

The equivalent input noise current of a valve is small enough to be negligible for RIAA amplifier design. The optimal anode bias current is the current that results in an optimal compromise between white noise and $1/f$ noise, both types becoming equivalent input voltage noise when you transfer them to the input.

For the most common types of audio triodes, Merlin Blencowe measured the noise and fitted the results to a simple noise model, see his paper in the November 2013 edition of the Journal of the Audio Engineering Society. This paper shows that the fit is sufficiently accurate for normal design work, as the fitting inaccuracy is much smaller than the sample-to-sample spread of the valves. Based on his article, the power spectral density of the equivalent input voltage noise is:

$$S(v_{n,eq}) = K \frac{I_A^2}{g_m^2 f} + 4k \frac{0.644}{\sigma} T_k \frac{1}{g_m} \quad (1)$$

K and σ are fitting parameters, which are tabulated for ECC81 (12AT7), ECC82 (12AU7), ECC83 (12AX7), ECC88 (6DJ8) and 6J52P valves in Merlin Blencowe's article. T_k is the cathode temperature, which Merlin Blencowe assumes to be 1000 K. If his temperature estimate is off, the σ value he found will automatically correct for it; that is, always take 1000 K when using his noise parameters. The other symbols have their usual meanings.

According to Child's law, the transconductance of a valve is ideally proportional to the third power root of the anode current. That is,

$$g_m = A \bullet \sqrt[3]{I_A} \quad (2)$$

where A is a constant that depends on the valve construction. Hence,

$$S(v_{n,eq}) = K \frac{I_A^{\frac{4}{3}}}{A^2 f} + 4k \frac{0.644}{\sigma} T_k \frac{1}{A I_A^{\frac{1}{3}}} \quad (3)$$

Taking the derivative to I_A and equating it to zero, the optimal anode current is found to be:

$$I_A = \left(\frac{0.644kAT_k}{\sigma K} \right)^{\frac{3}{5}} f^{\frac{3}{5}} \quad (4)$$

I've shown in an article in Electronics World October 2003 that for a given noise weighting you can calculate a kind of break-even frequency at which it doesn't matter for the total integrated noise whether the noise is white or $1/f$ or a bit of both. For IEC amended RIAA correction followed by A-weighting, this frequency is 1169 Hz.

The optimal anode current for minimal total weighted noise can now be estimated by plugging the break-even frequency into equation (4). A can be calculated from the valve datasheet using equation (2), σ and K are tabulated in Merlin Blencowe's article, T_k is assumed to be 1000 K and k is $1.38065 \bullet 10^{-23}$ J/K.

Merlin Blencowe's article also features graphs of the integrated noise from 200 Hz to 20 kHz versus the anode bias current. The noise optima in these graphs are not directly applicable to RIAA amplifier design because the break-even frequency is different. Assuming brick-wall filtering to make the math simple:

When the noise is $1/f$:

$$S(v_n) = \frac{K_1}{f} \quad (5)$$

the integrated noise from f_1 to f_2 is

$$\int_{f_1}^{f_2} S(v_n) df = \int_{f_1}^{f_2} \frac{K_1}{f} df = K_1 \ln\left(\frac{f_2}{f_1}\right) \quad (6)$$

When the noise is white

$$S(v_n) = K_2 \quad (7)$$

the integrated noise from f_1 to f_2 is

$$\int_{f_1}^{f_2} S(v_n) df = K_2 (f_2 - f_1) \quad (8)$$

The integrated noise is equal when

$$\frac{K_1}{K_2} = \frac{f_2 - f_1}{\ln\left(\frac{f_2}{f_1}\right)} \quad (9)$$

The noise densities are then equal at

$$f_{\text{break-even}} = \frac{f_2 - f_1}{\ln\left(\frac{f_2}{f_1}\right)} \quad (10)$$

With limits of 200 Hz and 20 kHz, the result is 4299.5 Hz. According to equation (4), this means that the optimal bias current for minimal RIAA- and A-weighted noise is about 45.78 % of the optimal current for uniform weighting from 200 Hz to 20 kHz.