Passive RIAA correction

Version 4, Marcel van de Gevel, 5 January 2024

1. No DC blocking (AC coupling) capacitor taken into account

The RIAA correction network of https://www.diyaudio.com/community/threads/tube-mc-phono-stage-riaa-calculator-formula-help.401437/post-7405500 but without its DC blocking capacitor is shown in Figure 1.

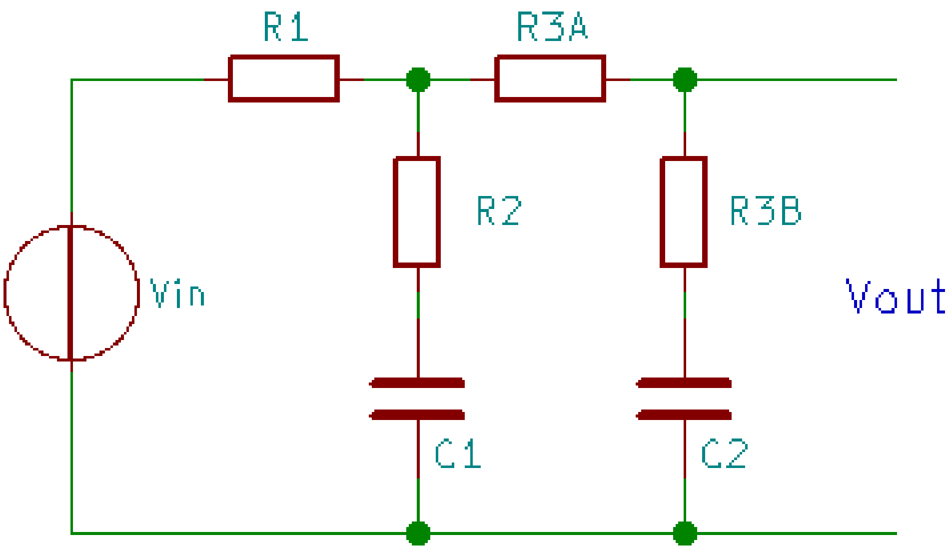

Figure 1: Unusual RIAA correction network. I have numbered the resistors unusually and introduce $R_3 = R_{3A} + R_{3B}$ to keep the equations simple.

Time constants corresponding to the required RIAA correction poles and zero:

$$\tau_{p1} = 3.18\,\text{ms} = 0.00318\,\text{s}$$

$$\tau_z = 318\,\mu\text{s} = 0.000318\,\text{s}$$

$$\tau_{p2} = 75\,\mu\text{s} = 0.000075\,\text{s}$$

The network has two zeros, so one too many. At $s = -\frac{1}{R_2 C_1}$, the impedance $\frac{1}{s C_1}$ of C_1 cancels R_2, causing a zero transfer. (The fact that this value of s doesn't correspond to any stationary sine wave doesn't change that.) For similar reasons, there is a zero at $s = -\frac{1}{R_{3B} C_2}$

The two poles of the network are much harder to find. As the poles of the input voltage to input current transfer are the exact same as the poles of the input voltage to anything else transfer, one can analyse the input admittance (reciprocal of the input impedance) to find the poles.

I haven't actually solved the poles, but rather calculated the characteristic polynomial. It is

$$s^2(R_1R_3+R_1R_2+R_2R_3)C_1C_2+s(R_1C_1+R_1C_2+R_2C_1+R_3C_2)+1 \quad \text{where} \quad R_3=R_{3A}+R_{3B}$$

To get the correct RIAA correction, the characteristic polynomial must be

$$(s\tau_{p1}+1)(s\tau_{p2}+1)=s^2\tau_{p1}\tau_{p2}+s(\tau_{p1}+\tau_{p2})+1$$

To keep the equations simple, I introduce $a=\tau_{p1}\tau_{p2}$ and $b=\tau_{p1}+\tau_{p2}$, so that the desired characteristic polynomial becomes as^2+bs+1

As there is one zero too many, there are two ways to solve this: either make R_2 or R_{3B} equal to zero.

