1.6 Horns

Despite the useful effects of mutual coupling, the radiation efficiency of even large loudspeaker diaphragms is small at low frequencies. For example, a diaphragm with a diameter of 250 mm has a radiation efficiency (proportional to the real part of the radiation impedance – see Section 1.2.7) of just 0.7% at 50 Hz when mounted in an infinite baffle, and half that when mounted in a cabinet. Sound power output is proportional to the product of the mean-squared velocity and the radiation efficiency, so a low radiation efficiency means that a high diaphragm velocity is required to radiate a given sound power. The only way in which the radiation efficiency can be increased is to increase the size of the radiating area, but larger diaphragms have more mass (if rigidity is to be maintained) which means that greater input forces are required to generate the necessary diaphragm velocity (see Chapter 2).

Electroacoustic efficiency is defined as the sound power output radiated by a loudspeaker per unit electrical power input. Because of the relatively high mass and small radiating area electro-acoustic efficiencies for typical loudspeaker drive-units in baffles or cabinets are of the order of only 1–5%. Horn loudspeakers combine the high radiation efficiency of a large diaphragm with the low mass of a small diaphragm in a single unit. This is achieved by coupling a small diaphragm to a large radiating area via a gradually tapering flare. This arrangement can result in electro-acoustic efficiencies of 10–50%, or ten times the power output of the direct-radiating loudspeaker for the same electrical input. Additionally, horns can be employed to control the directivity of a loudspeaker and this, along with the high sound power output capability, is why they are used extensively in public address loudspeaker systems.

The following sections describe, in a conceptual rather than mathematical way, how horns increase the radiation efficiency of loudspeakers, how they control directivity, and why there is often the need to compromise one aspect of the performance of a horn to enhance another.

1.6.1 The horn as a transformer

The discussion of near- and far-fields in Section 1.3.4 showed that, in the hydrodynamic near-field, the change in area of an acoustic wave as it propagates gives rise to a 'stretching pressure' which is additional to the pressure required for sound propagation. The stretching pressure does not contribute to sound propagation as it is in phase quadrature (90°) with the particle velocity, so the acoustic impedance in the near-field is dominated by reactance (see Section 1.2.7). As a consequence, large particle velocities are required to generate small sound pressures when the rate of change of area with distance of the acoustic wave is significant. It is this stretching phenomenon that is responsible for the low radiation efficiency of direct-radiating loudspeakers at low frequencies. Physically, one can imagine the air moving sideways out of the way, in response to the motion of the loudspeaker diaphragm, instead of moving backwards and forwards. In the hydrodynamic far-field, the stretching pressure is minimal, the acoustic impedance is dominated by resistance, and efficient sound propagation takes place. The only difference between the sound fields in the near- and far-fields is the rate of change of area with distance of the acoustic wave; the flare of a horn is a device for controlling this rate of change of area with distance, and hence the efficiency of sound propagation.

Horns are waveguides that have a cross-sectional area which increases, steadily or otherwise, from a small throat at one end to a large mouth at the other. An acoustic wave within a horn therefore has to expand as it propagates from throat to mouth. The manner in which acoustic waves propagate along a horn is so dependent upon the exact nature of this expansion that the acoustic performance of a horn can be radically changed by quite small changes in flare-shape. It is usually assumed in acoustics that changes in geometry that are small compared to the wavelength of the sound of interest do not have a large effect on the behaviour of the sound waves,

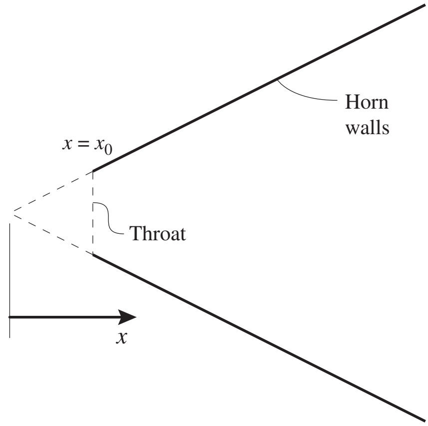

Figure 1.20. Geometry of a conical horn. The origin for the axial coordinate is usually taken as the imagined apex of the cone.

