Discrete-Element Line Arrays—Their Modeling and Optimization*

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A number of phenomena of line arrays made up of multiple-drive units are observed. Unique near-field and far-field simulation programs show the cause of each phenomenon, thus giving an understanding of the line array's true operation. Schemes for improving the line array's polar performance and frequency invariance are examined.

0 INTRODUCTION

The author was responsible for the design of loudspeaker systems while at McIntosh Laboratory in Binghamton, NY. The company has a long history of designing home loudspeakers with line arrays of multiple dome tweeters to take advantage of their very high power handling [16 or 23 1-in (25.4-mm) domes for current models].

Being initially unfamiliar with such arrays, I had discussions with the engineers that turned up a number of interesting observations:

- "Level is roughly uniform when the listener is within the endpoints of the array."
- "Longer arrays are more uniform in frequency response (versus listener height)."
- "Arrays can be very directional at high frequencies."
- "At high frequencies, the polar response broadens again."

To explore these sometimes contradictory observations, a number of measurements were taken of the longer of two arrays in production. This array consists of twenty-three 25-mm dome tweeters in a line, with a spacing of 80 mm between tweeters. Figs. 1–6 show the vertical polar responses of the array as measured from a distance of 4 m for frequencies of 0.5, 1, 2, 4.3, 8, and 16 kHz. (The reason for 4.3 kHz will be explained later.)

With these polar measurements we can see a number of interesting phenomena. At 0.5 kHz (Fig. 1) the array has a beamwidth of 25–30° between 6-dB down points. Rear firing energy is similar to front firing energy, and lobing is smooth. For higher frequencies, encroaching side ripple may vary the exact −6-dB beamwidth, but the overall directivity does not change substantially through 2 kHz. Then at 4.3 kHz (Fig. 4) lobes appear at ±90°. At higher frequencies (8 kHz, Fig. 5) these lobes swing toward the 0° angle and additional lobes form. At still higher frequencies (16 kHz, Fig. 6) the lobes all merge and the system takes on a rough, but distinctly less directional polar response.

Two of our initial observations are confirmed by these measurements. First the beamwidth is roughly constant within the endpoints, at least for the central beam. With 23 elements and an 80-mm spacing, the array has a total length of 1.76 m. Since the system was measured at a distance of 4 m, simple calculations show that it could be rotated 12.7° before the end tweeter would be the one firing perpendicularly at the microphone. Doubling this for a 25.4° swing "between endpoints" gave a rough correlation with the observed beamwidths. Second, we observe a very distinct broadening of the response at 8 and 16 kHz.

1 FAR-FIELD COMPUTER MODEL FOR POLAR RESPONSE OF ARRAY

To explore some of these phenomena, a means of computer simulation was desired. As a first approach it was felt that a program that would allow predicting the far-field polar response could answer a number of questions.

The concept of such a program is simple. In the far field an array's polar response is determined by the sum of the contributing vectors of each element. If the array's

* Presented at the 99th Convention of the Audio Engineering Society, New York, 1995 October 6–9; revised 1997 August 20.

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geometry is modeled and related to the radiated wavelength, then the phase of each vector can be predicted. If these vectors are summed with full regard to their phases, we should be able to simulate an array's response accurately for any observation angle. By recalculating the summation for a complete circle of observation angles, we can model the polar response.

For the program a number of assumptions were made:

- Each element is an omnidirectional point source.
- Each element has a magnitude of 1 referred to a distance of 1 m, unless a weighting factor is applied.
- The array is modeled as if viewed from infinity, but using vector strengths as if each element were always 1 m away, that is, the level is referenced to the combined level "at 1 m."

From these conditions we can state that the polar response of a single element is a circle at a level of 0 dB. (Most of the calculated polar responses plotted in this study are renormalized to have their maximum points 1 dB less than the radius of the graphs.)

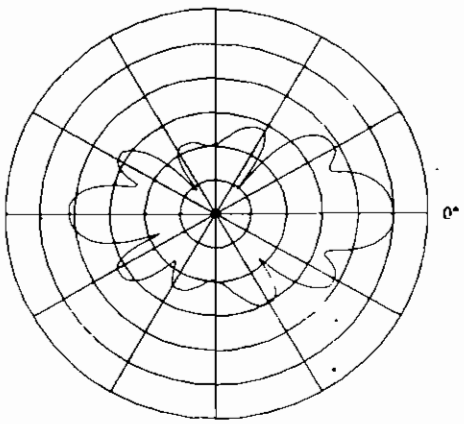

Fig. 1. Polar response of 23-tweeter array, 0.5 kHz.

