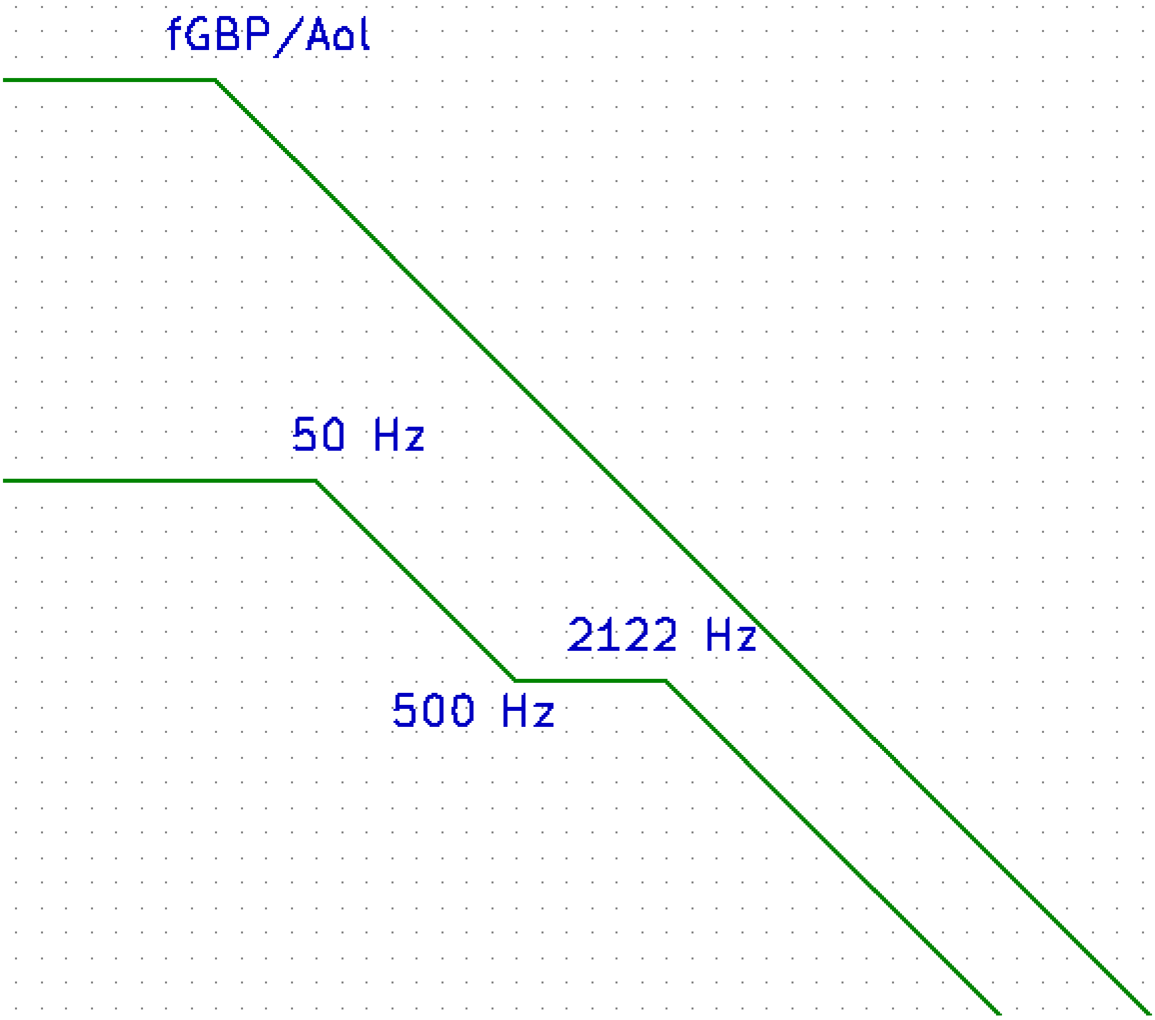
Approximately correcting for finite op-amp gain-bandwidth product in a single-loop RIAA amplifier

In one of the many phono preamplifier threads, EdGr pointed out that op-amp open-loop gain is not necessarily high enough to be considered infinite. This is particularly true at higher audio frequencies, where the open-loop gain drops due to the finite gain-bandwidth product. The gain-bandwidth product will normally not be accurately known anyway, as it depends on a fairly inaccurate on-chip capacitor and on the transconductance of the op-amp input stage, so an approximate correction should be good enough.

The Bode asymptotes of the open-loop gain of the operational amplifier look something like the upper trace in this plot, while the Bode asymptotes of the desired closed-loop gain look something like the lower trace. Although no scales have been drawn, the vertical scale is supposed to be a decibel scale and the horizontal scale a logarithmic frequency scale. Unless otherwise noted, the assumption will be made that the open-loop corner frequency of the op-amp f_{GBP}/A_{ol} lies below 50 Hz, as is often the case. f_{GBP} is the op-amp's gain-bandwidth product and A_{ol} its open-loop gain at 0 Hz.



