

RIAA correction with a DC servo loop

Version 1, Marcel van de Gevel, 3 December 2023

1. Introduction

When you put a DC feedback loop (DC servo) around a RIAA preamplifier, the feedback will shift the poles, especially the pole with the largest time constant. The zero stays exactly where it was.

The shift of the poles causes the RIAA correction to become inaccurate. There are several ways around this, such as:

1. Design the DC servo to have practically no loop gain left at 50 Hz and above, say magnitude of the loop gain ≤ 0.01 if you are willing to accept an error of the order of 1 %. For a first-order loop, this corresponds to a ≤ 0.5 Hz cut-off frequency, so the loop provides almost no subsonic filtering.
2. Precorrect the poles of the RIAA correction network, so they end up where they should after the DC feedback is applied.
3. Put zeros in the DC feedback network on top of the RIAA correction poles to keep them from shifting, that is, put an inverse RIAA network in the DC feedback loop.

Options 1 and 2 will be treated in this document for the simplest case: an amplifier with RIAA correction with a DC feedback consisting of one integrator (or one integrator and a frequency-independent attenuator). There is nothing wrong with option 3, but it is nonetheless outside the scope of this document.

By the way, passive RIAA correction with AC coupling leads to rather similar problems, but with more complicated equations. See <https://www.diyaudio.com/community/threads/tube-mc-phono-stage-riaa-calculator-formula-help.401437/post-7439558>

2. Precorrecting the poles

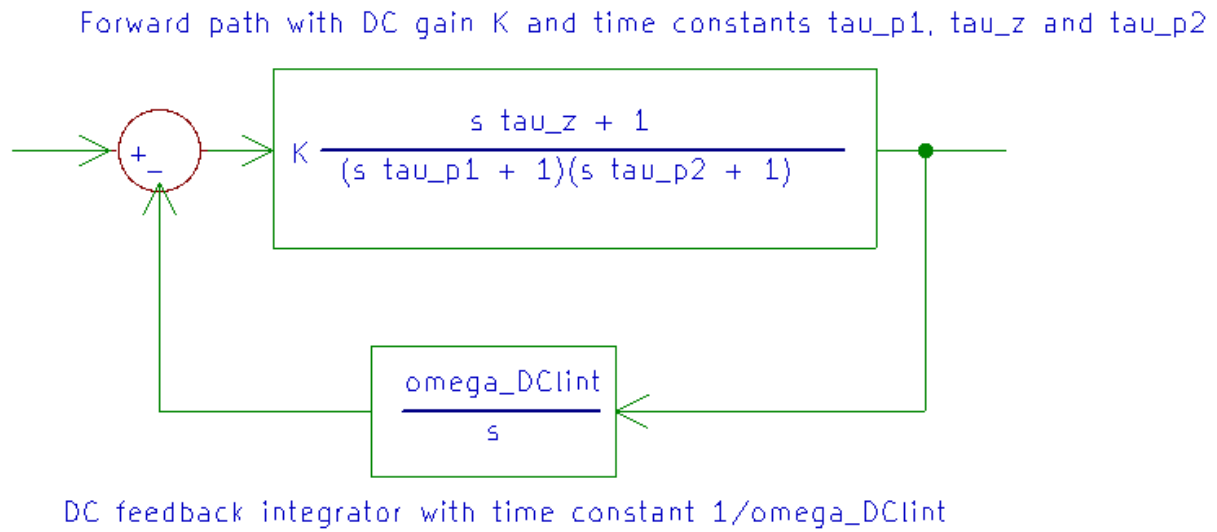


Figure 1: RIAA correction amplifier with a simple DC servo loop around it

In Figure 1, a RIAA correction amplifier with DC gain K gets a DC feedback around it via an integrator with radian bandwidth ω_{DCint} , that is, with a time constant $1/\omega_{DCint}$. If the DC feedback loop is to serve as a subsonic filter, a higher-order loop may be preferable, but as simple DC servos are quite popular and lead to the simplest equations, I assume this simple structure. (It should be noted that one can get a third-order Butterworth subsonic response by cascading the whole thing with a second-order filter with a quality factor of 1.)