so why should horns be any different? The answer lies in the stretching pressure argument above. The concept of a stretching pressure can be applied to horns by considering *flare-rate*. Flare-rate is defined as the rate of change of area with distance divided by the area, and usually has the symbol m:

$$m(x) = \frac{1}{S(x)} \frac{\mathrm{d}S(x)}{\mathrm{d}x} \tag{1.39}$$

where $S(x)$ is the cross-sectional area at axial position x. The simplest flare shape is the conical horn, which has straight sides in cross-section and a cross-sectional area defined by

$$S(x) = S(0)\left(\frac{x}{x_0}\right)^2 \tag{1.40}$$

where $S(0)$ is the area of the throat (at $x = 0$) and x_0 is the distance from the apex of the horn to the throat as shown in Fig. 1.20. The sound field within a conical horn can be thought of as part of a spherical wave field, and has a flare-rate which is dependent on distance from the apex:

$$m(x) = \frac{2}{x} \tag{1.41}$$

The flare rate in a conical horn (and in a spherical wave field) is therefore high for small x and low for large x. For a spherical wave field, the radius r at which the resistive and reactive components of the acoustic impedance are equal in magnitude is when $kr = 1$ (see Section 1.3.4), at which point the flare-rate is, with the substitution of x for r,

$$m = 2k \tag{1.42}$$

Thus for positions within a conical horn where $kx < 1$, the acoustic impedance is dominated by reactance and the propagation is near-field-like. For positions where $kx > 1$, the impedance is resistive and the propagation is far-field-like. The radial dependence of the flare-rate in a conical horn (and a spherical wave) gives rise to a gradual transition from the reactive, near-field dominated behaviour associated with the stretching pressure, to the resistive, radiating, far-field dominated propagation as a wave propagates from throat to mouth. The transition from near- to far-field dominance is gradual with increasing frequency and/or distance from apex, so distinct 'zones' of propagation are not clearly evident.

A common flare shape for loudspeaker horns is the exponential. An *exponential horn* has a cross-sectional area defined by

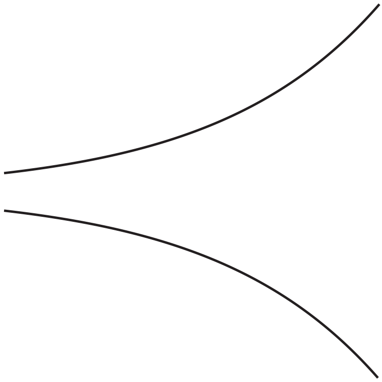

Figure 1.21. The flare shape of an exponential horn.

$$S(x) = S(0)\,\mathrm{e}^{mx} \tag{1.43}$$

The flare shape of the exponential horn is shown in Fig. 1.21. The flare-rate of an exponential horn is constant along the length of the horn ($m(x) = m$), giving rise to a behaviour that is quite different from the conical horn. With reference to equation (1.42), at frequencies where $k < m/2$, throughout the entire length of the horn, the reactive, near-field-type propagation dominates and, if the horn is sufficiently long, an almost totally reactive impedance exists everywhere. At frequencies where $k > m/2$, again throughout the entire length of the horn, the far-field-type propagation dominates leading to an almost totally resistive impedance everywhere. The frequency where $k = m/2$ is known as the *cut-off frequency* of an exponential horn and marks a sudden transition from inefficient sound propagation within the horn to efficient sound propagation. The cut-off frequency is then

$$k_c = \frac{m}{2}$$

Therefore

$$f_\mathrm{c} = \frac{mc}{4\pi}\ \text{(Hz)} \tag{1.44}$$

Physically, propagation within an exponential horn above cut-off is similar to a spherical wave of large radius, with minimal stretching pressure, and that below cut-off, similar to a spherical wave of small radius, dominated by the stretching pressure. The sharp cut-off phenomenon clearly occurs because the transition from one type of propagation to the other occurs simultaneously throughout the entire length of the horn as the frequency is raised through cut-off. The acoustic impedance at the throat of an infinite-length exponential horn is shown in Fig. 1.22, which clearly illustrates that, at frequencies below cut-off, the real part of the acoustic impedance is zero, which means that a source at the throat can generate no acoustic power (see Section 1.2.7). At frequencies above the cut-off frequency, the real part of the acoustic impedance is close to the characteristic impedance of air; a source at the throat therefore generates acoustic power with a radiation efficiency of 100%.

