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Analysis of Front Loaded Low Frequency Horn Loudspeakers

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ABSTRACT

The low frequency horn design procedures described by Keele and Leach are extended and generalized to cases where the horn is already specified, or where maximum output or the smoothest response is desired. The impact of finite-length horns is analysed. A more detailed analysis of the high frequency range is given, where it is shown how the voice coil inductance can be taken into account to create a third order low pass filter of specified shape. A new analysis of reactance annulling is presented that significantly improves the performance above cutoff for a certain class of horns. Large signal behavior is touched upon, and finally, an analysis of the sensitivity of driver and system parameters is given.

1 Introduction

About 40 years ago, Keele [1], Small [2] and Leach [3] presented papers on the design of low frequency horns using Thiele-Small parameters. The papers give good basic information, but are somewhat restricted in that they only consider maximum efficiency or maximum sensitivity designs for infinite horns. The purpose of this paper is to extend the work described in these papers.

In some cases, a horn is already available, and one wants to select or design a drive unit for it, giving a specified response. The horn may be short, with large ripples in the throat impedance, and a system design that minimizes the response variations may be desirable.

Reactance annulling is reviewed and it is found that the traditional method of reactance annulling has some

shortcomings when considering the frequency range above cutoff. While efficiency is maximized close to the horn cutoff frequency, it is in many cases reduced in the octave above it. A new method is presented to reduce this shortcoming.

While Small's 1977 paper includes some discussion of large signal behavior, his analysis is based on a driver loaded by a plane wave tube. The current paper will therefore look into the case where the driver is loaded by an infinite horn.

It has also been found that by including the voice coil inductance (and potentially an extra inductor) in the analysis, the high frequency response can be shaped to conform to known third-order low pass filter functions. A procedure for this is given.

None of the previously mentioned papers have included any analysis of the sensitivity of the system response to variation in driver and system parameters. This is quite important for all loudspeaker systems due to drive unit production tolerances, and a sensitivity of vented enclosures have previously been published by Keele [4]. The current paper therefore explores the sensitivity of the system performance to changes in C_{MS} , M_{MD} , R_E , Bl , front and rear chamber volumes, and horn throat area.

2 Front Loaded Horns

The front loaded horn is the most common of all horn loudspeaker configurations, as this is the configuration used in practically all compression driver-and-horn combinations in use. Many bass horns are also front loaded, with a closed rear chamber. A typical configuration of a front loaded horn is shown in figure 1, with the acoustic equivalent circuits shown in figure 2. The symbols are defined in [3], but a short summary is provided here.

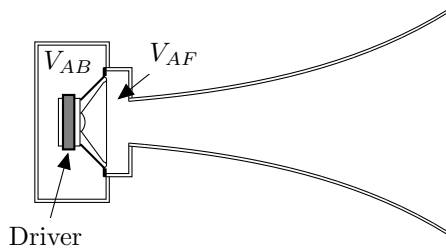


Fig. 1: Typical front loaded horn

$$\begin{aligned} p_{AE} &= \frac{e_g Bl}{S_d R_E}, & R_{AT} &= R_{AE} + R_{AM}, \\ R_{AE} &= \frac{(Bl)^2}{S_d^2 R_E}, & R_{AM} &= \frac{R_{MS}}{S_d^2} + R_{AB}, \\ R_{AL} &= \frac{\rho_0 c}{S_t}, & C_{AT} &= \frac{C_{AM} C_{AB}}{C_{AM} + C_{AB}} \end{aligned}$$

R_{AB} describes the loss in the rear chamber, $C_{AM} = S_d^2 C_{MS}$, $C_{AB} = V_{AB}/\rho_0 c^2$ is the compliance of the rear chamber, and

$$M_{AD} = \frac{M_{MD}}{S_d^2}, \quad C_{AE} = \frac{S_d^2 L_E}{(Bl)^2}.$$

The system Q is defined as

$$\begin{aligned} Q_{TC} &= \frac{1}{R_{AT} + R_{AL}} \sqrt{\frac{M_{AD}}{C_{AT}}} = \frac{\omega_0}{\omega_L + \omega_H} \\ &= \frac{1}{\omega_0 (R_{AE} + R_{AM} + R_{AL}) C_{AT}}. \end{aligned} \quad (1)$$

where the corner frequencies ω_H and ω_L are given by the simplified midband equivalent operating into a resistive acoustic load, and ω_0 is the resonance frequency of M_{AD} and C_{AT} . See [3].

It may also be useful to define a set of partial Q -factors:

$$\begin{aligned} Q_{EC} &= \frac{1}{\omega_0 R_{AE} C_{AT}}, & Q_{MC} &= \frac{1}{\omega_0 R_{AM} C_{AT}}, \\ Q_{LC} &= \frac{1}{\omega_0 R_{AL} C_{AT}}, & Q_{TC}^{-1} &= Q_{EC}^{-1} + Q_{MC}^{-1} + Q_{LC}^{-1}. \end{aligned}$$

Efficiency can be expressed by these Q -factors:

$$\eta_c = \frac{Q_{TC}}{Q_{LC}} \cdot \frac{Q_{TC}}{Q_{EC} - Q_{TC}}. \quad (2)$$

As show by Leach, the conversion efficiency is maximized if $R_{AL} = \sqrt{R_{AM} R_{AT}}$, and sensitivity is maximized if $R_{AL} = R_{AT}$.

