

**8.2.3 Bandpass network function** The complex-conjugate-pole-pair bandpass transfer function is

$$H(s) = \frac{H_o \alpha \omega_o s}{s^2 + \alpha \omega_o s + \omega_o^2}$$

where

$$\alpha = \frac{1}{Q} \quad \text{and} \quad Q = \frac{\omega_o}{\omega_2 - \omega_1} = \frac{f_o}{f_2 - f_1}$$

and where  $f_2$  and  $f_1$  are the frequencies where the magnitude response is  $-3$  dB from  $H_o$ , the passband gain which occurs at  $\omega_o = 2\pi f_o$ . The sinusoidal steady-state transfer function may be written in the form

$$H(j\omega) = \frac{H_o}{1 + jQ(\omega/\omega_o - \omega_o/\omega)}$$

Thus, the magnitude, phase, and delay functions are

$$\begin{aligned} G(\omega) &= \left[ \frac{H_o^2}{1 + Q^2(\omega/\omega_o - \omega_o/\omega)^2} \right]^{1/2} \\ &= \left[ \frac{H_o^2 \alpha^2 \omega_o^2 \omega^2}{\omega^4 + \omega^2 \omega_o^2 (\alpha^2 - 2) + \omega_o^4} \right]^{1/2} \end{aligned}$$

$$\phi(\omega) = \frac{\pi}{2} - \arctan \left( \frac{2Q\omega}{\omega_o} + \sqrt{4Q^2 - 1} \right) - \arctan \left( 2Q \frac{\omega}{\omega_o} - \sqrt{4Q^2 - 1} \right)$$

$$\tau(\omega) = \frac{2Q \cos^2 \phi}{\omega_o} + \frac{\sin 2\phi}{2\omega}$$

**Bandpass 1.** There are several configurations of the five elements which may be used to realize a bandpass function. One of the more practical configurations is the one shown in Fig. 8.5. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{-s(1/R_1 C_4)}{s^2 + s(1/R_5)(1/C_3 + 1/C_4) + (1/R_5 C_3 C_4)(1/R_1 + 1/R_2)}$$

In terms of our bandpass network function

$$H_o = \frac{1}{(R_1/R_5)(1 + C_4/C_3)}$$

$$\omega_o = \left[ \frac{1}{R_5 C_3 C_4} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{1/2}$$

$$\frac{1}{Q} = \alpha = \sqrt{\frac{1}{R_5(1/R_1 + 1/R_2)}} \left[ \sqrt{\frac{C_3}{C_4}} + \sqrt{\frac{C_4}{C_3}} \right]$$

$$\phi = \pi + \phi_{BP}$$

$$\tau = \tau_{BP}$$

Tuning this filter appears rather formidable. In practice  $R_1 \gg R_2$  and so  $R_2$  can be used to trim the  $Q$ . Then, to adjust the center frequency,  $R_2$  and  $R_5$  can be simultaneously adjusted by the same percentage with negligible effect on the  $Q$ .

From Operational Amplifiers,

Graham, Toby, Huelsman