In practice, horns have a finite length and, unless the mouth of the horn is large compared to a wavelength, an acoustic wave propagating towards the mouth sees a sudden change in acoustic impedance from that within the horn to that outside, and some of the wave is reflected back down the horn. A standing-wave field is set up between the forward propagating wave and its reflection (see Section 1.2.3), which leads to comb-filtering in the acoustic impedance. Figure 1.23 shows the radiation efficiency at the throat of a typical finite-length exponential horn. Also shown are

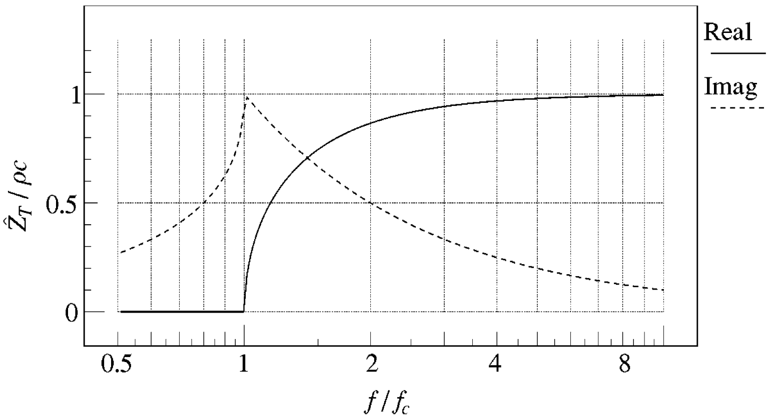

Figure 1.22. Acoustic impedance at the throat of an infinite-length exponential horn. f/f_c is the ratio of frequency to cut-off frequency and ρc is the characteristic impedance of air. No acoustic power can be radiated below the cut-off frequency as the real part of the acoustic impedance is zero.

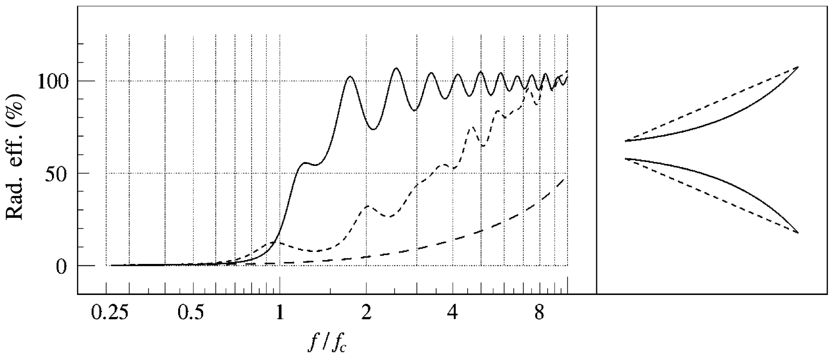

Figure 1.23. Radiation efficiency of an exponential horn (solid line) compared to that of a conical horn (short-dashed line) of the same overall size. Relatively small changes in the flare shape of a horn can have a large effect on the efficiency at low frequencies. The third curve (long-dashed line) is the radiation efficiency of a baffled piston having the same size as the throats of the horns.

the radiation efficiency of a conical horn having the same overall dimensions, and that of a piston the size of the throat mounted on an infinite baffle. The frequency scale is normalized to the cut-off frequency of the exponential horn. The comb-filtering, due to the standing wave field within the horn, can be seen, as can the improvement in radiation efficiency of the conical horn over the baffled piston, and of the exponential horn over the conical horn (at frequencies above cut-off).