When we choose to make $R_2 = 0$, equating the coefficients of the characteristic polynomial to there desired values and rearranging terms leads to

$$R_3=\frac{b\pm\sqrt{b^2-4a\frac{C_1+C_2}{C_1}}}{2C_2}$$

$$R_1=\frac{a}{R_3C_1C_2}$$

$$R_{3B}=\frac{\tau_z}{C_2}$$

$$R_{3A}=R_3-R_{3B}$$

When we choose to make $R_{3B} = 0$, I find these equations for the rest:

$$R_2=\frac{\tau_z}{C_1}$$

$$R_1=\frac{\frac{b-2\tau_z}{C_2}\pm\sqrt{\left(\frac{b-2\tau_z}{C_2}\right)^2-4\left(1+\frac{C_1}{C_2}\right)\frac{a+\tau_z(\tau_z-b)}{C_1C_2}}}{2\left(1+\frac{C_1}{C_2}\right)}$$

$$R_3=\frac{b-\tau_z-R_1(C_1+C_2)}{C_2}$$, which is also the value for R_{3A} when $R_{3B} = 0$.

The trick is to choose values for C_1 and C_2 that lead to realistic values for the resistors. There may be a bit more freedom to use standard capacitor values than with a more conventional RIAA correction network.

I have checked one numerical example with a pole-zero extraction program, which showed that the values were correct. Those values were:

R_1 = 119183.428858293 ohm
R_2 = 14454.5454545455 ohm

C_1 = 22 nF
R_{3A} = 23982.2825587722 ohm
R_{3B} = 0 ohm
C_2 = 2.2 nF

2. AC coupling capacitor included

The filter network is normally driven from an amplifier stage that has a certain output resistance and quite possibly an AC coupling (DC blocking) capacitor. It is easy to correct for output resistance when there is no AC coupling: take the Thévenin equivalent of the source and you see that its output resistance is in series with the input resistor of the filter. Just subtract the output resistance of the source from the calculated R_1, use the result as the value of the filter input resistor and Bob's your uncle.

Things get rather more complicated when there is an AC coupling capacitor of which the reactance is too large to be neglected, or when the output of the amplifying stage has considerable reactance for some other reason, for example because it is a triode stage with cathode decoupling and the cathode decoupling capacitor is not very large (outside the scope of this document). The circuit with AC coupling is shown in Figure 2.

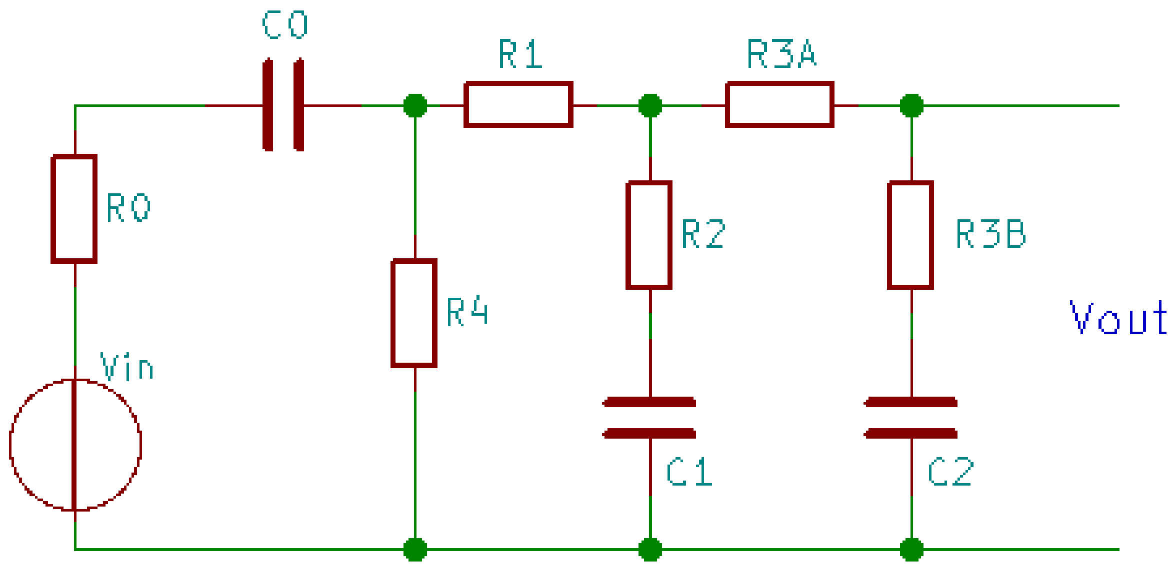

Figure 2: Passive RIAA correction with AC coupling. Again, R_3 = R_{3A} + R_{3B}.

