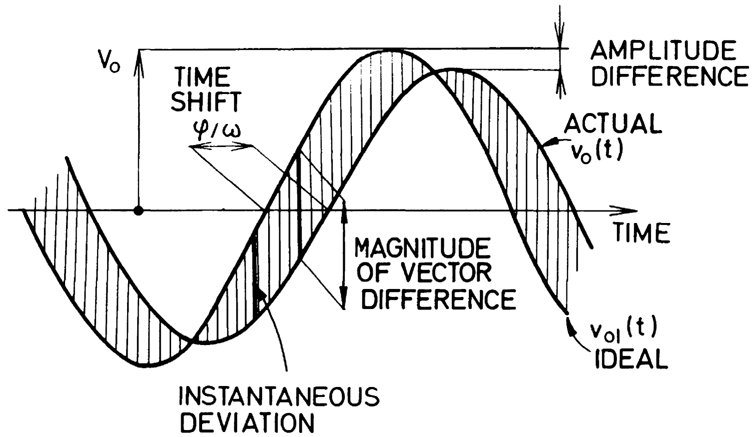

Figure 6–3. Interrelation between vector, amplitude, and phase errors in the time domain. (The phase error is represented by the time shift φ/ω.)

Confusing the vector error for amplitude error sometimes leads to misunderstanding; see, for example, Marzetta (3) and Mathews (4). To clarify the difference between the vector and amplitude errors, and to understand the relation between the vector and phase errors, the time-domain representation of Figure 6–3 may be useful.

In Figure 6–3, $v_{oI}(t)$ denotes the ideal output waveform and $v_o(t)$ represents the actual output waveform. If treated separately, only the difference in their amplitudes is observed, which is the essence of the amplitude error ϵ_A. An instantaneous confrontation of both waveforms yields different deviations, depending on the time of comparison, ranging from zero to a maximum value equal to the magnitude of the vector difference, which is the essence of the vector error ϵ_V. The cross-hatched difference in both waveforms is again a sine wave as a time expansion of the phasor $v_o - v_{oI}$.

With restriction to small errors, the largest deviation occurs near zero transition where linearization is possible:

$$v_{oI}(t) = V_o\omega t, \qquad v_o(t) = V_o(\omega t + \varphi).$$

The magnitude of the vector difference is simply $|v_o - v_{oI}| = V_o|\varphi|$ (the waveforms are drawn so that the phase-angle is negative) and the vector error $\epsilon_V = V_o|\varphi|/V_o = |\varphi|$ equals the phase error. For a numerical example, a vector error of

$$\epsilon_V = 0.0001 = 0.01\%$$

corresponds to a phase error of

$$|\varphi| = 0.0001 \text{ rad} = 0.0057°.$$

The interrelation of the vector and phase errors will be expressed more accurately in the next section.

6.1.2 Dynamic Errors of the Single-Pole Lag Network

Before starting analysis of the partial causes of error, we shall investigate the dynamic errors of a simple operational circuit with a normalized closed-loop gain G/G_I in the form

$$\frac{G}{G_I}(\mathrm{j}f) = \frac{1}{1 + \mathrm{j}f/f_H}, \tag{6.3}$$

suggesting the transfer function of a single-pole lag network. We are not interested in the frequency dependence of the ideal closed-loop gain G_I. We are concerned only with its deformation that arises from reality of the operational circuit and we represent this by a single lagging time constant corresponding to the upper frequency f_H. This idealized case covers a broad range of practical operational circuits. It includes the first-order resistive operational circuit (e.g., the voltage inverter and noninverting amplifier), and also the integrator in the frequency range $f \gg f_i/A_0$.

The individual dynamic errors will be easily found from their definitions [Eqs. (6.2a), (6.2b), and (6.2c)][2]:

$$\epsilon_V = \frac{f/f_H}{\sqrt{(1 + f^2/f_H^2)}}, \tag{6.4a}$$

$$-\epsilon_A = \frac{f^2/f_H^2}{\sqrt{(1 + f^2/f_H^2)}\,[1 + \sqrt{(1 + f^2/f_H^2)}]}, \tag{6.4b}$$

$$-\varphi = \arctan \frac{f}{f_H}, \tag{6.4c}$$

or,

$$\epsilon_V = \frac{f}{f_H}, \tag{6.5a}$$

$$-\epsilon_A = \frac{1}{2}\left(\frac{f}{f_H}\right)^2, \tag{6.5b}$$

$$-\varphi = \frac{f}{f_H} = \epsilon_V, \tag{6.5c}$$

for $f \ll f_H$. For a tenfold separation from the upper frequency f_H, i.e., for $f < f_H/10$, the inaccuracy of the approximate equations [Eqs. (6.5a), (6.5b), and (6.5c)] is less than 1% and thus it is immaterial in estimating the errors ϵ_V, ϵ_A, and φ. No other situation is worth consideration, anyway. At frequencies closer to the upper frequency, the dynamic errors increase to such an extent that the operational circuit becomes useless.