The exponential horn acts as an efficient impedance matching transformer at frequencies above cut-off by giving the small throat approximately the radiation efficiency of the large mouth. The power output of a source mounted at the throat of a horn is proportional to the product of its volume velocity and the radiation efficiency at the throat; thus, a small loudspeaker diaphragm mounted at the throat of an exponential horn can radiate low frequencies with high efficiency. Below cut-off, however, the horn flare effectively does nothing, and the radiation efficiency is then similar to the diaphragm mounted on an infinite baffle. This seemingly ideal situation is marred somewhat by the sheer physical size of horn flare required for the efficient radiation of low frequencies. The cut-off frequency is proportional to the flare rate of a horn, which in turn is a function of the throat and mouth sizes and the length of the horn, thus

$$S(L) = S(0)\,\mathrm{e}^{mL}$$

so

$$m = \frac{1}{L} \ln \left\{ \frac{S(L)}{S(0)} \right\} \tag{1.45}$$

where L is the length of the horn, and $\ln\{\,\}$ denotes the natural logarithm. For a given cut-off frequency and throat size, the length of the horn is determined by the size of the mouth. To avoid gross reflections from the mouth, leading to a strong standing wave field within the horn, and consequently an uneven frequency response, the mouth has to be sufficiently large to act as an efficient radiator of the lowest frequency of interest. In practice, this will be the case if the circumference of the mouth is larger than a wavelength. For the efficient radiation of low frequencies, the mouth is then very large. Also, a low cut-off frequency requires a low flare-rate which, along with the large mouth, requires a long horn. By way of example, a horn required to radiate sound efficiently down to 50 Hz from a loudspeaker with a diaphragm diameter of 200 mm would need a mouth diameter of over 2 metres, and would need to be over 3 metres long! Compromises in the flare-rate raise the cut-off frequency, and compromises in the mouth size gives rise to an uneven frequency response. Reference 5 is a classic paper on the optimum matching of mouth size and flare-rate.

A radiation efficiency of 100% is not usually sufficient to yield the very high electroacoustic efficiencies of 10% to 50% quoted in the introduction of this section. However, unlike 'real' efficiency figures, which compare power output with power input, the radiation efficiency can be greater than 100% as the figure is relative to the radiation of acoustic power into the characteristic impedance of air, ρc. Arranging for a source to see a radiation resistance greater than ρc results in radiation efficiencies greater than 100%. A technique known as compression is used to increase the radiation efficiency of horn drivers; all that is required is for the horn to have a throat that is smaller than the diaphragm of the driver, as shown in Fig. 1.24.

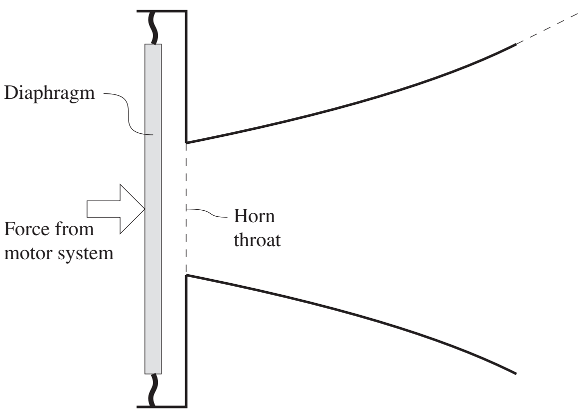

Figure 1.24. Representation of the principle behind the compression driver. Radiation efficiencies of greater than 100% can be achieved by making the horn throat smaller than the diaphragm.

Assuming that the cavity between the diaphragm and the throat is small compared to a wavelength, it can be shown that the acoustic impedance at the diaphragm is approximately that at the throat multiplied by the ratio of the diaphragm area to the throat area, known as the compression ratio

$$Z_{\mathrm{d}} \approx Z_{\mathrm{T}} \frac{S_{\mathrm{d}}}{S_{\mathrm{T}}} \tag{1.46}$$

where Z_{d} and S_{d} are the acoustic impedance and area at the diaphragm, and Z_{T} and S_{T} are the acoustic impedance and area at the throat. A compression ratio of 4:1 thus gives a radiation efficiency of 400% at the diaphragm. The 'trick' to achieving optimum electroacoustic efficiency is to match the acoustic impedance to the mechanical impedance (mass, damping, compliance etc.) of the driver. If the compression ratio is too high, the velocity of the diaphragm will be reduced by the additional acoustic load and the gain in efficiency is reduced. This can, however, have the benefit of 'smoothing' the frequency response irregularities brought about by insufficient mouth size, etc. Some dedicated compression drivers operate with compression ratios of 10:1 or more.

