

REACTANCE ANNULING FOR HORN LOUDSPEAKERS

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Loudspeaker performance can be improved by the proper choice of horn flare and driver resonance.

Sound pressure measurements being made on 3-channel Triplex in treated room.

TODAY the most important loudspeakers commercially are of the moving voice coil type, with permanent magnet or field coil excitation. They are used in two ways—as direct radiators and as horn-loaded radiators. Direct radiation is the term applied to a speaker with a cone which moves the air directly. Horn loading utilizes an expanding tube between the speaker and the air to match the normally high impedance of the speaker to the relatively low impedance of air. With the construction materials available at this stage of the art, it is not possible to design speakers which will match air impedance and still cover even a moderate frequency range. Thus, the transformerlike action of a horn is necessary for high efficiency performance. In addition, other benefits accrue from proper horn-loading design. The mechanism of operation of the two speaker applications is entirely different in nature. Output of direct radiators is useful only at the fundamental—or lowest—resonant frequency and above. At resonance, the stiffness of the cone suspension becomes equal, and opposite in sign, to the mass of the moving system

plus the mass of the air moved by it. Output thus is greatest at this point, on the basis of watts input. In commercial amplifiers of moderate or good regulation, however, output may be the same as or less than the output at higher frequencies. As speaker impedance rises at resonant frequency roughly in proportion to the magnet size, large magnet speakers draw less power and so the output is less accordingly.

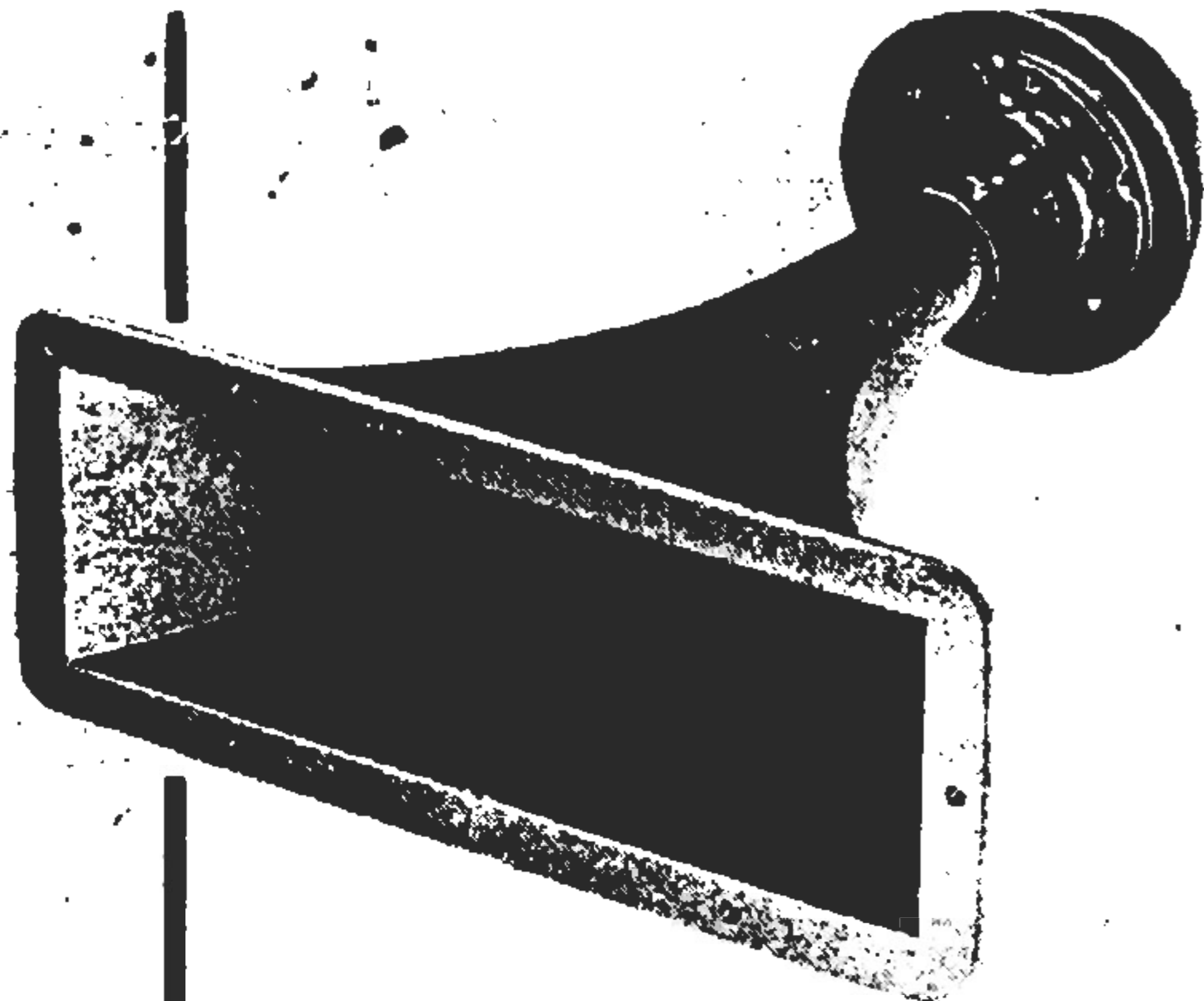
Above the resonant frequency, the moving system is mass-controlled, and the output would drop off at 6 db per octave due solely to this factor. However, there is an equal and compensating rise with frequency. Radiation resistance of the speaker is proportional to the square of the frequency so that the output remains constant. This relationship is valid up to the point where the effective diameter of the cone is equal to λ/π where λ is the wavelength. Beyond the frequency represented by this wavelength, output does drop, with sound pressure being held up somewhat by polar sharpening. The direct radiator is relatively lightly loaded, even so, and its output efficiency is low. The highest efficiency achieved at the present time is about 12%, and that can only be obtained with extra heavy magnets and voice coils. Paper cone direct-radiator speakers are often used with horn loading for low frequency operation. At higher frequencies, however, transition is made to smaller, lighter,

stiffer radiating elements which operate as “pistons” higher in frequency than paper cone speakers.