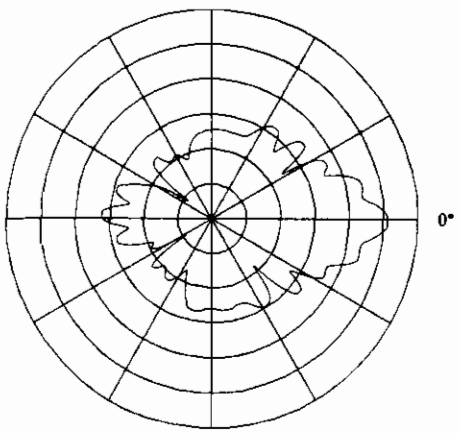

Fig. 2. Polar response of 23-tweeter array, 1 kHz.

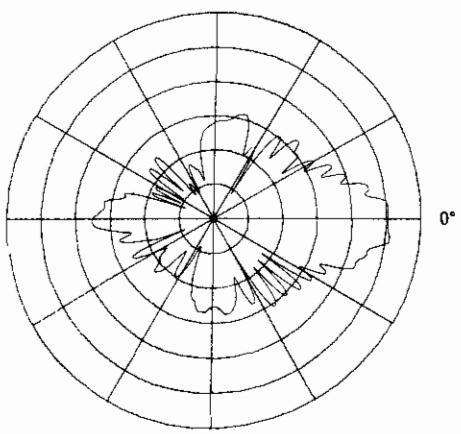

Fig. 3. Polar response of 23-tweeter array, 2 kHz.

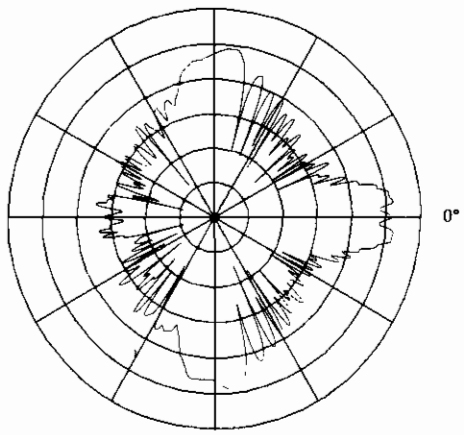

Fig. 4. Polar response of 23-tweeter array, 4.3 kHz.

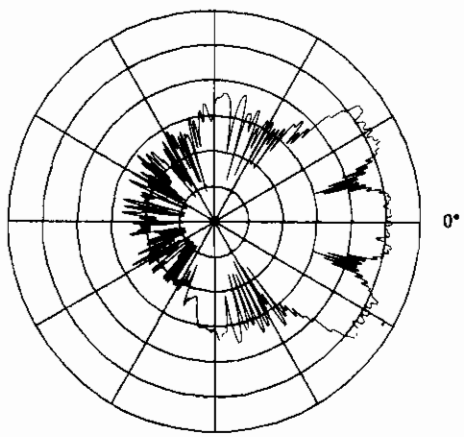

Fig. 5. Polar response of 23-tweeter array, 8 kHz.

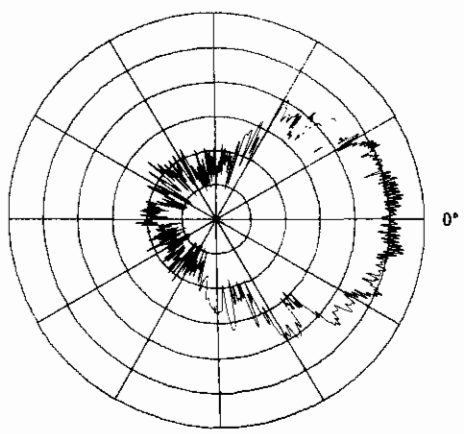

Fig. 6. Polar response of 23-tweeter array, 16 kHz.

Throughout this engineering report, it is assumed that rotation is around the center axis that bisects an array's length. For a home loudspeaker the typical usage would be with the long axis vertical. The polar response of interest would then be the vertical polar, with 0° defined as perpendicular from the array's midpoint.

Since the model presumes omnidirectional sources, it ignores the polar responses of real tweeters. At large observation distances each element would be viewed at the same radiation angle. That is to say that when the array is at 30°, the observer is on the 30° axis of each tweeter. Therefore the polar response (versus frequency) of a real tweeter (in decibels) could be added to the simulation of the idealized polar response. With typical tweeters this would reduce the outputs of all side and back lobes at most frequencies. Although this would make a more accurate model, the lack of consideration of the polar responses of real elements in no way detracts from the usefulness of the program.