1.6.2 Directivity control

In addition to their usefulness as acoustic transformers, horns can be used to control the directivity of a loudspeaker. Equation (1.26) and Fig. 1.5 in Section 1.3.6 show that the directivity of a piston in a baffle narrows as frequency is raised. For many loudspeaker applications, this frequency-dependent directivity is undesirable. In a public address system, for example (as discussed in Chapter 10), the sound radiated from a loudspeaker may be required to 'cover' a region of an audience without too much sound being radiated in other directions where it may increase reverberation. What is required in these circumstances is a loudspeaker with a directivity pattern that can be specified and that is independent of frequency. By attaching a specifically designed horn flare to a loudspeaker driver, this goal can be achieved over a wide range of frequencies.

Consider the simple, straight-sided horn shown in Fig. 1.20. The directivity of this horn can be divided into three frequency regions as shown in Fig. 1.25. At low

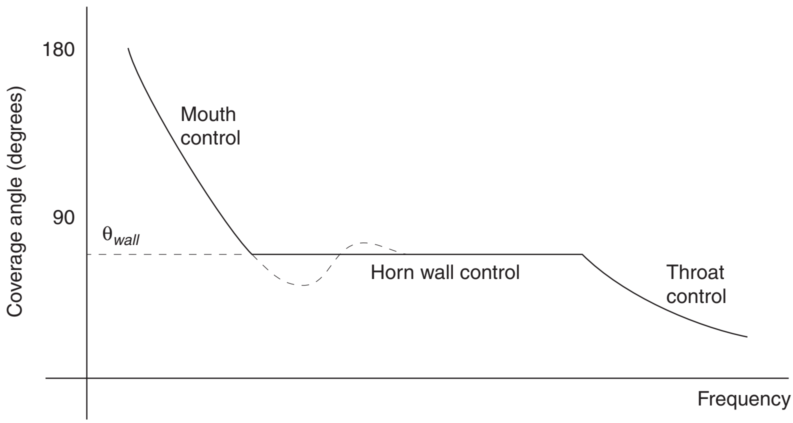

Figure 1.25. Simplified representation of the coverage angle of a straight-sided horn. At low frequencies, the coverage angle is determined by the size of the mouth, and at high frequencies by the size of the throat; the coverage angle in the frequency range between the two is fairly even with frequency and roughly equal to the angle between the horn walls (θ_{wall}). The dashed line shows a narrowing of the coverage angle at the lower end of the wall control frequency range which is often encountered in real horn designs.

frequencies, the coverage angle (see Section 1.3.6) reduces with increasing frequency in a manner determined by the size of the horn mouth, similar to a piston with the dimensions of the mouth. Above a certain frequency, the coverage angle is essentially constant with frequency and is equal to the angle of the horn walls. At high frequencies, the coverage angle again decreases with increasing frequency in a manner determined by the size of the throat, similar to a piston with dimensions of the throat. Thus the frequency range over which the coverage angle is constant is determined by the sizes of the mouth and of the throat of the horn. The coverage angle within this frequency range is determined by the angle of the horn walls. This behaviour is best understood by considering what happens as frequency is reduced. At very high frequencies, the throat beams with a coverage angle which is narrower than the horn walls as if the horn were not there. As frequency is lowered, the coverage angle (of the throat) widens to that of the horn walls and can go no wider. As frequency is further lowered, the coverage angle remains essentially the same as the horn walls until the mouth (as a source) begins to become 'compact' compared to a wavelength and the coverage angle is further increased, eventually becoming omni-directional at very low frequencies. The coverage angle shown in Fig. 1.25 is, of course, a simplification of the actual coverage angle of a horn. In practice, the mouth does not behave as a piston and there is almost always some narrowing of the directivity at the transition frequency between mouth control and horn wall control. A typical example of this is shown as a dashed line in Fig. 1.25. Different coverage angles in the vertical and horizontal planes can be achieved by setting the horn walls to different angles in the two planes.