The zeros are still exactly where they used to be, but the poles shift. Calculating the characteristic equation is still straightforward, although the number of terms gets a lot larger. The result I found is:

When the characteristic polynomial is written as $a_3 s^3 + a_2 s^2 + a_1 s + 1$, then

$$a_3 = R_1 R_3 R_4 C_0 C_1 C_2 + R_1 R_2 R_4 C_0 C_1 C_2 + R_2 R_3 R_4 C_0 C_1 C_2 + R_0 R_1 R_3 C_0 C_1 C_2 \\ + R_0 R_1 R_2 C_0 C_1 C_2 + R_0 R_2 R_3 C_0 C_1 C_2 + R_0 R_3 R_4 C_0 C_1 C_2 + R_0 R_2 R_4 C_0 C_1 C_2$$

$$a_2 = R_1 R_3 C_1 C_2 + R_1 R_2 C_1 C_2 + R_2 R_3 C_1 C_2 + R_3 R_4 C_1 C_2 + R_2 R_4 C_1 C_2 + R_1 R_4 C_0 C_1 \\ + R_1 R_4 C_0 C_2 + R_2 R_4 C_0 C_1 + R_3 R_4 C_0 C_2 + R_0 R_1 C_0 C_1 + R_0 R_1 C_0 C_2 + R_0 R_2 C_0 C_1 \\ + R_0 R_3 C_0 C_2 + R_0 R_4 C_0 C_1 + R_0 R_4 C_0 C_2$$

$$a_1 = R_0 C_0 + R_1 C_1 + R_1 C_2 + R_2 C_1 + R_3 C_2 + R_4 C_0 + R_4 C_1 + R_4 C_2$$

One could now assume the required pole positions to be known: two of them are the required RIAA correction poles, and one is the extra subsonic pole due to the AC coupling, for which one would have to pick a value. The desired values of a_3, a_2 and a_1 can then be calculated. After choosing convenient values for C_0, C_1 and C_2, realising that R_0 is the known output resistance of the first stage, and choosing between $R_2 = 0$ or $R_2 = \tau_z/C_1$, there would be three remaining unknowns: R_1, R_3 and R_4.

It should in principle be possible to solve the resulting system of three equations with three unknowns, but it looks like this would be rather unwieldy. I have therefore tried to find a suitable approximation instead.

3. AC coupling capacitor approximately included

The equations get much simpler without R_4, that is, dividing everything by R_4 and taking the limit for $R_4 \to \infty$. The extra pole then ends up at 0 and can then be factored out of the characteristic polynomial, reducing it to second order. The resistors R_0 and R_1 end up effectively in series, and can be replaced by one resistor.

Also dividing by $C_0 + C_1 + C_2$ to get the result in the form $a s^2 + b s + 1$, like in chapter 1, one has to calculate a and b such that

$$\lim_{R_4 \to \infty} \frac{a_3 s^3 + a_2 s^2 + a_1 s + 1}{R_4 s (C_0 + C_1 + C_2)} = a s^2 + b s + 1$$

holds for all s. Renaming $R_0 + R_1$ to R_{01}, the result is:

$$a = \frac{C_0 C_1 C_2}{C_0 + C_1 + C_2} (R_{01} R_3 + R_{01} R_2 + R_2 R_3)$$

$$b = \frac{(C_0 + C_1) C_2}{C_0 + C_1 + C_2} R_3 + \frac{(C_0 + C_2) C_1}{C_0 + C_1 + C_2} R_2 + \frac{(C_1 + C_2) C_0}{C_0 + C_1 + C_2} R_{01}$$