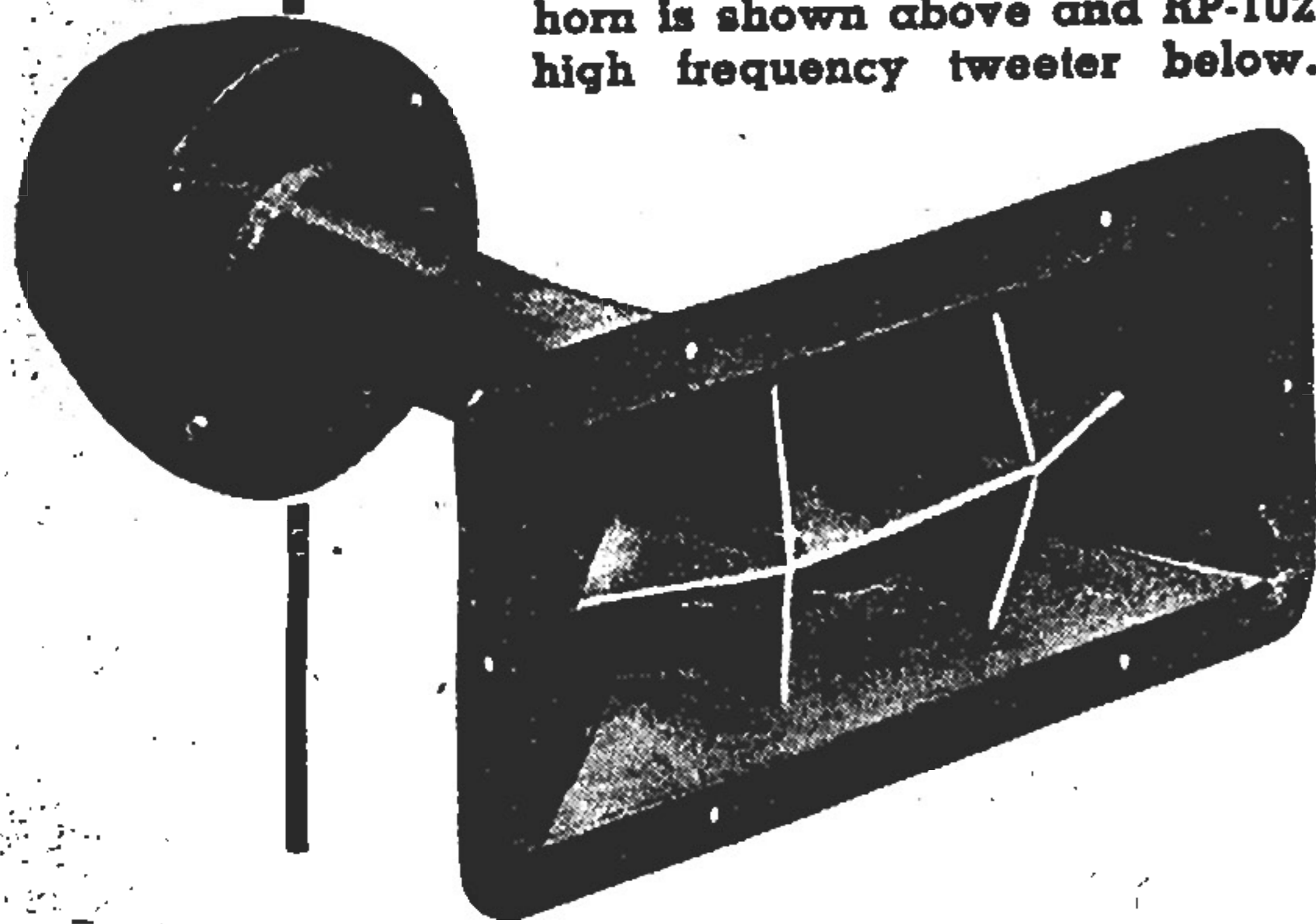
The general equivalent circuit of a horn-type speaker is shown in Fig. 1. On the left-hand side are the electrical elements of the horn unit. E_s is the constant voltage source with its associated internal impedance R_s , and R_v is the resistance of the voice coil. The inductance of the voice coil can be neglected at low frequencies. The transformer in the equivalent circuit represents the coupling between the electrical and mechanical sides. B is the flux density over the winding length of the coil and l is the total conductor length in centimeters. On the right-hand side of the transformer, R_m represents the losses of the mechanical system that are not useful in radiation. S_s is the stiffness of the suspension system, and can include any stiffness added by an external cavity. The mass M_s represents the total effective mass of the moving system and S_c indicates the stiffness of the air chamber between the driver diaphragm or cone and the throat of the horn. For most low frequency applications where the shunt reactance of S_c is sufficiently large, its effect may be neglected. The parameters R_m (resistance) and M_s (mass) of the horn proper are the values as viewed by the driver diaphragm.

At this point, it is illustrative to show the nature of the load presented

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RP-201 mid-channel (600-4000 cps) horn is shown above and RP-102 high frequency tweeter below.



to the diaphragm by the infinite horn, which is an idealized horn long enough so that no mouth reflections occur. A practical horn operates similarly, but with mouth reflections roughening the response if mouth area is too small. The parameters of the horn proper are represented at the right-hand side of Fig. 1 by R_h and M_h .