1.6.3 Horn design compromises

Sections 1.6.1 and 1.6.2 describe two different attributes of horn loudspeakers. Ideally, a horn would be designed to take advantage of both attributes, resulting in a high-efficiency loudspeaker with a smooth frequency response and constant directivity over a wide frequency range. However, very often a horn designed to optimize one aspect of performance must compromise other aspects. For example, the straight-sided horn in Fig. 1.20 may exhibit good directivity control but, being a conical-type horn, will not have the radiation efficiency of an exponential horn of the same size. The curved walls of an exponential horn, on the other hand, do not control directivity as well as straight-sided horns. Early attempts at achieving high efficiency and directivity control in one plane led to the design of the so-called *sectoral horn* or *radial horn* shown in Fig. 1.26. In this design, the two side-walls of the horn are straight, and set to the desired horizontal coverage angle. The vertical dimensions of the horn are then adjusted to yield an overall exponential flare. Whereas the goals of high efficiency and good horizontal directivity control can be achieved with a sectoral horn, the severely compromised vertical directivity can be a problem. Given

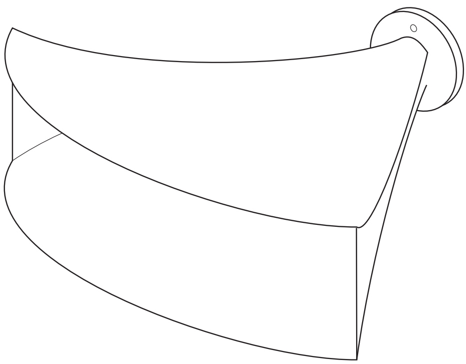

Figure 1.26. Sectoral or radial horn. The walls controlling the horizontal directivity are set to the desired coverage angle. The shape of the other two walls is adjusted to maintain an overall exponential flare resulting in less than ideal vertical directivity.

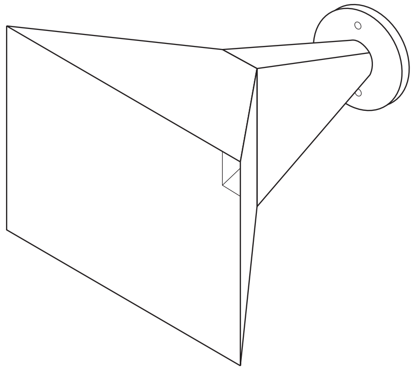

Figure 1.27. Constant directivity horn. Different horn wall angles in the two planes can be achieved using compound flares. Sharp discontinuities within the flare can set up strong standing-wave fields leading to an uneven frequency response.

that a minimum mouth dimension is required for directivity control down to a low frequency, setting the horizontal and vertical walls to different angles, for example 90° by 60°, means that different horn lengths are required in the two planes. To overcome this problem, later designs used compound flares[6] so that the exit angles of the horn walls can be different in the two planes, but the mouth dimensions and overall horn length remain the same. The so-called *constant directivity horn* (CD) is shown in Fig. 1.27. The sudden flare discontinuities introduced into the horn with these designs result in strong standing wave fields within the flare which can compromise frequency response smoothness. In fact, this is true of almost any flare discontinuity in almost any horn. Modern public address horn designs employ smooth transitions between the different flare sections and exponential throat sections to achieve a good overall compromise.

The control of directivity down to low frequencies requires a very large horn. For example, in a horn designed to communicate speech, directivity control may be desirable down to 250 Hz at a coverage angle of 60°. This can only be achieved with a horn mouth greater than 1.5 m across. The same horn may have an upper frequency limit of 8 kHz, which needs a throat no greater than 35 mm across. Maintaining 60° walls between throat and mouth then requires a horn length of about 1.3 m. Attempts to control directivity with smaller devices will almost always fail.

A large number of papers have been written on the subject of horn loudspeakers. As well as references 5 and 6 mentioned above, interested readers are referred to references 4 and 7 for a mathematical approach, 8 for an in-depth discussion on horns for high-quality applications and 9 for a very thorough list of historical references on the subject.