

factory if the local oscillation is within 5 to 10 cycles of the correct frequency. However, when both side bands are transmitted, the local oscillation must have exactly the proper frequency and even the correct phase, which makes double side-band carrier-suppression systems of transmission entirely impractical.¹

The waves that are sent out by carrier-suppression and single side-band systems of communication differ in appearance from a modulated wave in several respects as is apparent from Fig. 232, which shows the same signal transmitted by amplitude-modulation, carrier-suppression, and single side-band systems. The wave with carrier suppression differs from the modulated wave primarily in having an envelope that varies in

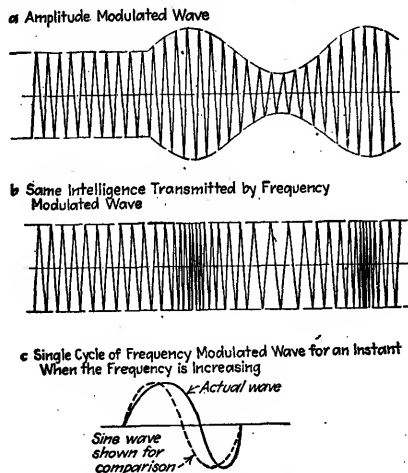


FIG. 233.—Character of waves produced by frequency modulation, together with large-scale reproduction of a single cycle, showing how the wave shapes are not sinusoidal.

amplitude at twice the modulation frequency as a result of action between the two side bands. The wave representing a single side band consists of a number of frequency components, one for each component in the original signal. Each of these components has an amplitude proportional to the amplitude of the corresponding signal component and a frequency differing from that of the carrier by the signal frequency. The result is that in a general way the envelope amplitude of the single side-band signal increases with the degree of modulation and varies in accordance with the difference frequencies formed by the various frequency components of the single side-band interacting with each other.

81. Frequency and Phase Modulation.—Instead of transmitting intelligence by varying the amplitude of the radiated wave, it is also possible to carry on communication by keeping the amplitude of the

¹ See R. V. L. Hartley, Relations of Carrier and Side Bands in Radio Transmission, *Proc. I.R.E.*, vol. 11, p. 34, February, 1923.

wave constant and varying the frequency in accordance with the signal to be transmitted. This is known as frequency modulation and results in a radiated wave having the appearance shown in Fig. 233b. The extent of the frequency variation in such a wave is made proportional to the amplitude of the signal, while the rate of frequency variation, *i.e.*, the number of times the frequency is changed between the minimum and maximum values per second, corresponds to the modulation frequency in amplitude modulation. Thus, if a 500-cycle sound wave is to be transmitted by frequency modulation of a 1,000,000-cycle carrier wave, this could be done by varying the transmitted frequency between 1,000,010 and 999,990 cycles, 500 times a second. If the pitch of the sound wave is increased to 1000 cycles, the carrier frequency would be varied between the same two limits 1000 times a second, while a sound wave of twice the intensity will be transmitted by varying the carrier frequency through twice the frequency range, *i.e.*, from 1,000,020 to 999,980 cycles in the above case.

Frequency-modulated waves can be readily produced by varying the capacity of the oscillator tuned circuit. The simplest way of doing this is to employ a small auxiliary condenser, one plate of which is a thin diaphragm that is vibrated by the voice currents in the same manner as is the diaphragm of a telephone receiver.¹ Reception is accomplished by converting the frequency-modulated wave into an amplitude-modulated wave by detuning the resonant circuits of the receiver so that the received frequency falls slightly to one side of the resonant point. As the frequency of the modulated wave varies, the response of the tuned circuit becomes alternately large and small, thus producing amplitude modulation.

Analysis of the Frequency-modulated Wave.—A superficial examination of frequency modulation might lead one to believe that intelligence could be transmitted in this way with an extremely narrow frequency band, since in the case cited above it appears that only 20 cycles band width is required to transmit the 500-cycle sound wave. This is not correct, however, because the variation in the frequency prevents the individual cycles from being exactly sinusoidal in shape. This is illustrated in Fig. 233c, where it is apparent that, since the changing frequency

¹ In the case of a high-power oscillator tube the diaphragm and its driving mechanism can be placed in an evacuated chamber, thereby enabling the arrangement to handle high voltages with small clearances between the plates.