The zero with a corner frequency of approximately 500 Hz (that is, the zero at s = -1/(318 µs)) is actually not affected at all by the finite op-amp gain-bandwidth product. In a single-loop phono amplifier, this zero is realized by putting a pole at s = -1/(318 µs) in the feedback network. Due to this pole, there is infinite feedback at s = -1/(318 µs), which makes the input-to-output transfer of the phono amplifier zero at s = -1/(318 µs), no matter whether the gain of the forward path is finite or infinite.

The distance between the upper and lower traces corresponds to the magnitude expressed in dB of the loop gain of the feedback amplifier, that is, of the open-loop gain divided by the attenuation of the feedback network. (Any effect of source and load impedances is implicitly assumed to be accounted for in the open-loop gain.) It is clear from the plot that the magnitude of the loop gain is more or less independent of frequency between 50 Hz and 500 Hz and above 2122 Hz, and that the magnitude of the loop gain above 2122 Hz is approximately 4.244 times smaller than between 50 Hz and 500 Hz.

As the magnitude of the loop gain is at its lowest above 2122 Hz, the error you get when you designed the feedback network assuming infinite loop gain will be the greatest above 2122 Hz. Denoting the (usually negative) loop gain as $A\beta$, the gain will (as usual) be $-A\beta/(1 - A\beta)$ times the reciprocal of the transfer of the feedback network, so $-A\beta/(1 - A\beta)$ times the desired gain when the feedback is designed for infinite loop gain.

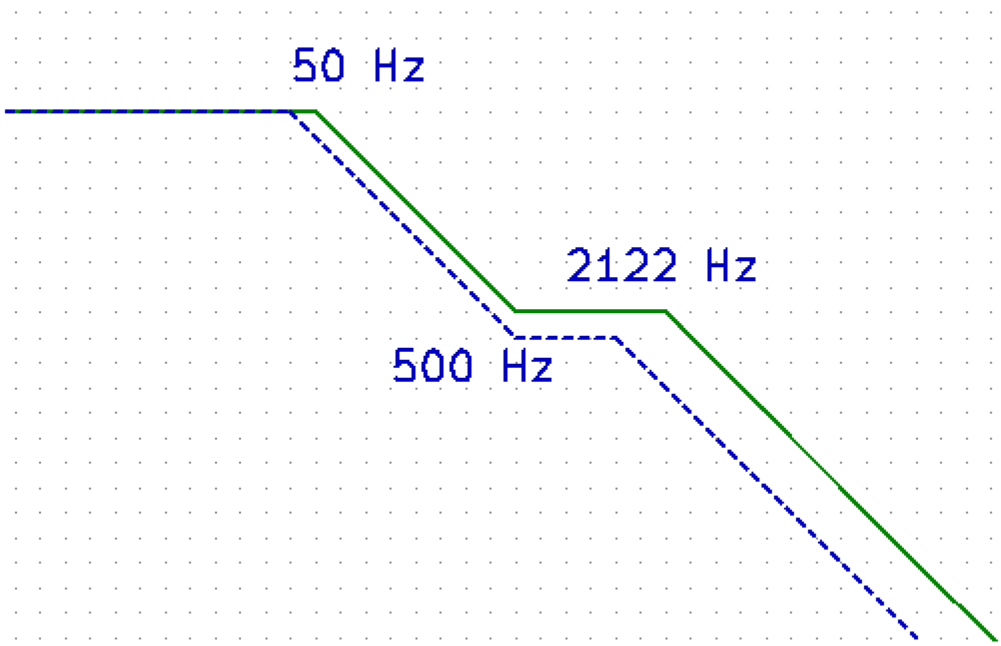
Practical example:
Suppose you want to make a phono amplifier with a gain of 200 (46 dB) at 1 kHz using an OPA134 with a gain-bandwidth product of 8 MHz.

The attenuation of the feedback network of 200 times at 1 kHz stays roughly constant up to 2122 Hz and then rolls off at a first-order rate, so the magnitude of the transfer of the feedback network is $f/(200 \bullet 2122\ \text{Hz})$ at audio frequencies above 2122 Hz.

The magnitude of the open-loop gain of an 8 MHz gain-bandwidth product op-amp is 8 MHz/f for frequencies in the region where the open-loop gain drops at a first-order rate.

Hence, when you use an OPA134 for a single-loop RIAA-corrected amplifier with a gain of 200 at 1 kHz, you have a magnitude of the loop gain of 8 MHz/(200 • 2122 Hz) ≈ 19 at audio frequencies above 2122 Hz. The phase shift of the open-loop gain is approximately -90º and the phase shift of the feedback network is approximately +90º well above 2122 Hz, so these cancel. As we apply negative feedback, $A\beta \approx -19$ and $-A\beta/(1 - A\beta) \approx 0.95$, resulting in a 5 % too low gain above 2122 Hz. (The error would have been smaller with a lower midband gain and/or a faster op-amp.)