As seen in the expression for horn air mass in Fig. 1, there is a factor T which determines the type of horn, whether conical, exponential or hyperbolic-exponential. The area expansion for the hyperbolic-exponential series is given by the relationship:

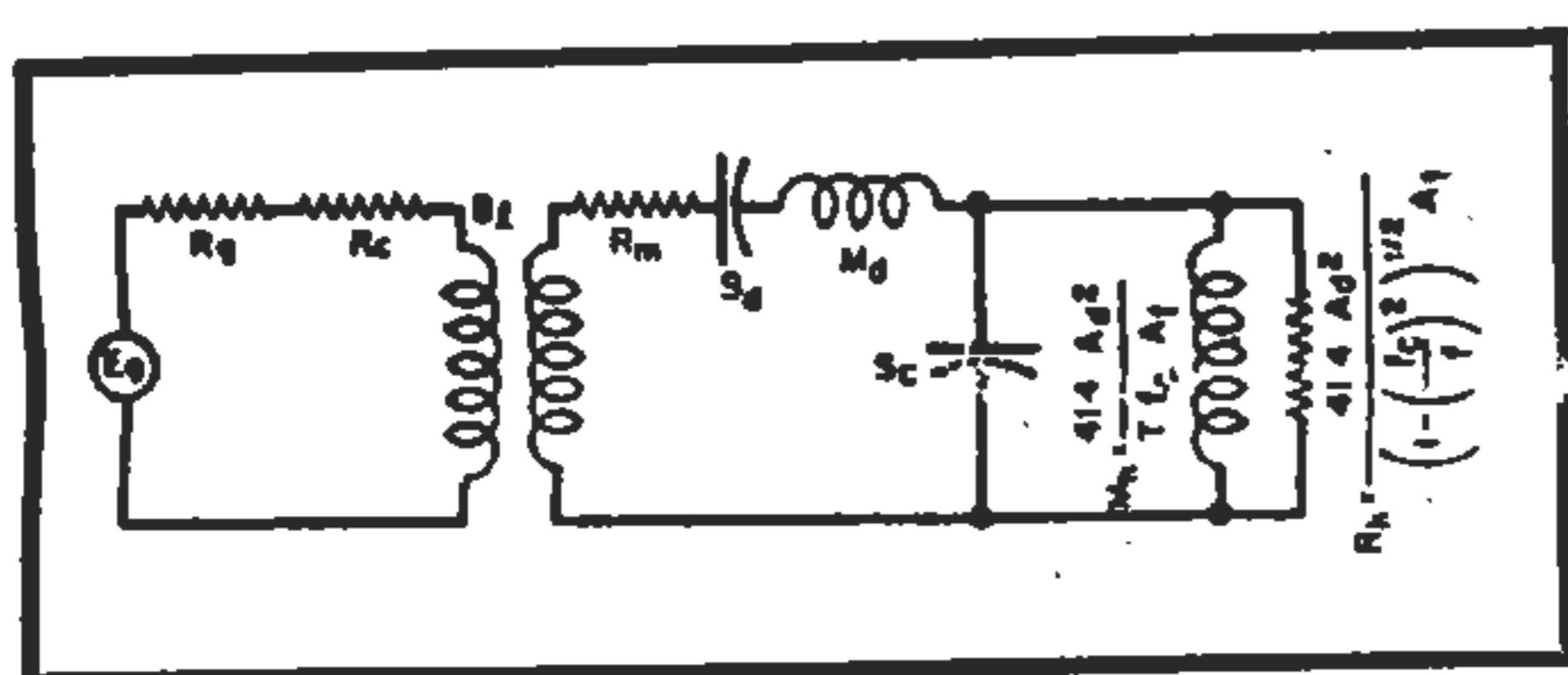
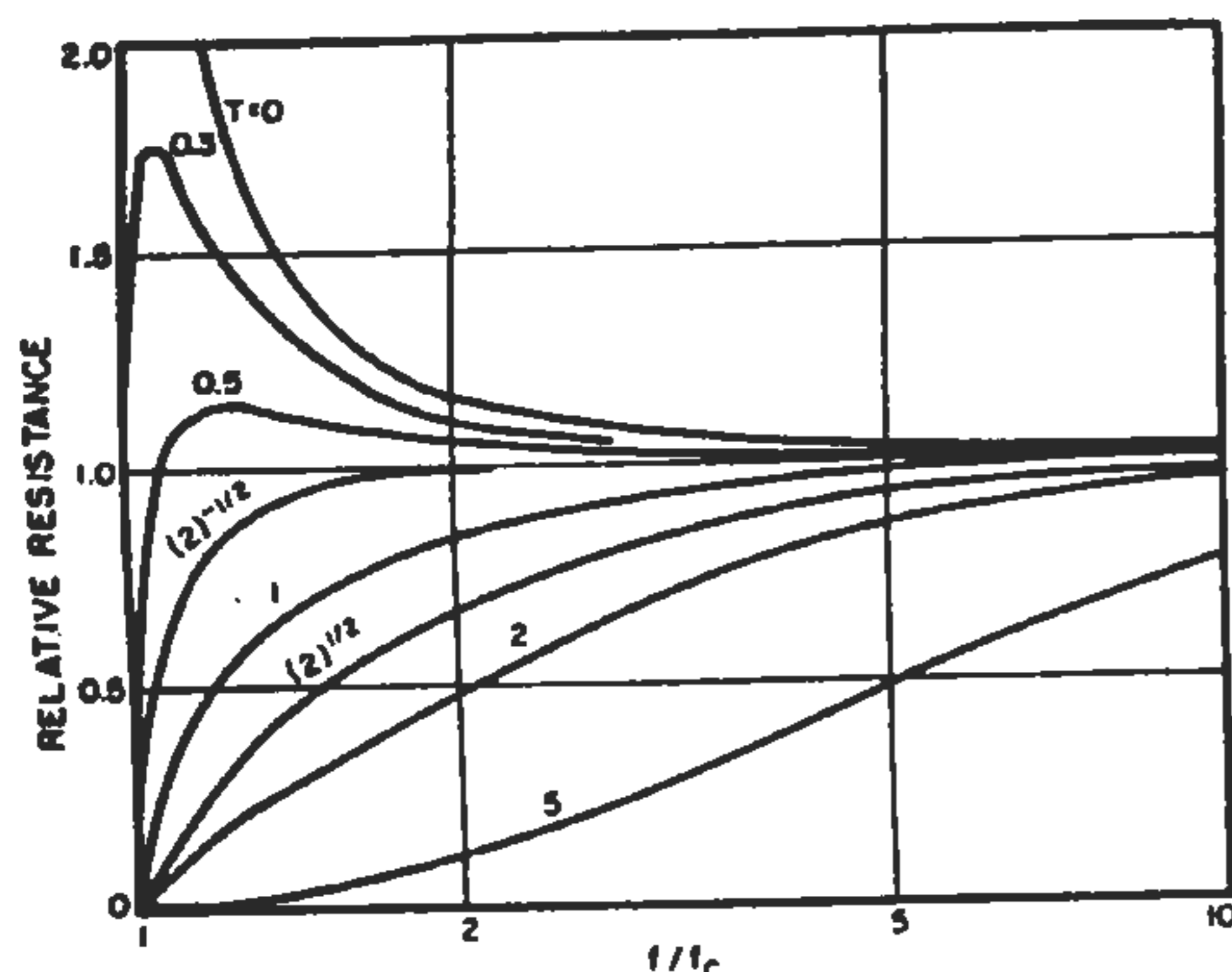


