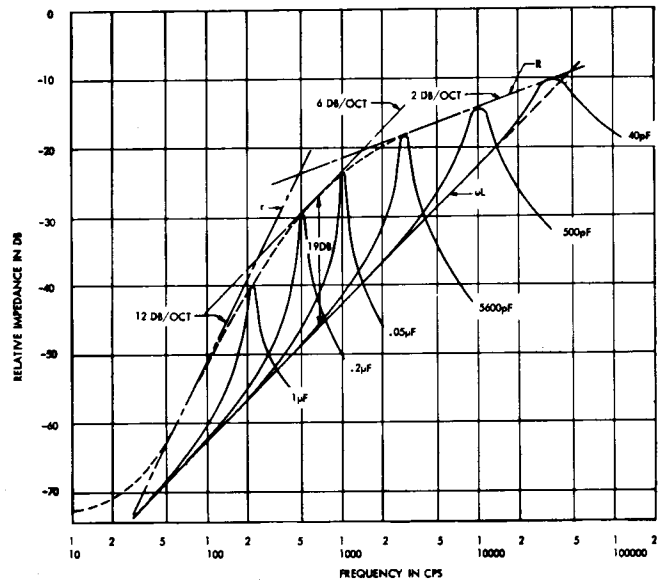


FIG. 11. Measured relative impedance of a 0.5 H playback head ($R = 130 \text{ kilohm}$ at 57 kc and $r = 100 \text{ ohm}$) with capacitance as parameter.

in rms fashion, since the laminations are independent, uncorrelated and non-concentric noise sources.

The permeability μ , the resistivity ρ , and the lamination thickness d of the laminations, and the frequency f will influence the effective R . Figure 11 shows a measurement of the impedance vs frequency for a sample reproducing head. The head is resonated with different low-loss capacitors. Note especially that R (the effective parallel resistance, which is of course also the impedance at resonance) vs frequency appears to have a slope of 2 db/oct. Two db/oct variation of R may also be expressed as 1 db/oct

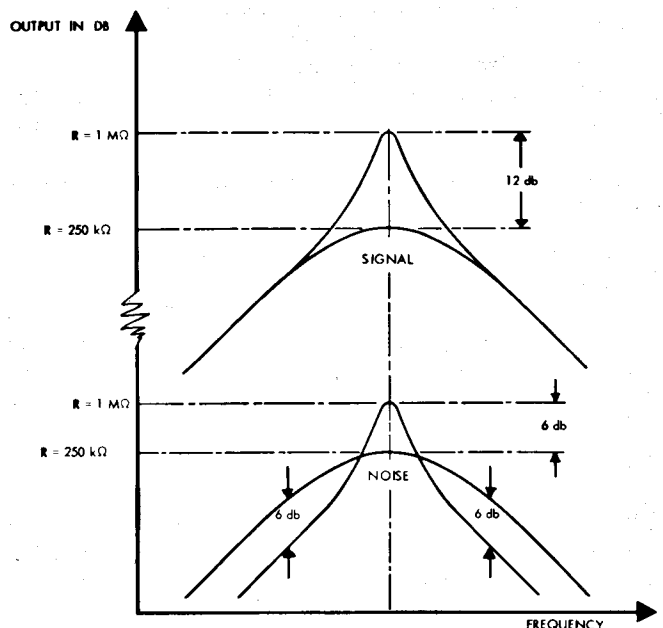


FIG. 12. Signal and noise frequency responses of a tape reproduce head with eddy current losses (R) as parameter.