Speaker distortion sensitivity under voltage and current drive.

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1. Voice coil velocity as a function of voltage or current

With reference to the linear speaker model depicted at in **Fig.1**, Appendix, we may write:

$$F = uZ_M \qquad [1.1]$$

Where *F* represents force on the voice coil, *u* its velocity, and Z_M the mechanical impedance of the voice coil, cone and suspension.
The electrical to mechanical energy conversion for its part, is performed by the motor assembly represented by a transformer with the difference that turns ratio *Bl* relates voltage to velocity and current to force as:

$$F = iBl \text{ and} \qquad [1.2]$$
$$V = uBl \qquad [1.3]$$

For the voltage drive mode, *V* results from the voltage divider formed by the voice coil series resistance and inductance lumped as $Z_S = R_e + sL_e$, and the equivalent electrical impedance as seen before the motor transformer. This impedance Z_E is derived in the Appendix.

$$V = V_i \frac{Z_E}{Z_S + Z_E} = V_i \frac{\frac{Bl^2}{Z_M}}{Z_S + \frac{Bl^2}{Z_M}} = V_i \frac{1}{\frac{Z_S Z_M}{Bl^2} + 1} \qquad [1.4]$$

So voice coil velocity *u* is obtained combining the above with [1.3]:

$$u = V_i \frac{1}{Bl} \frac{1}{\frac{Z_S Z_M}{Bl^2} + 1} = V_i \frac{1}{\frac{Z_S Z_M}{Bl} + Bl} \qquad [1.5]$$

For the current mode drive for its part, simply combining [1.1] and [1.2] we get:

$$u = i \frac{Bl}{Z_M} \qquad [1.6]$$

For the linear speaker model where parameters are constant, differences between voltage drive mode and current drive mode are:

For voltage mode, frequency response is determined by the voltage divider frequency behavior, which is more noticeable at the higher end. At low frequencies the series branch is less significant and voice coil velocity *u* follows input voltage more closely.

For current mode, voice coil velocity is strictly given by the mechanical impedance frequency dependence, so if the operating region includes speaker resonance, it will be strongly reflected.

2. Sensitivity definitions

We now address the issue of how, given the fact speaker parameters are not constants but dependent on temperature and voice coil position, voice coil velocity *u* deviates from its linear relationship with voltage and current.
One way of assessing the impact of variation of nominally constant parameters, is what is known as sensitivity analysis. The idea is to evaluate, given a certain variation of a selected parameter, how much this affects the output. Numerically this is expressed as the quotient of the output relative error divided by the parameter relative variation, that is, if:

$$u = u(p_1, p_2 ...) \qquad [2.1]$$

we define the sensitivity of *u* with respect to parameter p_1 as:

$$S_{p_1}^{u} = \frac{\Delta u / u}{\Delta p_1 / p_1} \qquad [2.2]$$

Being all other parameters held constant. This makes sense, the numerator is the relative deviation experienced by the (ideally constant) output, for a given variation of the (ideally constant also) parameter, note [2.2] can be put as:

$$\Delta u / u = S_{p_1}^{u} \, \Delta p_1 / p_1 \qquad [2.3]$$

For example, if sensitivity to p_1 were 10%, then a 1% variation of p_1 should be reflected as a 0.1% variation in *u*. The larger the sensitivity as calculated, the more output will be perturbed by variations of the parameter to blame, so from the standpoint of linearity we want all sensitivities to be as small a possible. With some informal manipulations, [2.2] can be more conveniently expressed as:

$$S_{p_1}^{u} = \frac{du / u}{dp_1 / p_1} = \frac{\partial u}{\partial p_1} \frac{p_1}{u} \qquad [2.4]$$

That is, sensitivity can be evaluated by taking the (partial) derivative of output *u* with respect to the relevant parameter and this is what we are doing next.

3. Voltage mode sensitivity

From [1.5], we deduce we must study sensitivity of *u* with respect to *Bl*, Z_M and Z_S. For *Bl* we have:

$$S_{Bl}^{u} = \frac{\partial u}{\partial Bl}\frac{Bl}{u} = \frac{\frac{Z_M Z_S}{Bl^2} - 1}{\frac{Z_M Z_S}{Bl^2} + 1} \qquad [3.1]$$

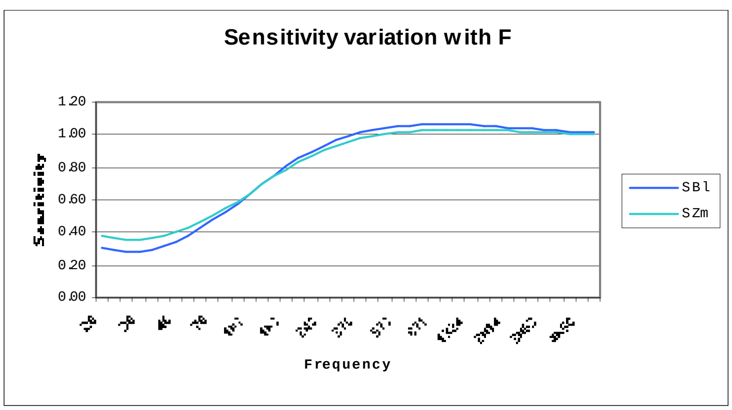
For Z_M in turn:

$$S_{Z_M}^{u} = \frac{\partial u}{\partial Z_M}\frac{Z_M}{u} = -\frac{\frac{Z_M Z_S}{Bl^2}}{\frac{Z_M Z_S}{Bl^2} + 1} \qquad [3.2]$$

