

$$(1) y = (a^2 + \tan^2 \theta * x^2)^{1/2}$$

Define

$$c = \tan^2 \theta \quad (\text{a constant})$$

then,

$$\begin{aligned} dy/dx &= 1/2 * (a^2 + c * x^2)^{-1/2} * 2 * c * x \\ &= c * x * (a^2 + c * x^2)^{-1/2} \end{aligned}$$

Solve for $dy/dx = \tan \alpha = b$, where α is the exit angle of CD,

$$b = c * x * (a^2 + c * x^2)^{-1/2}$$

Square both sides

$$b^2 = c^2 * x^2 * (a^2 + c * x^2)^{-1}$$

then

$$c * x^2 = c^2 * x^2 / b^2 - a^2$$

$$a^2 = c^2 * x^2 / b^2 - c * x^2$$

$$a^2 = (c^2 / b^2 - c) * x^2$$

$$x^2 = a^2 / (c^2 / b^2 - c)$$

$$x^2 = a^2 * b^2 / (c^2 - c * b^2)$$

$$x = a * b / (c^2 - c * b^2)^{1/2}$$

substituting back in our constants,

$$x_c = a * \tan(\alpha) / (\tan^4 \theta - \tan^2 \theta * \tan^2 \alpha)^{1/2}$$

let $d = \tan(\alpha) / (\tan^4 \theta - \tan^2 \theta * \tan^2 \alpha)^{1/2}$ and note that d is a function of α & θ alone.

$$x_c = a * d$$

we want to find a such that $y_c = 1$.

$$1 = (a^2 + \tan^2 \theta * (a * d)^2)^{1/2}$$

$$1 = a^2 + \tan^2\theta * a^2 * d^2$$

$$1 = a^2 * (1 + \tan^2\theta * d^2)$$

$$a = 1 / (1 + \tan^2\theta * d^2)^{1/2}$$

and note that a is a function of α & θ alone. Therefore x_c is a function of α & θ alone.

$$a = 1 / (1 + \tan^2\alpha / (\tan^2\theta - \tan^2\alpha))^{1/2}$$

so

$$x_c = 1 / (1 + \tan^2\alpha / (\tan^2\theta - \tan^2\alpha))^{1/2} * \tan(\alpha) / (\tan^4\theta - \tan^2\theta * \tan^2\alpha)^{1/2}$$

$$= \tan(\alpha) / [(1 + \tan^2\alpha / (\tan^2\theta - \tan^2\alpha)) * (\tan^4\theta - \tan^2\theta * \tan^2\alpha)]^{1/2}$$