In some designs, the correction signal of the DC feedback loop is not injected straight at the input, like in Figure 1, but somewhere after the first stage. As long as the gain from the input of the first stage to the injection point is flat over the audio band, one can calculate the correction signal back to an equivalent correction signal injected at the input by dividing it by that gain. As a result, ω_{DCint} then scales down with the gain. For example, when the integrator has a time constant of 0.1 s and the gain from the input to the injection point is 20, it is equivalent to having an integrator with a time constant of 2 s injecting straight at the input, so $\omega_{DCint} = 0.5$ rad/s. A disadvantage of injecting the correction signal somewhere after the first stage is that the open-loop inaccuracy and non-linearity of the first stage and of whatever other stages there are between the input and the injection point affect the bandwidth and linearity of the DC loop.

It should also be noted that instead of using an integrator circuit with an RC time equal to $1/\omega_{DCint}$, one can also use a shorter RC time and an attenuator between the integrator and the subtraction point. Advantages are that you can then get away with smaller capacitors and that the noise of the integrator is attenuated. A disadvantage is the reduced offset and subsonic signal handling, but considering how small cartridge signals are, you can use a lot of attenuation before that is likely to become a problem.

Straightforward feedback theory shows that the transfer function of Figure 1 is

$$H(s) = \frac{K \frac{s\tau_z + 1}{(s\tau_{p1} + 1)(s\tau_{p2} + 1)}}{1 + K\omega_{\text{DClint}} \frac{s\tau_z + 1}{s(s\tau_{p1} + 1)(s\tau_{p2} + 1)}}$$

which can also be written as

$$H(s) = \frac{1}{\omega_{\text{DClint}}} \cdot \frac{s(s\tau_z + 1)}{s^3 \frac{\tau_{p1}\tau_{p2}}{K\omega_{\text{DClint}}} + s^2 \frac{\tau_{p1} + \tau_{p2}}{K\omega_{\text{DClint}}} + s\left(\tau_z + \frac{1}{K\omega_{\text{DClint}}}\right) + 1}$$

When the desired time constants of the poles are $\tau_{p\text{RIAA1}}$, $\tau_{p\text{RIAA2}}$, and $\tau_{p\text{subs}}$, the desired denominator of the second factor (desired characteristic polynomial) is

$$(s\tau_{p\text{RIAA1}} + 1)(s\tau_{p\text{RIAA2}} + 1)(s\tau_{p\text{subs}} + 1) = s^3 \tau_{p\text{RIAA1}} \tau_{p\text{RIAA2}} \tau_{p\text{subs}} + s^2 (\tau_{p\text{RIAA1}} \tau_{p\text{RIAA2}} + \tau_{p\text{RIAA1}} \tau_{p\text{subs}} + \tau_{p\text{RIAA2}} \tau_{p\text{subs}}) + s(\tau_{p\text{RIAA1}} + \tau_{p\text{RIAA2}} + \tau_{p\text{subs}}) + 1$$

The trick is to choose τ_{p1} , τ_{p2} and ω_{DClint} such that the characteristic polynomial becomes what it should be. You can do that by equating the coefficients of the polynomials, the actual and the desired ones.

That is,

$$\tau_z + \frac{1}{K\omega_{\text{DClint}}} = \tau_{p\text{RIAA1}} + \tau_{p\text{RIAA2}} + \tau_{p\text{subs}}$$

so ω_{DClint} must be

$$\omega_{\text{DClint}} = \frac{1}{K} \cdot \frac{1}{\tau_{p\text{RIAA1}} + \tau_{p\text{RIAA2}} + \tau_{p\text{subs}} - \tau_z}$$

Regarding τ_{p1} and τ_{p2} , to keep the equations managable, I introduce

$$x = \tau_{p1} + \tau_{p2}$$

and

$$y = \tau_{p1} \tau_{p2}$$

Hence,

$$x = K\omega_{\text{DClint}} (\tau_{p\text{RIAA1}} \tau_{p\text{RIAA2}} + \tau_{p\text{RIAA1}} \tau_{p\text{subs}} + \tau_{p\text{RIAA2}} \tau_{p\text{subs}})$$

or, equivalently,

$$x = \frac{\tau_{p\text{RIAA1}} \tau_{p\text{RIAA2}} + \tau_{p\text{RIAA1}} \tau_{p\text{subs}} + \tau_{p\text{RIAA2}} \tau_{p\text{subs}}}{\tau_{p\text{RIAA1}} + \tau_{p\text{RIAA2}} + \tau_{p\text{subs}} - \tau_z}$$

and

$$y = K\omega_{\text{DClint}} \tau_{p\text{RIAA1}} \tau_{p\text{RIAA2}} \tau_{p\text{subs}}$$

or, equivalently,

$$y = \frac{\tau_{p\text{RIAA1}} \tau_{p\text{RIAA2}} \tau_{p\text{subs}}}{\tau_{p\text{RIAA1}} + \tau_{p\text{RIAA2}} + \tau_{p\text{subs}} - \tau_z}$$

