End Correction at a Flue Pipe Mouth

http://www.fonema.se/mouthcorr/mouthcorr.htm

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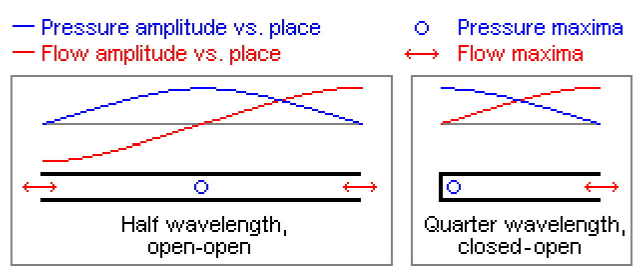
Abstract

A simple notion is that the fundamental resonance of a pipe occurs when the sound wavelength is half or a quarter of the resonator length. It is however well recognized that the practical frequency comes out lower than this, you have to apply an end correction, the pipe appears to be acoustically somewhat longer than its physical length. A formula for the basic mechanism behind this is theoretically derived, then expanded into the case where the open end area is made smaller than the pipe cross section.

The end correction was experimentally determined for several pipes with mouths extending 360 and 90 degrees of the circumference. Formulas are given to compute the end correction, using optimal coefficients found from these measurements.

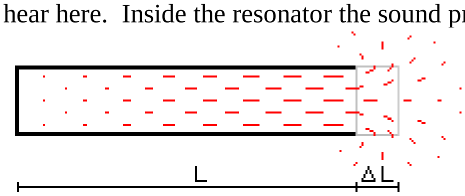
Introduction

The pitch f of flue instruments like the flute, organ pipe, or whistle is predominantly controlled by a resonator length L. This length is closely connected to the wavelength $\lambda = c/f$, where c is the speed of sound. Examples of the most common cases are open and stoppered organ pipes, commonly characterized by their working length as being half and quarter wavelength, respectively. When such pipes oscillate at their fundamental pitch their internal standing wave patterns of acoustic pressure and flow are classically illustrated this simplified way.



Let us initially inspect a quarter wavelength resonator tube of length L and area A, closed at one end. This tube has a total acoustical mass $M^* = \rho L/A$ and acoustical capacitance $C^* = AL/(\rho c^2)$, where ρ is density of the air and c is its speed of sound. The effective resonating mass and capacitance are less than these total ones. They are reduced by the factor $2/\pi$ due to the sinusoidal distribution within the tube of flow and pressure. (This factor is the area of a sine quadrant, divided by the area of its circumscribed rectangle). Thus the effective mass is $M = 2\rho L/(\pi A)$ and effective capacitance $C = 2AL/(\pi\rho c^2)$. Inserting these into the resonance formula $f = 1/(2\pi\sqrt{MC})$ and cleaning up, leads into the well known quarter wave expression $f = c/(4L)$ for the fundamental resonance.

The flue end of a pipe where the air jet blows across the mouth is an open end of the resonator, having an acoustic flow maximum. The acoustic pressure at this place in principle is minimum. Still it is far from zero, it is the sound pressure you can hear here. Inside the resonator the sound pressure is considerably higher, its magnitude approaches the blowing pressure.



The air immediately outside the end of the pipe takes part in the acoustic oscillation. This air makes the pipe appear to be acoustically somewhat longer than its physical length. This apparent length increase ΔL is called the end correction. To compute the resonance frequency the length measure one should use is the sum of physical length plus the end correction.

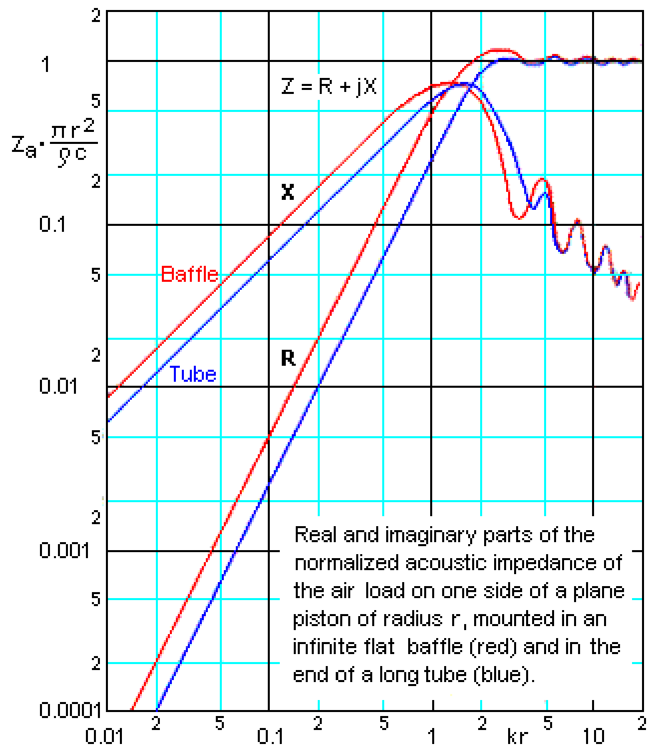
Theory

A theoretical basis for computation of the end correction is the 'radiation acoustic impedance of a circular piston', reproduced here. This impedance tells the ratio of acoustic pressure at the piston, divided by the flow rate induced by it. The piston does not physically exist, it is an abstract theoretical vehicle to state that one assumes the air speed to be the same at all places across the tube end. This is a good approximation, but not exactly true in reality, since air viscosity reduces the flow rate in the boundary layer very close to the tube surface.

Two different cases are illustrated here. In blue for a free tube end and in red for a baffled tube, i.e. when a wall limits the external sound to spread only into a half space rather than all around.

There is a critical frequency, typically taken as *kr*=1, which implies that wavelength equals the circumference of the piston/tube. At higher frequencies, or greater *r*, the tube end tends to impedance match the ambience such that the tube does not act as a resonator, but just a transmission line. The scale of whistles and organ pipes is always such that *kr* is substantially less than unity, so what applies to our problem is the left half of the diagram.

The impedance *Z* is composed from two parts, the real resistance *R* and the imaginary reactance *X*.



Frequency is plotted on a normalized scale, where $kr=2\pi fr/c=2\pi r/\lambda$.
After Beranek: Acoustics, McGraw-Hill 1954, pp 119, 122.

R tells the in-phase component of the pressure to flow ratio. A given flow *U* will develop a power $W=RU^2$ that is lost from the resonator and is radiated into the ambient space to become a useful sound. Normally this 'radiation resistance' is a major determinant of the resonator Q value.

The reactance $X=2\pi f M$ is the quadrature component. The nice thing is that *X* turns out proportional to frequency for $kr<1$, hence its corresponding acoustical mass *M* is constant within this $kr<1$ range. This *M* is the effective measure of the co-oscillating air outside the tube opening. From the upward sloping blue *X* contour in the diagram, leaving out trivial intermediate formula manipulations, we arrive at the most commonly cited expression

$\Delta L = 0.6 * r = 0.3 * D$, where the equivalent tube radius is $r=\sqrt{A/\pi}$.

Since it is rather immaterial whether the tube is circular, quadratic, or even moderately oblong rectangular, let us here stick to the correction expressed in terms of the tube area, such that it amounts to

$\Delta L = 0.34\sqrt{A}$

For a baffled tube end we similarly find a slightly larger value, namely 0.48 times the square root of tube area.