Surprising conclusions follow from Eqs. (6.5a), (6.5b), and (6.5c):

1. The vector error ϵ_V increases in proportion to frequency, achieving appreciable values even far below the upper frequency f_H. A distinct paradox arises. The operation within the vector error of 0.01% at a frequency f_1 requires the upper frequency f_H to be at least 10,000 f_1. To be specific, processing a low-frequency signal of 100 Hz by a noninverting amplifier with a gain of $G_I = 10$ requires the upper frequency f_H (identical with crossover frequency f_c) to be at least 1 MHz and the necessary operational amplifier unity-gain frequency f_t to be at least 10 MHz! Little can be done to moderate this discrepancy (see Section 6.3.4). Here, the general conflict between accuracy and speed manifests itself perhaps most dramatically.

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[2] Should calculation of Eq. (6.4) present any difficulty, the detailed derivation of dynamic errors in Section 6.3.1 can be used as a guide.

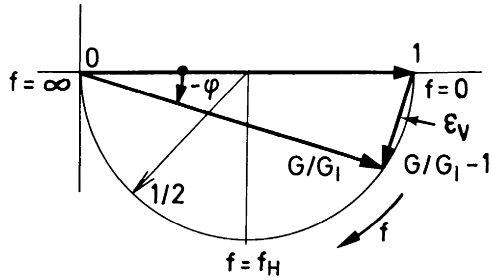

Figure 6–4. Normalized gain G/G_I of a single-pole lag network with upper frequency f_H.

2. Substantially more favorable results follow from judging the accuracy according to the amplitude error ϵ_A. To achieve the same error of 0.01% at a frequenty f_1, an upper frequency of $f_H = (100/\sqrt{2})\, f_1 = 71\, f_1$ will suffice. The same noninverting amplifier as above will process sine-wave signals with an amplitude error of less than 0.01% up to a frequency of 14 kHz.
3. The phase error φ expressed in radians coincides with the vector error ϵ_V. (It coincides except for the sign. The vector error, being the magnitude of the difference phasor, is always a positive number.)

This last conclusion also follows from Figure 6–4. Varying the frequency from zero to infinity causes the end point of the vector of normalized closed-loop gain G/G_I, according to Eq. (6.3), to move along a half circle below the vector 1 as a diameter. At low frequencies, both the vectors 1 and G/G_I are close to each other, their difference vector $G/G_I - 1$ is perpendicular to them, and its length $\epsilon_V = |G/G_I - 1|$ can be thought of as a measure of the phase angle $-\varphi$ in radians. Rotation of the vector G/G_I (i.e., the phase error φ) appears to be the main cause of the vector error ϵ_V, while contraction of the vector G/G_I (i.e., the amplitude error ϵ_A) is negligible in comparison. Although Eqs. (6.5a) and (6.5b) relate to a grossly simplified model, they are most useful and we shall often refer to them in subsequent sections.

6.1.3 Effect of Static Error

The preceding conclusions are illustrative, but conditional. Eq. (6.3) does not describe a general case since it implicitly assumes coincidence of the actual and ideal closed-loop gains at $f = 0$, that is, it assumes zero static error. Generalization will be achieved by the modified expression

$$\frac{G}{G_I} = \frac{\alpha}{1 + \mathrm{j}f/f_H}, \qquad \alpha \neq 1. \tag{6.6}$$

With a real constant α close to $+1$ and for $f \ll f_H$, the dynamic errors are

$$\epsilon_V = \sqrt{\left[(\alpha - 1)^2 + \left(\frac{f}{f_H}\right)^2\right]}, \tag{6.7a}$$

$$-\epsilon_A = 1 - \alpha + \frac{1}{2}\left(\frac{f}{f_H}\right)^2, \tag{6.7b}$$

$$-\varphi = \frac{f}{f_H}. \tag{6.7c}$$

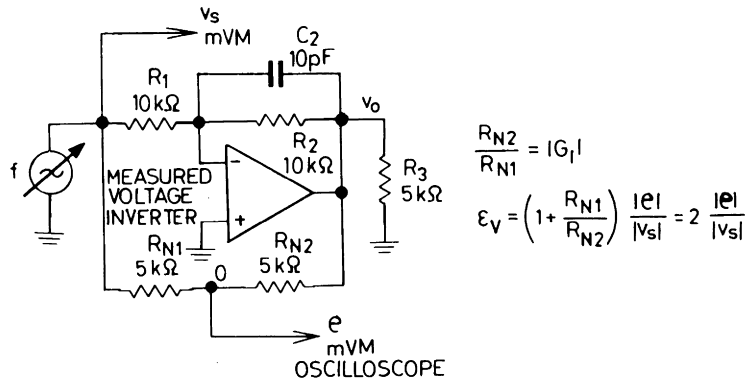

Figure 6–25. Measurement of the voltage-inverter vector error.