In the horizontal plane the apparent width of the array is one tweeter. It is assumed (and generally observed) that the horizontal polar response is therefore no different for a vertical array than it is for one tweeter.

For an array viewed from a large distance and rotated around its midpoint, tweeters on one end advance while those on the other end recede. The amounts of advancement and recession are related to the amount of rotation and in proportion to the distance from the axis of rotation. Our model predicts the advancement and the recession for each element and relates that to the intended wavelength (thus defining degrees of advance or retard). Through array rotation each element becomes a rotating unit vector. The amount of vector rotation (in radians) is equal to the number of wavelengths of advancement or recession. The in-phase and quadrature components of each element are summed independently for each observation angle, yielding the far-field polar response.

The formula is, in simplified form,

response at angle α =

$$\sqrt{\left(\Sigma \frac{\cos 2\pi nd}{\lambda \sin \alpha}\right)^2 + \left(\Sigma \frac{\sin 2\pi nd}{\lambda \sin \alpha}\right)^2}$$

where

n = element number
d = element-to-element spacing
λ = wavelength of frequency of interest
α = angle relative to axis of normal radiation.

1.1 Results of Far-Field Polar Model

Olson shows various examples of the calculated polar response of short arrays [1]. The model was confirmed against these examples with good agreement. Fig. 7 shows Olson's predicted polar responses of two elements spaced 5/2 wavelengths apart. In total we see five lobes in the half-circle of radiation with no energy in the right-angle directions. Our prediction of the same example (Fig. 8) shows a similar polar pattern. Several points should be noted. All of our predicted polar responses are based on omnidirectional radiators and will thus show equal front and rear radiation patterns. Also, 0° is to the right on our plots. Our plots are logarithmic in scale (Olson's are linear). The scale has 10 dB per ring and coincides with B + K polar charting paper.

Having perhaps too much confidence in the accuracy of our plots, we proceeded to model the 23-tweeter array, previously measured in Figs. 1–6. These are shown as Figs. 9–14. Results of the simulations are mixed. At the lowest frequencies the comparisons are good, but as the frequency increases, the simulated polars show distinctly narrower responses around the axial lobes. This discrepancy will be resolved later in this report. One phenomenon, the large secondary lobe at a right angle to the array (radiating off the array's end) was confirmed. As observed, this lobe would swing forward for higher frequencies. At still higher frequencies, second and third sidelobes formed with shapes similar to the original one.

1.2 What Is the Cause of These Lobes?

Consider viewing our array in line with its long axis, 90° up or down from the normal listening axis (Fig. 15). At very low frequencies array dimensions are insignificant and all the elements are in phase to the observer. However, as the frequency rises, the element spacing becomes significant and interunit phase shifts mount.

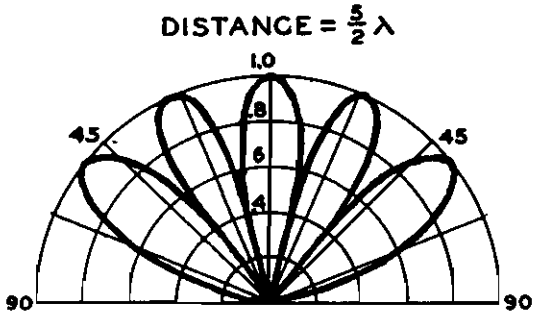

Fig. 7. Calculated polar response of two elements, 5/2 λ spacing. (From [1].)

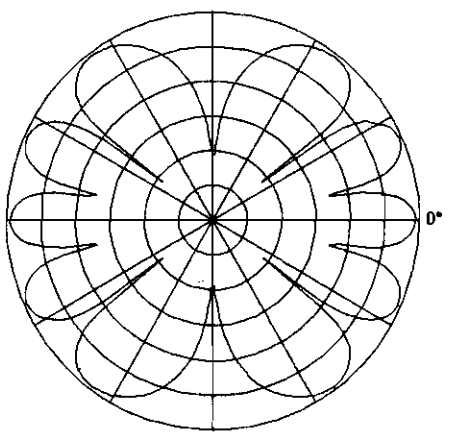

Fig. 8. Author's example, similar to Fig. 7.

As an example, when the unit-to-unit spacing is ¼ wavelength, then relative to the nearest unit, each receding unit has an additional 90° phase shift. Every four units the phase shift cycles through a complete phase rotation with (for the far-field model) no net contribution from those four units. (The first and third units cancel each other, as do the second and fourth.) If the number of elements in our array is an integral multiple of 4, the array has no net output at the 90° angle (for this frequency).