Fig. 1. General equivalent circuit of a horn-loaded loudspeaker.

Fig. 2. Resistive component of speaker impedance vs. frequency for various values of parameter T .



$$A_x = A_0 \left(\cosh \frac{X}{X_0} + T \sinh \frac{X}{X_0} \right)^2 \quad (1)$$

The most generally useful type of horn is designed with a T smaller than unity, as will be shown later.

If the horn parameters M_h and R_h are expanded in series form, the following relationship occurs for horn impedance as viewed by the diaphragm:

$$Z_h = 267 \frac{A_d}{A_0} \left[\frac{\left(1 - \frac{1}{\mu^2}\right)^{1/2} + j \frac{T}{\mu}}{1 - \left(\frac{1 - T^2}{\mu^2}\right)} \right] \quad (2)$$

A plot of the resistive component against the relative frequency (as a ratio of frequency to cutoff frequency), with T as the parameter, is shown in Fig. 2. It can be seen that the T values near 0.5 give the flattest characteristic down closer to cutoff than the other types, this flatness being desirable because output is then more nearly independent of frequency. It is this control of the resistive component of horn impedance that makes horns having values of T less than unity more useful generally than the exponential horn ($T = 1$). Horns with values of T greater than one are seldom used, because of poor throat resistance characteristics. Plane wave theory of infinite horns predicts zero resistance at cutoff as shown by Fig. 2. Actually, however, for finite horns the resistance is not zero, and the useful range of the horn extends to 10% or 20% below cutoff.

Figure 3 shows the reactance characteristics of horns with various T values, including the exponential. Horn reactance is positive or masslike, but differs from the usual concept of an inductance or mass to the extent that its value does not increase steadily with frequency. Instead, the ultimate value approaches zero at high frequencies; and all horns will act the same at sufficiently high frequencies, presenting only resistive loads.

These unusual horn reactance characteristics can be matched to the driver reactance characteristics to get maximum possible efficiency at the lower end of the passband. Such matching is best done by the reactance annulling principle² which treats the driver and horn as a passband system in such a way as to cancel out mutually all or portions of the undesirable—but inevitable—reactive elements always present in the two components. The need for this cancellation effect can be seen from the acoustic output of a horn loudspeaker system:

$$P_a = \frac{(B1)^2 E_d^2 R_h \times 10^{-9}}{[R_c R_h + (B1)^2 \times 10^{-9}]^2 + R_c^2 [\omega (M_d + M_h) - S/\omega]^2} \quad (3)$$

This equation shows that the mode

A_x	= area at point x in inches
A	= throat area in square inches
x	= distance from horn throat in inches
x_0	= $2155/f_c$
f_c	= cutoff frequency
A_d	= effective diaphragm area in square inches
μ	= f/f_c
ω	= $2\pi f$
ω_c	= angular cutoff frequency of horn
f_r	= unloaded driver resonant frequency

Table 1. Definitions of symbols.

of operation of a horn system is different from that of a direct radiator. At medium and higher frequencies, R_h is constant, and with this condition applying, it can be seen that the output rolls off at 6 db per octave because the system is mass-controlled, as a result of the frequency-dependent terms in the denominator. Output independent of frequency is desirable, which in this case implies resistance control, dictating that the frequency-dependent terms be minimized. It is, of course, possible to cancel frequency-dependent terms by reactance annulling at only one frequency. However, their net value is considerably reduced at other frequencies so that the ratio of resistance to reactance is made much more favorable, and efficiency thereby increased. Exact cancellation is placed at the cutoff frequency of the horn, because resistance there is relatively low, and this reactance cancellation is especially important for a favorable ratio. For zero reactance at cutoff frequency:

$$\omega_c (M_d + M_h) - \frac{S_d}{\omega_c} = 0 \quad (4)$$

which can be reduced to the following relation between speaker resonant frequency and horn cutoff frequency, in terms of the system constants:

$$f_r = \left[f_c^2 + \frac{42.7 A_d^2 f_c}{T A_0 M_d} \right]^{1/2} \quad (5)$$

It can be seen from this equation that the speaker resonant frequency must always be larger than the horn cutoff frequency. This relationship, while all-important in horn speaker design, is not generally known.

One of the major differences between the operation of a cone speaker as a direct radiator in contrast to its use as a horn unit driver is pointed up here. As a direct radiator, the cone speaker cannot be operated satisfactorily below its resonant frequency, due to low output and high distortion. As a horn unit driver, it should be operated below resonant frequency over a substantial part of its low frequency range.