Another very practical method makes use of the fact that the sending-end reactance of a transmission line one-eighth wave length long depends on the receiving-end resistance, and this resistance can be supplied by the plate circuit of a tube and varied by applying the signal to the grid. See Austin V. Eastman and Earl D. Scott, Transmission Lines as Frequency Modulators, *Proc. I.R.E.*, vol. 22, p. 878, July, 1934.

causes the time required to complete one-quarter cycle to differ from the time required by the next quarter cycle, the actual wave contains more than a single frequency. In fact, exact analysis shows that the frequency-modulated wave not only contains the same side-band frequencies as does the amplitude-modulated wave but also has higher order side bands that differ from the carrier frequency by integral multiples of the modulation frequency. Thus, when a carrier wave of frequency f_o is frequency modulated at a rate of f_s cycles per second, the resultant wave contains components having frequencies of f_o , $f_o + f_s$, $f_o - f_s$, $f_o + 2f_s$, $f_o - 2f_s$, $f_o + 3f_s$, $f_o - 3f_s$, etc.

The exact nature of a frequency-modulated wave can be determined by writing down the equation giving the instantaneous wave amplitude and then determining the frequency components contained in the result. The frequency-modulated wave can be readily shown to be expressible by the following equation¹

$$i = I_m \sin (\omega t + \Phi + m_f \sin vt)$$

where ω and v are 2π times the carrier and audio frequencies, respectively, Φ is an arbitrary phase constant, and m_f (called the modulation index) is the ratio

$$\frac{\text{Variation in carrier frequency away from average carrier frequency}}{\text{Audio frequency}}$$

By making use of Bessel's functions it can be readily demonstrated that this wave consists of a carrier wave plus a series of side bands such as

¹ See Hans Roder, Amplitude, Phase, and Frequency Modulation, *Proc. I.R.E.*, vol. 19, p. 2145, December, 1931; Balth. van der Pol, Frequency Modulation, *Proc. I.R.E.*, vol. 18, p. 1194, July, 1930; John R. Carson, Notes on the Theory of Modulation, *Proc. I.R.E.*, vol. 10, p. 57, February, 1922.

The equation of the frequency-modulated wave is derived as follows: If the current is defined by the relation $i = A \sin \phi$, then the frequency at any instant is $(d\phi/dt)/2\pi$, and $\phi = \Phi + \int 2\pi f dt$, where Φ is an arbitrary phase constant. In the case of frequency modulation, the instantaneous frequency f is $f = f_o(1 + k \cos vt)$, where k_f is a constant. Substituting this in the integral giving ϕ , and denoting $(\omega t + \Phi)$ by $\omega_o t$, results in

$$i = A_o \sin (\omega_o t + m_f \sin vt)t$$

where

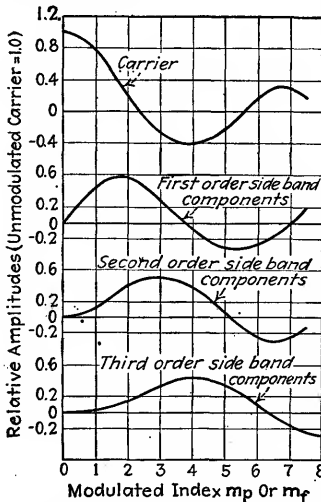
$$m_f = \frac{\text{variation in radio frequency away from the mean}}{\text{audio frequency}}$$

This can then be rewritten as

$$i = A_o \{ I_o(m_f) \sin (\omega_o t) + I_1(m_f) [\sin (\omega_o + v)t - \sin (\omega_o - v)t] + I_2(m_f) [\sin (\omega_o + 2v)t + \sin (\omega_o - 2v)t] + \dots \}$$

where $I_n(x)$ means the Bessel function of the first kind and the n th order.

described above. Since Bessel's functions are not familiar to most engineers, the results to which they lead have been plotted in Fig. 234, which shows the amplitude of the first-, second-, and third-order side-band components, *i.e.*, the side bands that differ from the carrier frequency by f_s , $2f_s$, and $3f_s$. These curves



where I is the amplitude of the wave, ω and ν are 2π times the radio and audio frequencies, respectively, Φ is an arbitrary constant, and m_p the angle in radians through which the phase is displaced about the average phase. This equation is seen to be identical with that for the frequency-modulated wave, the only difference being in the interpretation of the modulation index m , which in the case of phase modulation depends only on the amplitude of the modulation and is independent of the frequency of the audio signal. It is hence apparent that the phase-modulated wave contains the same side-band components as does the frequency-modulated wave, and, if the modulation indexes in the two cases are the same, the relative amplitudes of these different components will also be the same. As long as the modulation index is less than unity (*i.e.*, phase shifts less than 57.3°), only the first-order side-band components are of appreciable magnitude, but each additional 57.3° of phase shift will add another pair of important side-band components. The relative amplitude of the carrier and the first three side bands can be obtained from Fig. 234 for any given modulation index.

