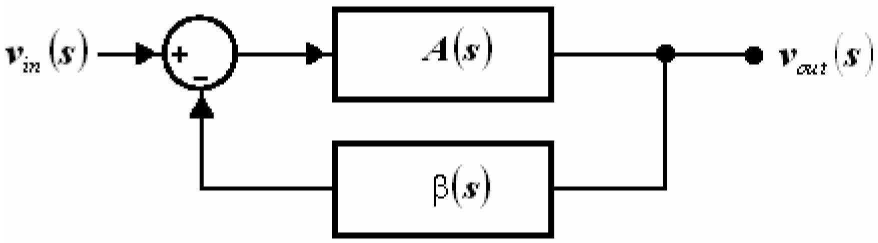
Measuring Loop Gain By Successive Voltage And Current Injection At An Arbitrary Impedance Test-Point. (Due To R. D. Middlebrook).

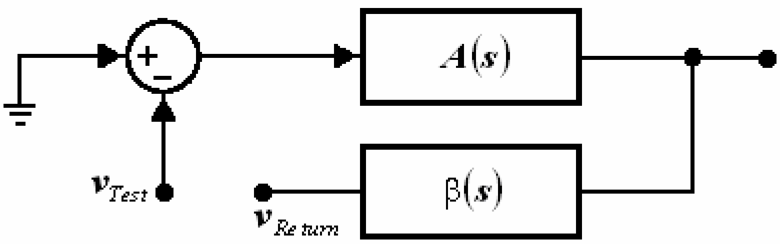


$$\frac{v_{out}}{v_{in}}(s) = \frac{A(s)}{1 + A(s)\cdot\beta(s)}$$

$\Longrightarrow$

Loop Gain $T(s) = +A(s)\cdot\beta(s)$

(a)



Voltage Return Ratio $T_v(s) = \dfrac{v_{Return}}{v_{Test}} = -A(s)\cdot\beta(s)$

$\Longrightarrow$

Loop Gain $T(s) = -T_v(s)$

(b)

Figure 1. Relationship between loop gain and return ratio.

For negative feedback, loop gain (or loop transmission) $T(s)$ is always positive (**fig. 1a**). Since the ratio of signal returned by the loop in response to a test signal (voltage or current) is negative (**fig. 1b**), the return ratio $T_v(s)$ (or $T_i(s)$ for current stimulus) is 180^0 out of phase with the actual loop gain.

The loop transmission path is arbitrarily represented (**fig. 2**) as a non-ideal voltage controlled current source (VCCS). Identical results may be obtained by assuming a non-ideal VCVS, CCCS, or CCVS.

The reversed polarity of the returned signal v_y with respect to the input stimulus v_x is reflected in the direction of the voltage arrows (**fig. 2**).

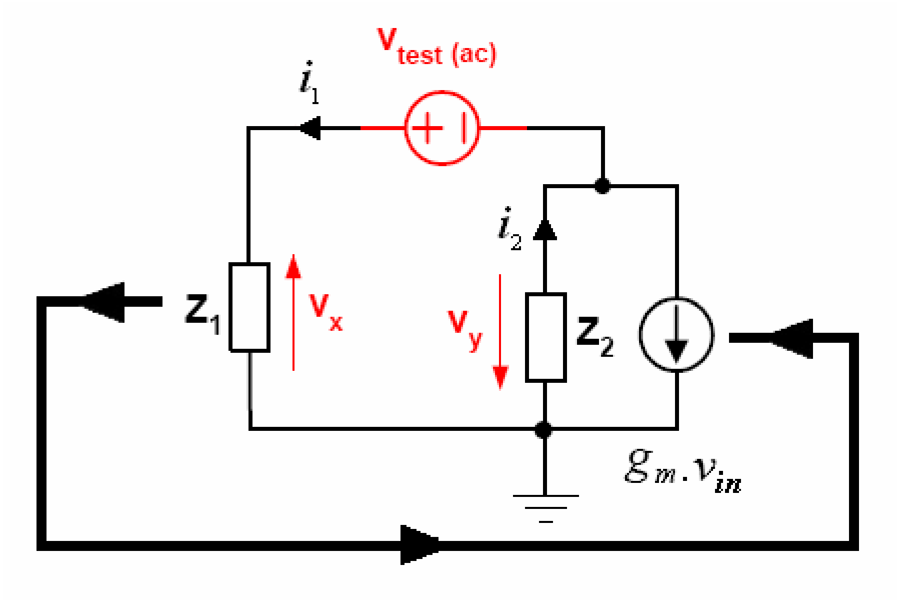


Figure 2. Voltage Injection.