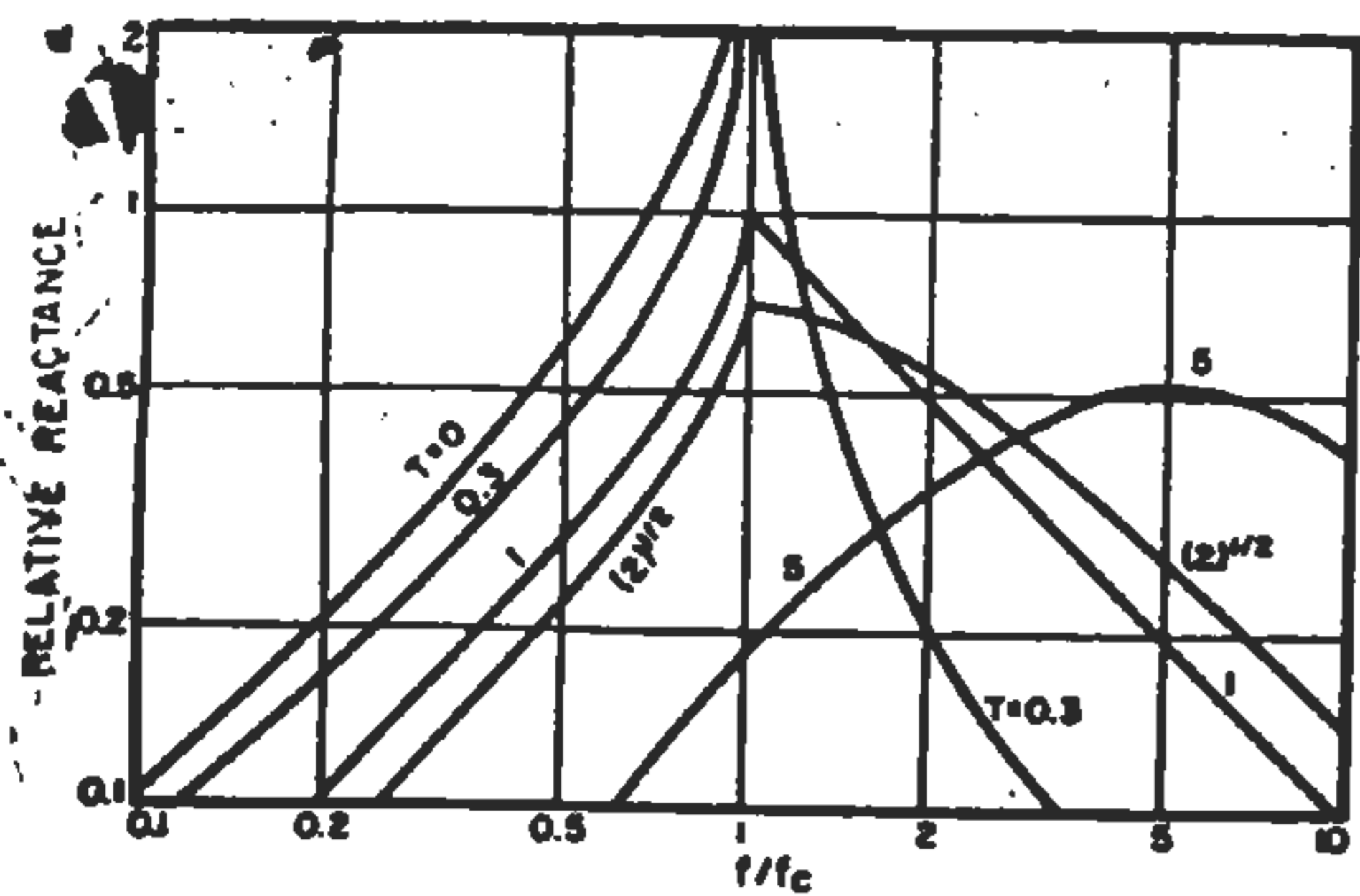


Fig. 3. Reactance characteristics of horns having various values of T .

How this comes about is shown by Fig. 4, which is universal in that it applies to any speaker. Speaker reactance X_s appears as a series resonant circuit, being inductive in nature above resonant frequency, zero at resonant frequency, and capacitive below resonant frequency. In the example, horn reactance is that developed by a flare of $T = 0.7$, as shown by curve X_h . The net—or sum—reactance is X_n , which falls fairly close to a zero value over a very wide operating range, giving a favorable resistance-to-reactance ratio for best efficiency. The ratio of resonance to cutoff frequency, in the case of example given, is 2.5 to 1.

Thus, in the example, the system is essentially resistance-controlled between the horn cutoff frequency and resonant frequency, making it possible to achieve maximum efficiency between these frequencies. If the speaker resonance were placed lower, say at horn cutoff, net reactance at and above cutoff would be relatively large, causing a resultant reduction in efficiency. This condition would be further aggravated, as may be expected from the foregoing, if the driver resonant frequency were moved

still further down so that it fell below horn cutoff.

The practical result of reactance annulling in a middle frequency horn-type speaker is shown in Fig. 5. Curve A represents the sound pressure performance of the unit when the resonant frequency of the moving system is properly placed with respect to cutoff, at about 1100 cycles. Curve B represents sound pressure with resonant frequency placed too low, at about 400 cycles.

Figure 6 is a chart based on Eqt. (5). It indicates the relationship that must exist between the driver resonant frequency and horn cutoff frequency, in terms of the moving system and horn constants, to obtain reactance annulling at horn cutoff frequency.