The required a and b follow from the required RIAA correction, like in chapter 1. One can again choose between $R_2 = 0$ or $R_2 = \tau_z/C_1$, and pick convenient values for the capacitances. The remaining unknowns are then R_{01} and R_3. Solving these results in

$$R_{01}=\frac{b(C_0+C_1+C_2)-(C_0+C_2)C_1R_2-(C_0+C_1)C_2R_3}{C_0(C_1+C_2)}$$

and

$$R_3=\frac{b(C_0+C_1+C_2)-2R_2C_1C_2\pm\sqrt{\left(b(C_0+C_1+C_2)-2R_2C_1C_2\right)^2-4C_2(C_0+C_1)\left(a(C_0+C_1+C_2)\frac{C_1+C_2}{C_1C_2}+R_2^2C_1(C_0+C_2)-bR_2(C_0+C_1+C_2)\right)}}{2C_2(C_0+C_1)}$$

In practice, R_4 needs to have a finite value to bias the next stage. Comparing the admittances of the two circuits in Figure 3,

$$Y_1=\frac{1}{R_4}+\frac{sC_0}{sR_0C_0+1}=\frac{s(R_0+R_4)C_0+1}{sR_0C_0R_4+R_4}$$

and

$$Y_2=\frac{sC_5}{sR_5C_5+1}=\frac{sR_4C_5}{sR_5C_5R_4+R_4}$$

The denominators are equal for all s when $R_5C_5=R_0C_0$ and the asymptotes for $s\rightarrow\infty$ are equal when $R_5=\frac{R_0R_4}{R_0+R_4}$. Hence, $C_5=\frac{R_0+R_4}{R_4}C_0$. The only remaining difference is the location of the zero of the admittance, either at $s=0$ or at $s=-\frac{1}{(R_0+R_4)C_0}$.

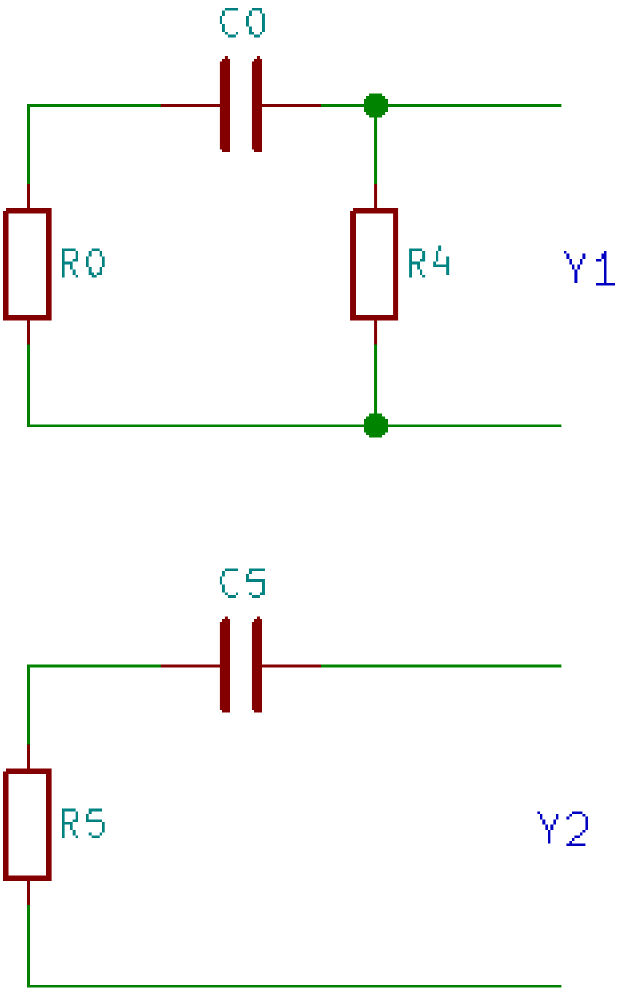

Figure 3: AC coupling and a simplified circuit

Hence, the approximation can be improved by replacing C_0 in the earlier equations with C_5 and R_0 with R_5. Therefore, R_{01} will be renamed to R_{51}, the sum of R_5 and R_1. See also Figure 4.