The actual loop gain (in the absence of v_{test}) is obtained by inspection:

$$T(s) = g_m \cdot \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} \tag{1}$$

From fig 2:

$$v_y = i_2 \cdot Z_2 \tag{2}$$

And:

$$i_2 - g_m \cdot v_x - i_1 = 0$$

$\Rightarrow$

$$i_2 = g_m \cdot v_x + \frac{v_x}{Z_1} \tag{3}$$

(3) into (2):

$$v_y = v_x \cdot \left(g_m \cdot Z_2 + \frac{Z_2}{Z_1} \right)$$

The voltage ratio $T_v(s)$ is defined as (**fig. 1b**):

$$T_v(s) = \frac{v_y}{v_x}$$

$\Rightarrow$

$$T_v(s) = \left(g_m \cdot Z_2 + \frac{Z_2}{Z_1} \right) \tag{4}$$

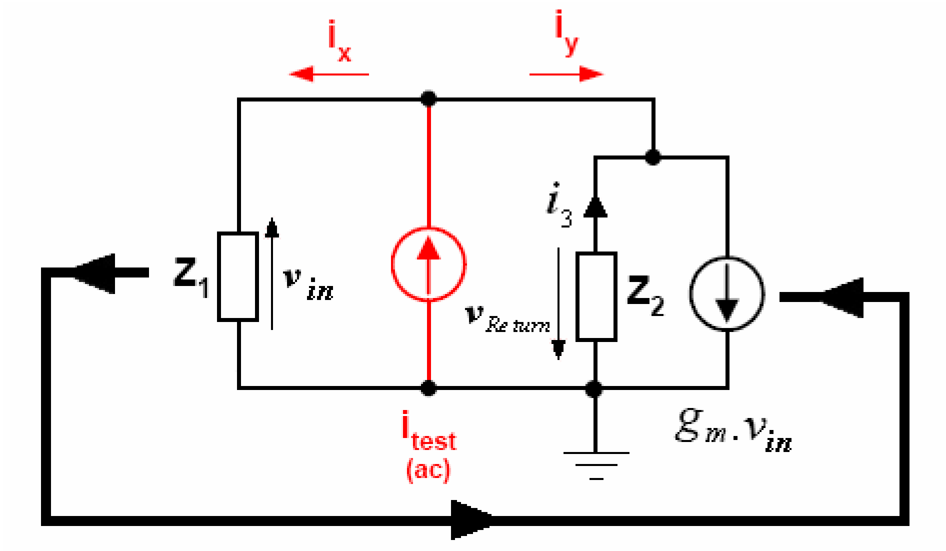


Figure 3. Current Injection.

For current injection, the reversed polarity of the returned signal i_y with respect to the input stimulus i_x is reflected in the direction of the current arrows (**fig. 3**).

$$i_y + i_3 - g_m \cdot v_{in} = 0$$

$\Rightarrow$

$$i_y = g_m \cdot v_{in} - \frac{v_{return}}{Z_2} \tag{5}$$

But:

$$v_{return} = -v_{in} = -i_x \cdot Z_1 \tag{6}$$

(6) into (5):

$$i_y = i_x \cdot \left(g_m \cdot Z_1 + \frac{Z_1}{Z_2} \right) \tag{7}$$

The current return ratio $T_i(s)$ is defined as:

$$T_i(s) = \frac{i_y}{i_x}$$

$\Rightarrow$

$$T_i(s) = \left(g_m \cdot Z_1 + \frac{Z_1}{Z_2} \right) \tag{8}$$

Express $T_v(s)$ in (4) in terms of $T(s)$ in (1) by eliminating g_m:

$$T_v(s) = T(s) \cdot \left(1 + \frac{Z_2}{Z_1}\right) + \frac{Z_2}{Z_1}$$

$\Rightarrow$

$$\frac{T_v(s) - T(s)}{T(s) + 1} = \frac{Z_2}{Z_1} \tag{10}$$

Express $T_i(s)$ in (8) in terms of $T(s)$ in (1) by eliminating g_m:

$$T_i(s) = T(s) \cdot \left(1 + \frac{Z_1}{Z_2}\right) + \frac{Z_1}{Z_2}$$

$\Rightarrow$

$$\frac{T_i(s) - T(s)}{T(s) + 1} = \frac{Z_1}{Z_2} \tag{11}$$

From (10) and (11):

$$\frac{T_v(s) - T(s)}{T(s) + 1} = \frac{T(s) + 1}{T_i(s) - T(s)}$$

$\Rightarrow$

$$T(s) = \frac{T_i(s) \cdot T_v(s) - 1}{T_i(s) + T_v(s) + 2} \tag{12}$$

Note that the alternative conventions shown in **fig. 4** and **fig. 5** give the same result.