For a given driver, the moving system mass and effective area are fixed, so the parameters that are variable are the cutoff frequency, throat size, horn T and driver resonant frequency. The cutoff frequency is chosen on the basis of the lowest frequency to be passed, but in some cases a compromise must be made between cutoff frequency and allowable size. The throat size cannot be arbitrarily chosen, as it determines the efficiency and impedance of the horn system. If it is assumed that the driver and horn will be matched so as to minimize the net mechanical reactance between cutoff and driver resonance, the driver appears as a generator having an internal impedance equal to the mechanical losses plus the impedance reflected from the electrical side.

On this basis, the required throat area in square inches is given by:

$$A_t = \frac{267 A_s^2}{R_m + \frac{(B1)^2 \times 10^{-9}}{R_c + R_e}} \quad (6)$$

Reactance and resistance measurements of drivers and horns are made with bridge and vacuum tank.

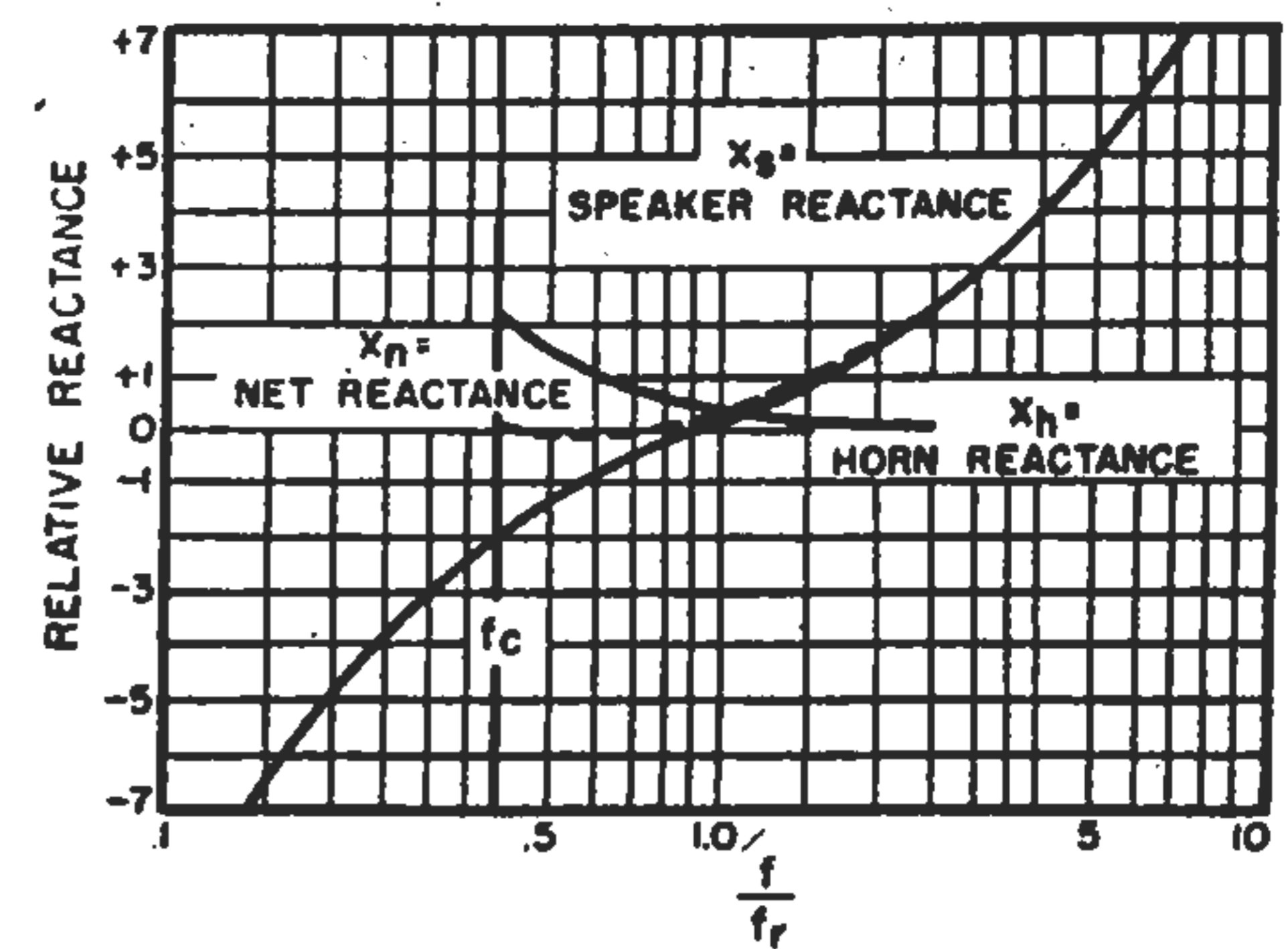


Fig. 4. Relationship between speaker reactance and horn reactance; $T = 0.7$.