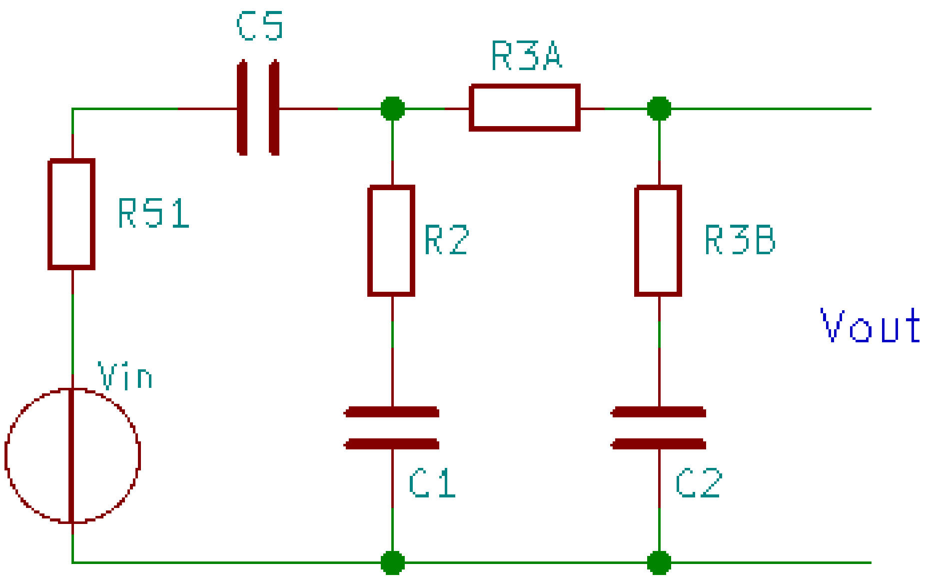

Figure 4: Simplified RIAA correction with an AC coupling capacitor. R_{51} is the sum of R_5 and R_1.

3.1. Resulting method for calculating the values

Calculate the output resistance R_0 of the driving stage, see Figure 2.

Calculate $a=\tau_{p1}\tau_{p2}$ and $b=\tau_{p1}+\tau_{p2}$, like in chapter 1.

Choose convenient values for C_0, C_1 and C_2, see Figure 2. If negative or complex values for the resistors are later calculated, the choice of capacitances may have to be revisited.

Choose the value for the bias resistor R_4.

Make a choice between $R_2=\frac{\tau_z}{C_1}$ or $R_2 = 0$.

Calculate $C_5=\frac{R_0+R_4}{R_4}C_0$

Calculate $R_5=\frac{R_0 R_4}{R_0+R_4}$, the parallel value of the output resistance of the first stage and the bias resistor R_4.

Calculate

$$R_3=\frac{b(C_1+C_2+C_5)-2R_2C_1C_2\pm\sqrt{\left(b(C_1+C_2+C_5)-2R_2C_1C_2\right)^2-4C_2(C_1+C_5)\left(a(C_1+C_2+C_5)\frac{C_1+C_2}{C_1C_2}+R_2^2C_1(C_2+C_5)-bR_2(C_1+C_2+C_5)\right)}}{2C_2(C_1+C_5)}$$

Calculate

$$R_{51} = \frac{b(C_1 + C_2 + C_5) - (C_2 + C_5) C_1 R_2 - (C_1 + C_5) C_2 R_3}{C_5 (C_1 + C_2)}$$

Calculate

$$R_1 = R_{51} - R_5$$

If you chose $R_2 = \frac{\tau_z}{C_1}$, then $R_{3A} = R_3$ and $R_{3B} = 0$.