Combinations of Phase, Amplitude, and Frequency Modulation.—Frequency and phase modulation are often combined with amplitude modulation as undesirable by-products. For example in the plate-modulated oscillator the plate-supply voltage of the oscillator tube is varied in accordance with the intelligence being transmitted, and, since the generated frequency depends more or less upon the plate voltage, the oscillations actually generated possess both frequency and amplitude modulation. For this reason modulated oscillators are practically never used in radio communication.

Combined phase and amplitude modulation can occur in a number of ways. Thus, if the tank circuit of a modulated amplifier or linear amplifier is not tuned exactly to resonance, there will be a phase shift that will vary with the modulation because the effective tube resistance (and hence the phase shift) varies with the amplitude modulation. Another source of combined amplitude and phase modulation is energy transfer between the carrier wave after modulation and the unmodulated exciting frequency. Such coupling causes a phase shift in the exciting voltage applied to the modulated amplifier, and, as the phase shift depends upon the amount of energy transfer, it will vary with the amplitude modulation.¹

Phase and frequency modulation that occurs as a by-product of amplitude modulation is very undesirable in radio transmitters. This is because such modulation produces high-order side-band frequencies, which represent energy radiated upon adjacent frequency bands and

¹ For further information see W. A. Fitch, Phase Shift in Radio Transmitters, *Proc. I.R.E.*, vol. 20, p. 863, May, 1932.

which may interfere with other communications. Phase modulation is particularly bad in this respect because the modulation index m_p of phase modulation is independent of the frequency of modulation, whereas with frequency modulation the index m_f is inversely proportional to the modulation frequency and so tends to be low when the modulation frequency is high enough to make the second- and third-order side bands lie in adjacent channels.

Problems

1. The equation of a modulated wave is

$$e = 25(1 + 0.7 \cos 5000t - 0.3 \cos 10,000t) \sin 5 \times 10^4 t$$

a. What frequency components does the modulated wave consist of, and what is the amplitude of each?

b. Sketch the modulation envelope and evaluate the degree of modulation for the peaks and troughs.

2. Write the equation of a 100-volt carrier wave of 1000 kc when modulated 40 per cent at 400 cycles.

3. Calculate the total band width to which a radio receiver should respond for satisfactory reception of (a) ordinary broadcast signals, (b) perfect reproduction of speech and music, (c) telegraph code signals sent at 30 words per minute, (d) radio signals that represent radio extensions of wire telephone systems:

4. The Class C amplifier of the example in Sec. 61 (page 328) is to be completely modulated, using plate modulation with a transformer-coupled modulator such as shown in Fig. 215c.

a. Specify the undistorted audio power that the modulator must develop and the effective load impedance that the tube offers to the modulator.

b. Design a suitable modulator using commercial tubes in a push-pull circuit. The design includes the selection of suitable tubes, specification of operating conditions, and a statement as to the turn ratio of the output transformer.

5. a. In Prob. 4, obtain a linearity curve by calculating and plotting the output when the total plate voltage (*i.e.*, direct-current voltage plus modulating voltage) is 0, 0.293, 1.0, 1.707, and 2 times the direct-current voltage.

b. From these results calculate the second-, third-, and fourth-harmonic distortion.

6. a. Design a grid-modulated amplifier (100 per cent modulation) using the Type 800 tube of Fig. 75 ($\mu = 15$) for a plate-supply voltage of 1000 and permitting the grid to go a reasonable amount positive. In this design specify the proper voltages for the grid, the approximate load impedance, the expected carrier power, and the approximate grid current to be expected at the crest of the modulation cycle.

b. Calculate the maximum permissible internal impedance of the source of modulating voltage for a second-harmonic distortion not to exceed 5 per cent as a result of grid current [using Eq. (156)], and, from this and the audio voltage required, design the audio-modulating system.

7. The average power loss at the screen grid is much higher in a suppressor-grid modulated amplifier than the screen loss when the same tube is used as a Class C amplifier. Explain.

8. Explain the action of a van der Bijl modulated Class A amplifier by a mathematical analysis based on the equivalent circuit of Fig. 151 and the related analysis in Sec. 55, using only the first- and second-order effects.