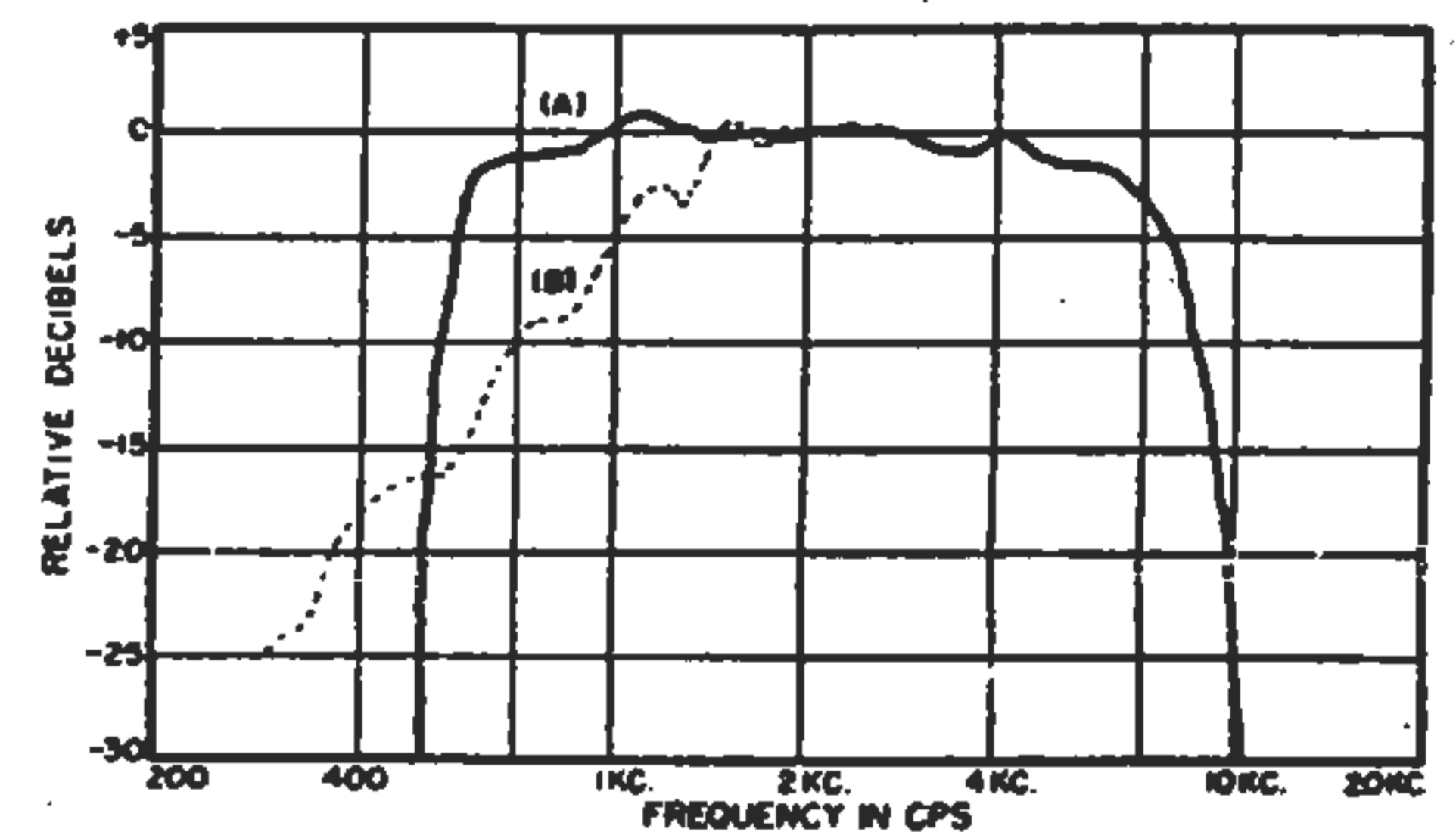


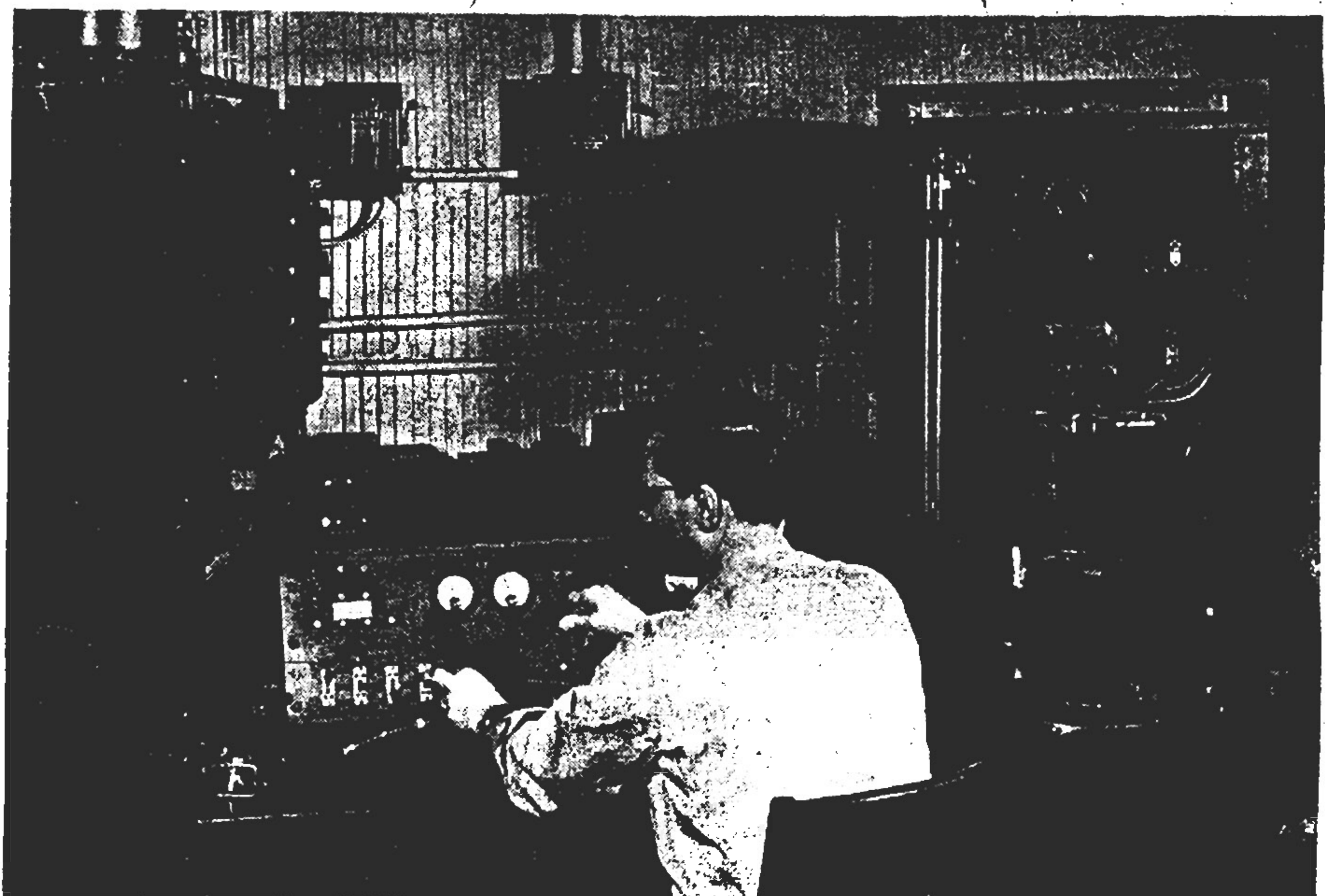
Fig. 5. Sound pressure performance of middle frequency horn-type speaker (A) with resonant frequency of moving system properly placed with respect to cutoff (about 1100 cps), and (B) with resonant frequency low (about 400 cps).