If you chose $R_2 = 0$, then $R_{3B} = \frac{\tau_z}{C_2}$ and $R_{3A} = R_3 - R_{3B}$

3.2. Example checked with a pole-zero extraction program

Using

R_0 = 25 kΩ

C_0 = 220 nF

C_1 = 22 nF

C_2 = 2.2 nF

R_4 = 470 kΩ

the calculations result in this as the only option without negative resistances:

R_1 = 109171.187490245 Ω

R_2 = 14454.5454545455 Ω

R_{3A} = 23894.8717329408 Ω

R_{3B} = 0 Ω

The extracted pole and zero locations are:

Poles:

-8.332 rad/s

-313.815 rad/s

-13333 rad/s

Zeros:

-3145 rad/s

-5.24 frad/s (that is, zero with a numerical error)

The lowest RIAA correction pole is at -313.815 rad/s, meaning its time constant is about 0.207 % too large.

4. AC coupling included, with the resistor to ground (or a DC bias voltage) at the end

There may be practical reasons for wanting the DC bias resistor at the end of the network, rather than at the beginning. For example, because one has a PCB with space for such a resistor at the end of the network, or to reduce the number of possible failures that could lead to an incorrect DC voltage at the output. For simplicity, I will assume that $R_{3B} = 0$. We then end up with the lower circuit of Figure 5.

