

Matched-Delay Crossover Tutorial

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Abstract

FIR Gaussian-derived MDS crossovers are compared with linear-phase FIR Linkwitz-Riley-derived MDS crossovers of orders 2, 4, and 8; effects of impulse response truncation are included. IIR Bessel-derived MDS crossovers are compared with IIR Butterworth-derived and Linkwitz-Riley-derived MDS crossovers, for even orders 2 through 10. The two-stage MDS crossover is introduced; it increases the highpass transition band slope of the MDS crossover. The high-order “Gaussian-like” lowpass filter is introduced and applied to the MDS crossover; it is mostly of academic interest.

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