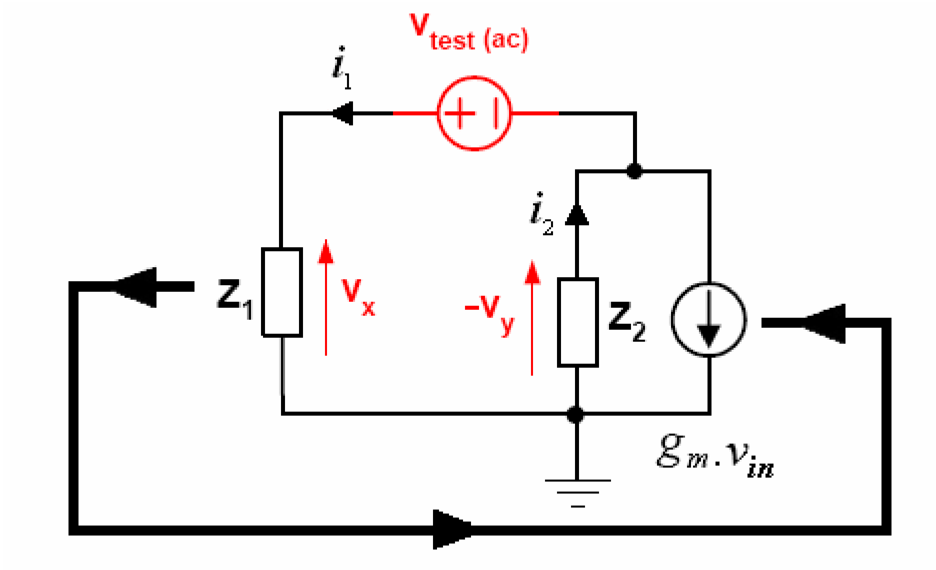


Figure 4. Alternative Voltage Arrow Convention For Voltage Injection.

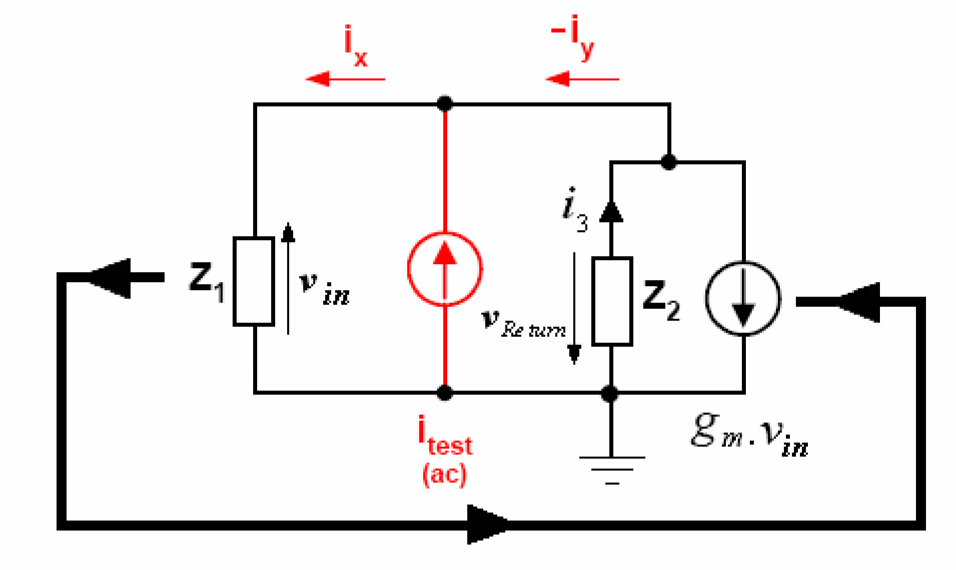


Figure 5. Alternative Current Arrow Convention For Current Injection.