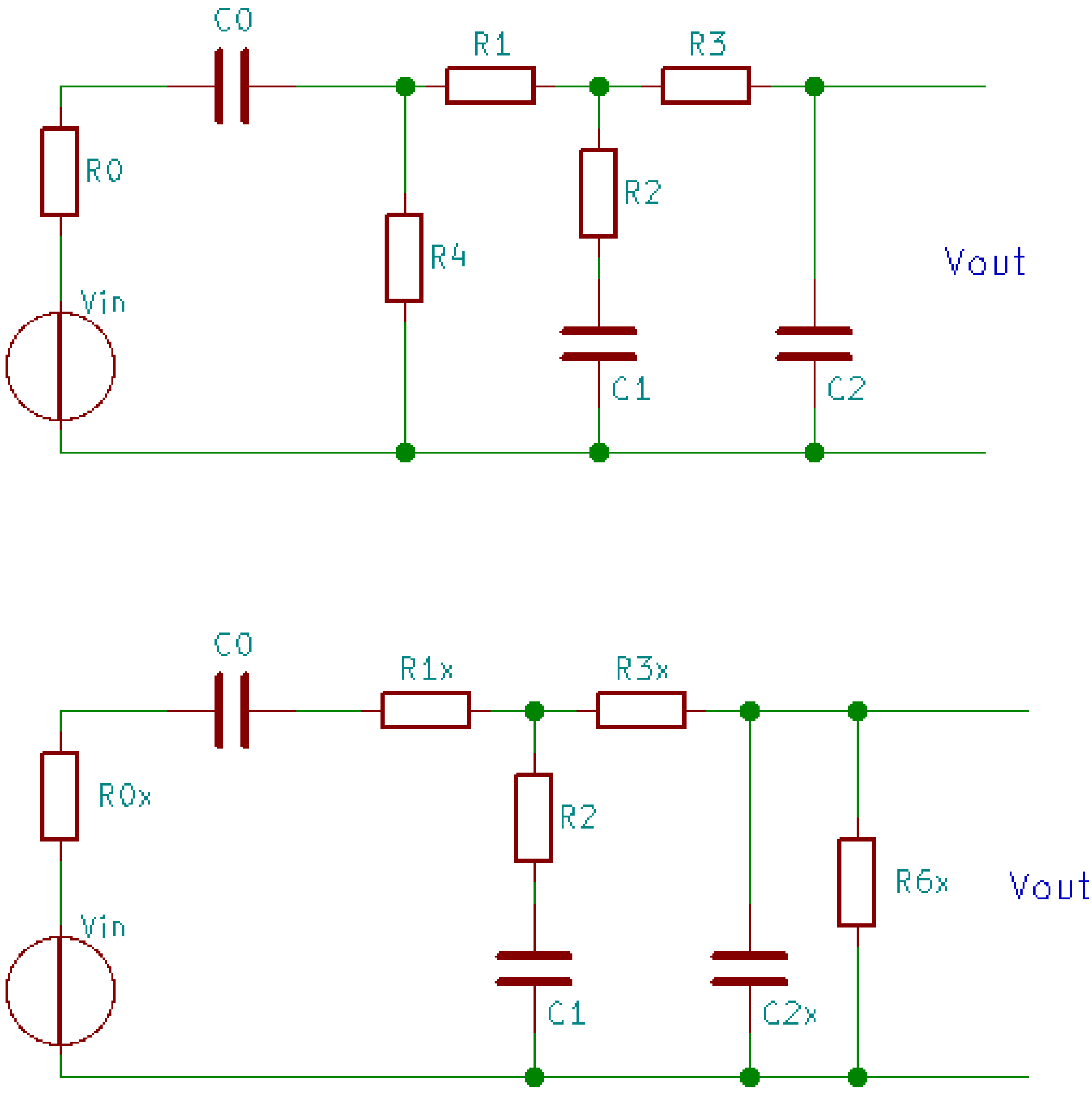

Figure 5: Passive RIAA with the DC bias resistor right after the AC coupling (upper schematic) or at the end (lower schematic)

Calculating the characteristic polynomial of this circuit again leads to a third-order equation with lots of terms; I gave up calculating it halfway. Unlike in chapter 3, I also haven't found a satisfactory approximation to reduce it to a simpler second-order system and end up with standard-value capacitors and only awkward values for the resistors. I have only found an iterative approximate solution for that, one that is based on transforming the circuit into something that looks more like the circuit in the upper part of Figure 5 (which is obviously the circuit of the previous chapters).

When you look at the input admittance of the upper part of Figure 5 and choose $R_1 = 0$, the subcircuit consisting of R_3, R_4 and C_2 can be replaced with a terminal equivalent that looks like R_{3x}, C_{2x} and R_{6x} in the lower part. This is illustrated in Figure 6.

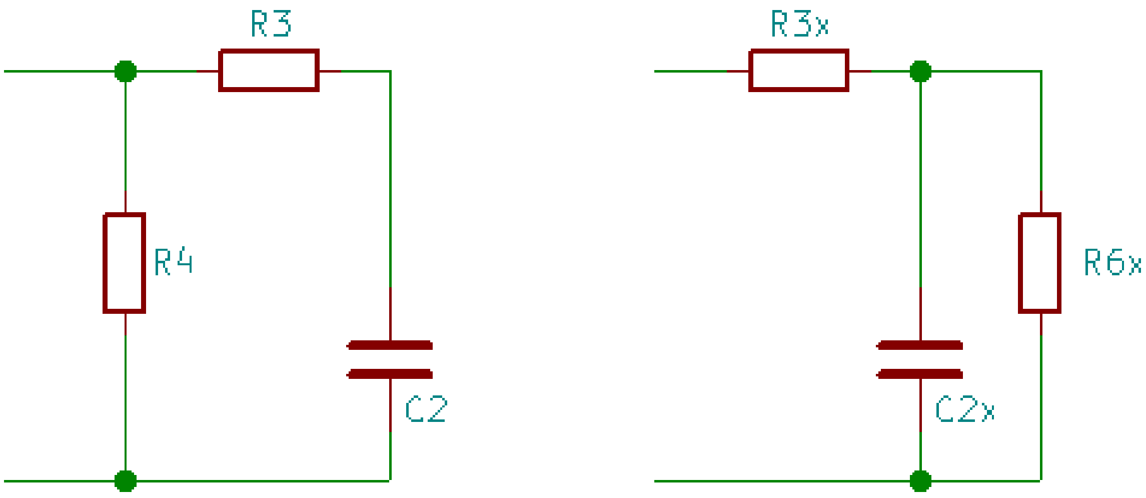

Figure 6: Two subcircuits that can be dimensioned to have equal impedances

When you calculate the impedances of both networks in Figure 6, it turns out that they are equal when

$$R_{3x}=\frac{R_3 R_4}{R_3+R_4}$$

$$R_{6x}=R_4-R_{3x}$$

$$C_{2x}=\frac{R_3+R_4}{R_{6x}}C_2$$

Hence, when you manage to dimension the network in the upper part of Figure 5 properly under the constraint $R_1 = 0$, then you can transform it to correct values for the network in the lower part of Figure 5. The capacitance of C_{2x} in the lower part of Figure 5 may get an awkward value, though, which can be solved by tweaking the chosen C_2.