The percentage error between 50 Hz and 500 Hz is 4.244 times smaller than above 2122 Hz when the open-loop corner frequency is zero. That is, when the open-loop corner frequency is zero and A_{ol} is therefore infinite (to keep the gain-bandwidth product constant), there will be a 0 % error at 0 Hz, -5 %/4.244 ≈ -1.178 % error between 50 Hz and 500 Hz and a -5 % error above 2122 Hz. Knowing that the zero doesn't move at all, this must mean that corner frequency of the first pole is about 1.178 % too low, while the second pole's corner frequency is about 5 % - 1.178 % = 3.822 % too low.



When the open-loop corner frequency is non-zero and, hence, A_{ol} is finite, you actually get a compensation effect. The gain at 0 Hz is then already too low. In fact, the first RIAA correction pole will be spot on when the open-loop corner frequency happens to be equal to the RIAA corner frequency of 50.0487... Hz. The low-frequency gain is then as much off as the gain between 50 Hz and 500 Hz, so the corner frequency of the first pole ends up precisely where it should.

All in all, the finite gain-bandwidth product normally mostly affects the second RIAA pole that is supposed to lie at s = -1/(75 μs) = -13.3333... krad/s, with a corner frequency of approximately 2122 Hz. Assuming an open-loop bandwidth of somewhere between 0 and 50 Hz, the percentage shift of the first pole can vary between 0 % and 4.244 times smaller than the percentage shift of the second pole.

Calling the gain at 1 kHz K, the magnitude of the loop gain above 2122 Hz is $f_{GBP}/(K \cdot 2122\text{ Hz})$ and the error percentage of the closed-loop gain above 2122 Hz is $-100\,\%/(1 + f_{GBP}/(K \cdot 2122\text{ Hz}))$. The shift of the second pole is only 3.244/4.244 times this value, as just explained, or using the official time constants rather than the rounded frequencies, (318 - 75)/318 = 243/318 times this value.

Hence, the shift of the second pole is

$$-100\,\% \cdot \frac{243}{318\left(1+\dfrac{f_{GBP}}{K \cdot 2122\,\text{Hz}}\right)}$$

and the shift of the first pole is somewhere between 0 and

$$-100\,\% \cdot \frac{75}{318\left(1+\dfrac{f_{GBP}}{K \cdot 2122\,\text{Hz}}\right)}$$

The shift of the zero is precisely 0 %.

You can precorrect for these errors by reducing the time constants of the feedback network by the same percentages, that is:

$$75\,\mu\mathrm{s}\cdot\left(1-\frac{243}{318\left(1+\frac{f_{\mathrm{GBP}}}{K\cdot 2122\,\mathrm{Hz}}\right)}\right)$$

318 µs

$$3180\,\mu\mathrm{s}\cdot\left(1-\frac{75}{318\left(1+\frac{f_{\mathrm{GBP}}}{K\cdot 2122\,\mathrm{Hz}}\right)}\right)$$

Single-loop amplifier with subsonic filter

For the single-loop amplifier with subsonic filter of https://www.diyaudio.com/community/threads/single-stage-active-riaa-correction-with-second-or-third-order-butterworth-high-pass-included.413649/ , no exact procedure has been found to size the feedback network to get the desired time constants, so you have to iterate anyway.

According to the approximate equations given in the thread, without precorrection for finite gain-bandwidth product,

$(R_8 + R_9)(C_5 + C_6) \approx 318$ µs

$(R_8 + R_9)C_6 \approx 75$ µs

so

$C_5/C_6 \approx$ 318 µs/75 µs - 1 = 3.24

which is very close to 22/6.8, a ratio of two E6 values. However, experiments with a pole-zero extraction program show that with infinite loop gain at all frequencies, a ratio of 6.8/2.2 = 3.09090909... usually leads to more accurate pole positions than the theoretical ratio of 3.24. This must be due to everything that was neglected when deriving the approximate equations.

When the 75 µs is reduced to

$$75\,\mu\mathrm{s}\cdot\left(1-\frac{243}{318\left(1+\frac{f_{\mathrm{GBP}}}{K\cdot 2122\,\mathrm{Hz}}\right)}\right)$$

to correct for finite gain-bandwidth product, you can reduce C_6 to

$$C_6 = C_{6\text{, infinite GBP}}\cdot\left(1-\frac{243}{318\left(1+\frac{f_{\mathrm{GBP}}}{K\cdot 2122\,\mathrm{Hz}}\right)}\right)$$

and add the capacitance that you removed from C_6 to C_5 to keep the factor $C_5 + C_6$ in the expression for the zero the same.

This will probably lead to inconvenient values for the capacitors. You can then try to scale the impedance of the whole feedback network to get back to convenient values again. It should be noted that some E6 values have ratios just above 6.8/2.2 = 3.090909..., such as 4.7/1.5 = 3.13333... and 15/4.7 = 3.19148936...