which has some relation to the dynamic error. With negligible loading of the artificial summing junction (if needed, a buffer emitter follower can be inserted),

$$e = v_s \frac{R_{N2}}{R_{N1} + R_{N2}} + v_o \frac{R_{N1}}{R_{N1} + R_{N2}} = -v_s \frac{R_{N2}}{R_{N1} + R_{N2}} \left(\frac{G}{G_I} - 1 \right).$$

In this expression, we have denoted $v_o/v_s = G$ and $-R_{N2}/R_{N1} = G_I$ as actual and ideal closed-loop gain, respectively. The term inside the parentheses is the dynamic error [Eq. (6.1)]. The desired vector error $\epsilon_V = |G/G_I - 1|$ is obtained by taking its magnitude,

$$\epsilon_V = \left(1 + \frac{R_{N1}}{R_{N2}} \right) \frac{|e|}{|v_s|}. \tag{6.38}$$

The indicated absolute values are to emphasize that only the *amplitudes* of voltages v_s and e, as measured by common ac millivoltmeters, are needed for evaluation of the vector error. The oscilloscope serves only as a monitor.

The amplitude of the sine-wave generator adapts to frequency. It should be large enough to prevent the voltage e from being lost in noise, and it should be small enough to keep the amplifier operating in the linear region (limited output voltage swing and rate). The most reliable indicator of correct measurement is a mutual proportionality between readings of $|v_s|$ and $|e|$ when the excitation amplitude is reduced to one-half.

A test circuit modified for measuring the vector error of a noninverting amplifier is shown in Figure 6–26. The input voltage v_s is applied to the amplifier under test as well as to an auxiliary voltage inverter modeling the ideal closed-loop gain $G_I = R_{N2}/R_{N1} + 1$. Both output voltages, v_o and $-G_I v_s$, are summed on the normal resistors R_N and their sum is evaluated as an error voltage e. The vector error is then calculated from

$$\epsilon_V = \frac{2}{R_{N2}/R_{N1} + 1} \frac{|e|}{|v_s|}. \tag{6.39}$$

Dynamic errors of the auxiliary inverter must be negligible. Since both amplifiers operate with about the same feedback factor, it is sufficient for the auxiliary inverter to specify an operational amplifier with a tenfold unity-gain frequency.

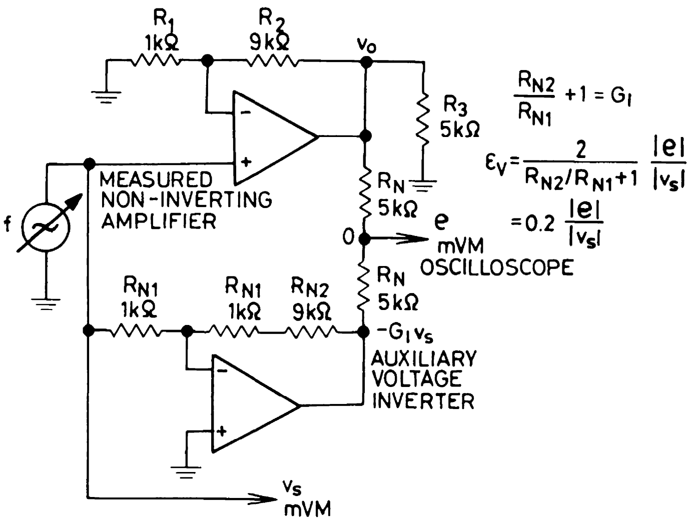

Figure 6–26. Measurement of the noninverting amplifier vector error.

6.5 Summary

1. Two kinds of errors are encountered in operational circuits: multiplicative and additive.
2. The multiplicative error of a linear operational circuit is the relative error of its closed-loop gain,

$$\epsilon = \frac{G}{G_I} - 1.$$

3. The vector error,

$$\epsilon_V = \left|\frac{G}{G_I} - 1\right|,$$

determines the accuracy of those operational circuits that process instantaneous values of general analog signals.
4. The amplitude error,

$$\epsilon_A = \left|\frac{G}{G_I}\right| - 1,$$

determines the accuracy of those operational circuits that process analog sine-wave signals fully characterized by their amplitudes.
5. The phase error,

$$\varphi = \arg\frac{G}{G_I},$$

has no meaning by itself. It is often used (sometimes justifiably, sometimes not) as an equivalent of the vector error.
6. The static error,

$$\epsilon_0 = \frac{G}{G_I}(j_0) - 1,$$

is the dc value of the amplitude error and (taken absolutely) the vector error.