At ½ wavelength spacing similar results are found. Each receding element has an additional 180° phase shift. If there is an even number of elements, they will all cancel completely in the far field. At these two frequencies we can predict complete nulls at 90° in the polar curves. At intermediate frequencies there may not

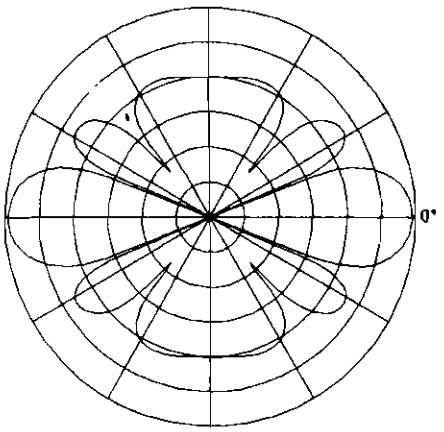

Fig. 9. Calculated far-field polar response of 23-element array, 0.5 kHz.

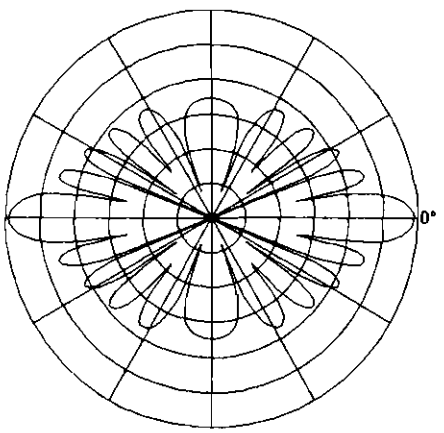

Fig. 10. Calculated far-field polar response of 23-element array, 1 kHz.

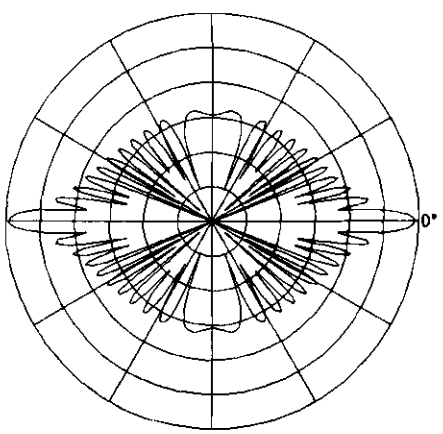

Fig. 11. Calculated far-field polar response of 23-element array, 2 kHz.

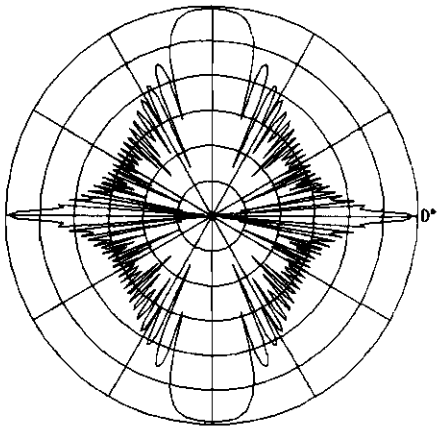

Fig. 12. Calculated far-field polar response of 23-element array, 4.3 kHz.

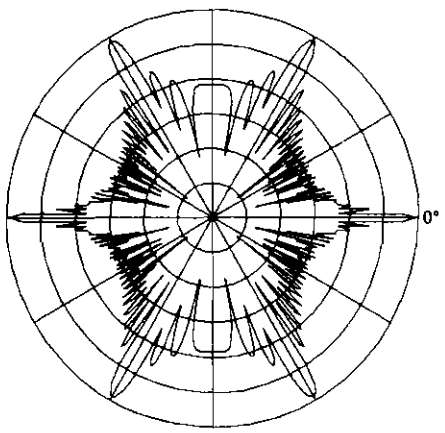

Fig. 13. Calculated far-field polar response of 23-element array, 4970 Hz.

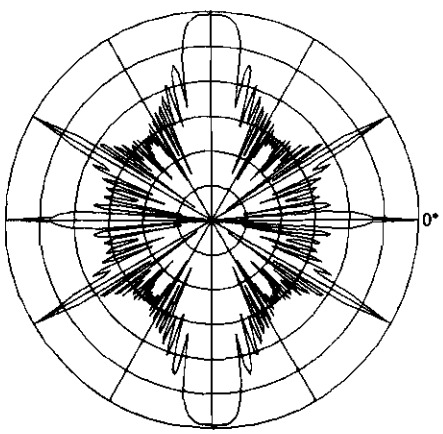

Fig. 14. Calculated far-field polar response of 23-element array, 8600 Hz.