R_m can be determined by vacuum measurements, but in most cases can be neglected. The effective diameter used to obtain the area A_t is roughly 80% of the nominal speaker size.

Because of possible throat air overload, the minimum permissible area of the throat may have to be limited if high power operation over a wide band is required of a single horn unit. With

(Continued on page 35)

Magnetic measuring equipment consists of ballistic galvanometer, calibrated by potentiometer and standard cell, and magnetizing system with control and switching equipment, analyzing flux paths in iron and air.



by the **Thiokol Chemical Corporation**, 780 N. Clinton Ave., Trenton 7, N. J. The booklet gives starting information on the formulation of binders for laminates from combinations of "Thiokol" liquid polymers with several liquid epoxy resins by wet lay-up methods.

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SHOCK AND VIBRATION ISOLATORS

Features and uses of the Series 5200 shock and high vibration isolators are discussed in **Barry Corporation's** Product Bulletin #541. Data on isolator loads, shock transmissibility, vibration transmissibility and coupled natural frequencies are given in graph form.

Complete specification data is also given in this brochure, copies of which may be obtained from Department #541, **Barry Corporation**, 1000 Pleasant St., Watertown, Mass.

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INSTRUMENT COMPONENTS

PIC Design Corp., 160 Atlantic Ave., Lynbrook, L. I., N. Y., has announced a new 64-page catalog which illustrates its complete line of precision shafts, gears, collars, couplings, differentials, breadboard equipment, etc. All items are completely dimensioned for design and detail layout, and complete tolerance specifications enable the designer or engineer to mount associated equipment.

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Various magnetic tape equipment developed by **The Davies Laboratories Incorporated**, 4705 Queensbury Rd., Riverdale, Md., for use in the instrumentation field is presented in Bulletin 54-D. Entitled "Magnetic Tape Data Recording Equipment," it covers such items as compensation, recording and playback heads, portable recorders, laboratory recording-reproducing equipment, tape transports, FM discriminators, and playback amplifiers.

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Advantages of printed circuitry for electrical and electronic systems are covered thoroughly in a 12-page bulletin published by the **National Vulcanized Fibre Company**, Wilmington 99, Delaware. Titled "Mechanize Your Wiring. . . With Copper-Clad Phenolite," the bulletin highlights the numerous potentials printed circuitry offers design engineers and manufacturers. Of particular interest is a discussion of the economies of printed circuit design and construction as compared with conventional hand wiring methods.

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Reactance Annulling

(Continued from page 17)

the preceding parameters determined, the only remaining variables are the driver resonant frequency and the T of the horn. The resonant frequency of the driver is not necessarily fixed by the elements of the moving system but may be suitably adjusted in the case of many systems. If the f_r/f_s ratio is fixed by other considerations, the required T can be obtained from the chart. If this ratio is not fixed, then a T value of .5 to 0.7 would be a satisfactory choice. The resistive and reactive characteristics associated with various T values give the designer an additional tool for matching the characteristics of more complicated driver systems than those considered here.

A common method of constructing low frequency horns is to take off sound from one side and bury the other side in a sealed cavity. While somewhat more expensive and space-consuming because of the cavity required, there can be one design advantage in this arrangement. It is possible to obtain stiffness control of the woofer by judicious use of the air in the cavity. For proper reactance annulling, the cavity size is made such that its stiffness addition to the speaker gives the proper resonant frequency. Implied here is the fact that the speaker resonant frequency must initially be placed considerably lower than the cavity resonance to allow latitude of adjustment. The great advantage of this method of operation is that the addition of the air stiffness allows a lower resonance speaker, so that speaker suspension nonlinearities are of a lower order. The disadvantage is that higher frequencies have difficulty in transmission through the relatively great path length of the horn. In addition, sometimes it is necessary to fold the horn, and then transmission of highs is impossible above a certain frequency. Experimentally, the cavity size can be determined by adjusting it until the first major speaker impedance rise occurs at the horn cutoff point. This value is quite high and cannot be mistaken easily.

When radiation from both sides of the cone is used, the speaker stiffness can be adjusted experimentally by adding stiffening lacquer to the speaker suspension. This system of front and rear take-off is commonly used to provide frequencies higher than can be transmitted, say, through a rear loading horn only. Front radiation may be direct to the air, or horn loading for the front also may be used.

Photographs at the top of page 16 show **Jensen** commercial high-fidelity horn-loading speaker units which employ reactance annulling through use

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of low T hyperbolic-exponential flares. These units are manufactured and sold under the **Hypex** trademark for wide-range sound reproduction.

REFERENCES:

1. Salmon, V., U. S. Patent 2,338,262.
2. Plach, Daniel J., "Design Factors in Horn-Type Speakers," *J.A.E.S.*, Vol. 4, October, 1953.

Fig. 6. Chart based on Eq. (5).