You can only choose $R_1 = 0$ in the upper part of Figure 5 when you use a somewhat different procedure than in chapter 3 to dimension R_0 and the rest of the network. In particular, R_0 is now an unknown that needs to be solved rather than the known output impedance of the driving stage. Physically, R_0 can now be seen as the series connection of the known output impedance of the driving stage and a series resistor that gets whatever value is needed. To simplify the calculations, we will further assume that $\frac{R_0+R_4}{R_4}$ is chosen to be some fixed value.

4.1. Iterative and approximate design procedure

The procedure is now as follows:

Calculate $a=\tau_{p1}\tau_{p2}$ and $b=\tau_{p1}+\tau_{p2}$, like in chapter 1.

Choose convenient values for C_0, C_1 and C_2, see the upper part of Figure 5. If negative or complex values for the resistors are later calculated, the choice of capacitances may have to be revisited.

Choose a value for $\alpha=\frac{R_0+R_4}{R_4}$, normally this will be a value somewhere between 1 and 1.5.

Calculate $R_2=\frac{\tau_z}{C_1}$.

Calculate $C_5=\alpha C_0$

Calculate

$$R_3=\frac{b(C_1+C_2+C_5)-2R_2C_1C_2\pm\sqrt{(b(C_1+C_2+C_5)-2R_2C_1C_2)^2-4C_2(C_1+C_5)\left(a(C_1+C_2+C_5)\frac{C_1+C_2}{C_1C_2}+R_2^2C_1(C_2+C_5)-bR_2(C_1+C_2+C_5)\right)}}{2C_2(C_1+C_5)}$$

Calculate

$$R_5=\frac{b(C_1+C_2+C_5)-(C_2+C_5)C_1R_2-(C_1+C_5)C_2R_3}{C_5(C_1+C_2)}$$

(In chapter 3, this used to be the equation for R_{51}, but when R_1 = 0, R_5 = R_{51}.)

Now calculate

$$R_0=\alpha R_5$$

$$R_4=\frac{R_0}{\alpha-1}$$

and at last

$$R_{3x}=\frac{R_3R_4}{R_3+R_4}$$

$$R_{6x}=R_4-R_{3x}$$

$$C_{2x}=\frac{R_3+R_4}{R_{6x}}C_2$$

If the value of C_{2x} is inconvenient, tweak C_2 a bit and do the whole calculation again.

In the lower network of Figure 5, R_{0x} and R_{1x} are effectively in series and together act as R_0. That is,

Calculate the output impedance of the driving stage, this is now called R_{0x}.

Finally, calculate $R_{1x}=R_0-R_{0x}$

4.2. Example checked with a pole-zero extraction program

When

R_{0x} = 38461.5384615385 Ω

choosing

C_0 = 100 nF

C_1 = 22 nF

C_2 = 872.5 pF (value tweaked to get a nice C_{2x})

α = 1.1662 (value tweaked to get a nice R_{6x})

the calculations result in this as the only option without negative resistances:

R_{1x} = 139715.07245752 Ω

R_2 = 14454.5454545455 Ω

R_{3x} = 70674.1409656935 Ω

R_{6x} = 1001387.29657377 Ω

C_{2x} = 1.000001449 nF

The extracted pole and zero locations are:

Poles:

-6.821 rad/s

-308.296 rad/s

-13333 rad/s

Zeros:

-3145 rad/s

+16.526 frad/s (that is, zero with a numerical error)

The lowest RIAA correction pole is at -308.296 rad/s, meaning its time constant is about 2.001 % too large. This is worse than in chapter 3. Trying again with C_0 = 220 nF to increase the ratio between C_0 and the other capacitances shows that this reduces the error to just over 1 %, still worse than chapter 3.