And for Z_S by symmetry:

$$S_{Z_S}^{u} = \frac{\partial u}{\partial Z_S}\frac{Z_S}{u} = -\frac{\frac{Z_M Z_S}{Bl^2}}{\frac{Z_M Z_S}{Bl^2} + 1} = S_{Z_M}^{u} \qquad [3.3]$$

All sensitivities are complex frequency dependent magnitudes, about which we are interested in evaluating their modules within the frequency range of interest.
We now take as example the Scan Speak driver used earlier for composite 2-way impedance computation. For the woofer model 18W8545K we have:



Mms = 0.0205 Kg
Cms = 0.001576 m/N
Rms = 6.86 N/s
Bl = 8.2 N/A
Re = 5.5 ohm
Le = 0.4 mHy

From *Mms*, *Cms* and *Rms*, we compute Z_M from [A.1], and from *Re* and *Le* we get Z_S.

Putting all together, we obtain the above graph. Interestingly, sensitivity is lower at low frequencies which are the most troublesome in that both cone excursion and voice coil instantaneous temperature variations are most significant.

4. Current mode sensitivity

We could do the same now with [1.6], but we can advance by inspection that sensitivity to *Bl* and Z_M should be unity, for *u* is directly or inversely proportionally related to each, while it should be 0 with respect to Z_S for it does not appear in the expression for *u*.

$$S_{Bl}^{u} = \frac{\partial u}{\partial Bl}\frac{Bl}{u} = 1 \qquad [4.1]$$

For Z_M:

$$S_{Z_M}^{u} = \frac{\partial u}{\partial Z_M}\frac{Z_M}{u} = -1 \qquad [4.2]$$

And for Z_S:

$$S_{Z_S}^{u} = 0 \qquad [4.3]$$

Comparing this with the variable values plotted above, it can be seen there is no significant advantage from the standpoint of sensitivity for current mode drive at least for this particular unit. On the contrary, to the benefit of a better low frequency behavior as indicated earlier, the voltage mode lower low frequency sensitivity to variations in *Bl* and Z_M is added, while there remains a sensitivity to Z_S not present in the current mode that eventually may be troublesome at high power levels.

Appendix

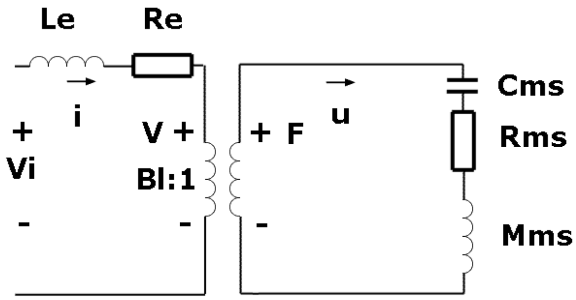


Fig. 1 Speaker electromechanical linear model

Fig 1 depicts a typical speaker electromechanical model. voice coil velocity relates with force, through moving mass, frictional losses and suspension compliance. The electrically equivalent concepts of inductance, resistance and capacitance are included in the model to signify the homology among mechanical and electrical equations. Mass exerts an inertial force according to Newton's law proportional to the derivative of speed (acceleration) while elasticity contributes with a force proportional to the integral of speed (displacement), and friction in turn counteracts with a force proportional to speed. All this forces are additive, and following standard Laplace transform, total mechanical impedance Z_M can be represented as a series combination expressed as electrical equivalents:

$$F = uZ_M = u\left[R_{MS} + sM_{MS} + \frac{1}{sC_{MS}}\right] \qquad \text{[A1]}$$

Solving for electrical impedance *V/i* from [2] , [3] and [A1] we get:

$$Z_E = \frac{V}{i} = \frac{Bl^2}{Z_M} = Bl^2 \frac{1}{R_{MS} + sM_{MS} + \frac{1}{sC_{MS}}} \qquad \text{[A2]}$$

For reasons that will be readily apparent, we can rewrite [A2] as:

$$Z_E = \frac{1}{\frac{R_{MS}}{Bl^2} + s\frac{M_{MS}}{Bl^2} + \frac{1}{sC_{MS}Bl^2}} \qquad \text{[A3]}$$

Recognizing now the denominator (with dimensions of admittance) as a parallel resonant circuit, we finally define the electrical equivalent of mechanical components as:

$R_{ES} = \frac{Bl^2}{R_{MS}}$, $C_{MES} = \frac{M_{MS}}{Bl^2}$ and

$L_{CES} = Bl^2 C_{MS}$

[A4]

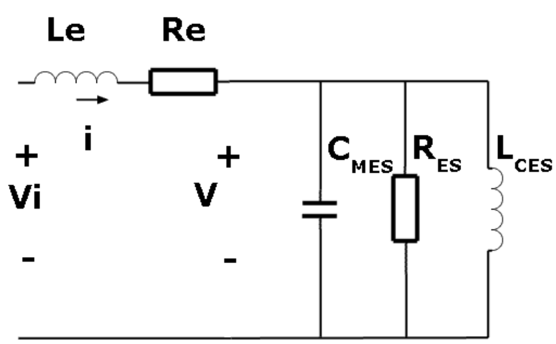


Fig. 2 Speaker electrical linear model

The final electrical equivalent circuit of **Fig 2** results from adding in series the voice coil inductance and resistance L_E and R_E . Enclosure loading can at this point be added as further circuit components, but they are not relevant for the objective we are dealing with.