In the limit for $\tau_{p\text{subs}} \rightarrow \infty$, x approaches $\tau_{p\text{RIAA1}} + \tau_{p\text{RIAA2}}$, and y approaches $\tau_{p\text{RIAA1}}\tau_{p\text{RIAA2}}$, which simply confirms that you don't need to precorrect the poles when the DC loop is very slow.

Anyway, as

$$x = \tau_{p1} + \tau_{p2}$$

and

$$y = \tau_{p1} \tau_{p2}$$

we have

$$\tau_{p2} = x - \tau_{p1}$$

$$\tau_{p1}(x - \tau_{p1}) = y$$

$$\tau_{p1}(\tau_{p1} - x) + y = 0$$

$$\tau_{p1}^2 - x\tau_{p1} + y = 0$$

$$\tau_{p1} = \frac{x \pm \sqrt{x^2 - 4y}}{2}$$

and

$$\tau_{p2} = \frac{x \mp \sqrt{x^2 - 4y}}{2}$$

Example 1:

Suppose we have a DC gain $K = 1000$ and want a DC servo with a cut-off frequency of 0.5 Hz:

$$\tau_{p\text{subs}} \approx 0.3183098862 \text{ s}$$

$$\tau_{p\text{RIAA1}} = 3.18 \text{ ms}$$

$$\tau_z = 318 \text{ } \mu\text{s}$$

$$\tau_{p\text{RIAA2}} = 75 \text{ } \mu\text{s}$$

$$K = 1000$$

from which one can calculate that

$\omega_{\text{DCint}} \approx 0.003112870639 \text{ rad/s}$, corresponding to an integrator time constant of more than five minutes, namely 321.2468862 s. Fortunately, one can also use an RC time of, for example, 3.212468862 s and attenuate the integrator output 100 times before subtracting it at the input.

$$x \approx 0.003225983579 \text{ s}$$

$$y \approx 2.363195135 \bullet 10^{-7} \text{ s}^2$$

$$\tau_{p1} \approx 3.150984962 \text{ ms (0.9124225806 \% less than the ordinary 3.18 ms)}$$

$$\tau_{p2} \approx 74.99861673 \text{ }\mu\text{s (only 0.0018443545 \% less than the ordinary 75 }\mu\text{s)}$$

Failing to correct for the shifting poles will therefore cause an error of less than 1 %.

Example 2:

Suppose we have a DC gain $K = 1000$ and want a DC servo with a cut-off frequency of 10 Hz:

$$\tau_{p\text{subs}} \approx 15.9154943 \text{ ms}$$

$$\tau_{p\text{RIAA1}} = 3.18 \text{ ms}$$

$$\tau_z = 318 \text{ }\mu\text{s}$$

$$\tau_{p\text{RIAA2}} = 75 \text{ }\mu\text{s}$$

$$K = 1000$$

from which one can calculate that

$\omega_{\text{DCint}} \approx 0.05304337896 \text{ rad/s}$, corresponding to an integrator time constant of more than five minutes, namely 18.85249431 s. Fortunately, one can still use a shorter RC time constant and attenuate the integrator output before subtracting it at the input.

$$x \approx 0.002760559591 \text{ s}$$

$$y \approx 2.013444656 \bullet 10^{-7} \text{ s}^2$$

$$\tau_{p1} \approx 2.685587364 \text{ ms (15.54756718 \% less than the ordinary 3.18 ms)}$$

$$\tau_{p2} \approx 74.97222707 \text{ }\mu\text{s (only 0.03703057768 \% less than the ordinary 75 }\mu\text{s)}$$

That is, due to the effect of the 10 Hz DC servo loop, the largest time constant of the RIAA correction network will therefore have to be reduced by over 15 %.