It will be useful in later calculations to also define an impedance matching parameter β as

$$\beta = \frac{R_{AL}}{R_{AT}} = \frac{S_d}{S_t} \cdot \frac{\rho_0 c S_d n R_E}{(Bl)^2 + R_E S_d^2 R_{AM}} \quad (3)$$

which we can see is proportional with the compression ratio S_d/S_t .

3 High-Frequency Range with Inductance Included

Leach provides an analysis of the high frequency range of the horn speaker neglecting the voice coil inductance, and shows how C_{AF} can be chosen to maximize the upper corner frequency. If the voice coil inductance is not negligible, the resulting high frequency equivalent circuit will look like figure 3. This circuit can be recognized as a third order lowpass filter, and in some cases it may be desirable to adjust the components of this filter to obtain a third order response at a given cutoff frequency, for instance to roll off the response of a bass horn in a multiway system.

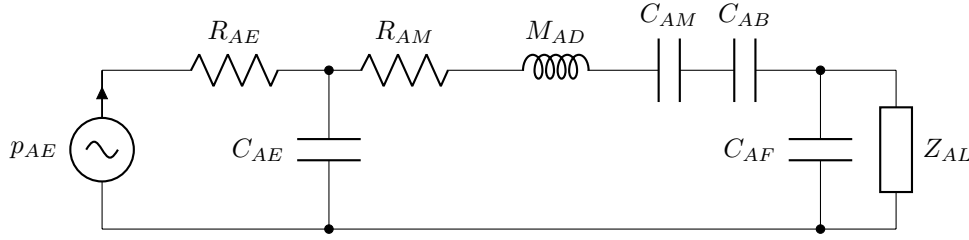


Fig. 2: Front loaded horn loudspeaker equivalent diagram, acoustic domain.

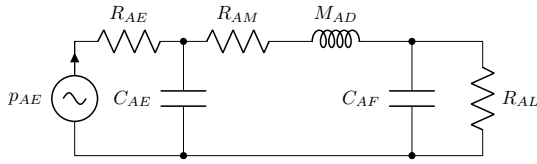


Fig. 3: High Frequency acoustical equivalent circuit with electrical inductance included.

Since tabulated values for normalized filters of several types exist, designing the horn loudspeaker to approximate one of the standard filter responses is relatively easy. Tabulated values are given in standard text books for filters with equal and unequal termination resistances [5], see figure 4a.

If unequal termination of the filter is required, Bartlett's Bisection Theorem may be applied. This theorem states that if a symmetrical filter is bisected, as in figure 4b, and one half is impedance scaled, the response of the filter will not change. A method for this will be given below. It is assumed that R_{AM} is negligible.

In order to use tabulated filter values, the values must be denormalized, as the filters are tabulated for a frequency of 1 rad/s and termination impedances of 1Ω. The process is described in [5].

Let C_1 , L_2 and C_3 be the tabulated, normalized filter values, see figure 4a. With $\beta = R_{AL}/R_{AE}$, the transformed, normalized inductance L to use in the calculations can be found using Bartlett's Bisection Theorem as

$$L = L_2 \frac{1}{2} (1 + \beta), \quad (4)$$

which gives

$$M_{AT} = \frac{R_{AE} L_2^{\frac{1}{2}} (1 + \beta)}{\omega_3}. \quad (5)$$

Similarly,

$$C_{AE} = \frac{C_1}{\omega_3 R_{AE}}, \quad C_{AF} = \frac{C_3}{\omega_3 \beta R_{AE}}. \quad (6)$$

From the definitions of C_{AE} and C_{AF} , it follows that

$$L_E = \frac{C_1 R_E}{\omega_3}, \quad C_{AF} = \frac{C_3 S_d^2 R_E}{\omega_3 \beta (Bl)^2} = \frac{C_3 S_t}{\rho_0 c \omega_3} \quad (7)$$

In most cases, the diaphragm mass is fixed, and either ω_3 or β must be adjusted to get the desired filter function. For a given diaphragm mass, these are as follows. For a given β ,

$$\omega_3 = \frac{R_{AE} L_2 (1 + \beta)}{2 M_{AD}} = \frac{(Bl)^2 L_2 (1 + \beta)}{2 R_E M_{MD}}, \quad (8)$$

and for a given ω_3 the necessary β is

$$\beta = \frac{2 M_{AD} \omega_3}{R_{AE} L_2} - 1 = \frac{2 \omega_3 M_{MD} R_E}{(Bl)^2 L_2} - 1. \quad (9)$$

A few examples of third order responses are given in figure 5, with the component values given in table 1. The driver parameters are $Bl = 18 \text{ Tm}$, $R_E = 6 \Omega$, $S_d = 350 \text{ cm}^2$, $C_{MS} = 2 \cdot 10^{-4} \text{ m/N}$, $M_{MD} = 64.5 \text{ g}$ and $R_{MS} = 2 \text{ Ns/m}$. Two of the curves show the response without C_{AF} , and with the optimum C_{AF} for a second order response, illustrating the difference between second and third order responses using the same driver. It can be seen that the Butterworth type filter response gives a -3 dB point close to ω_H , the mass corner frequency, while the Chebychev type filter responses either produces a much lower corner frequency, or requires a higher compression ratio.

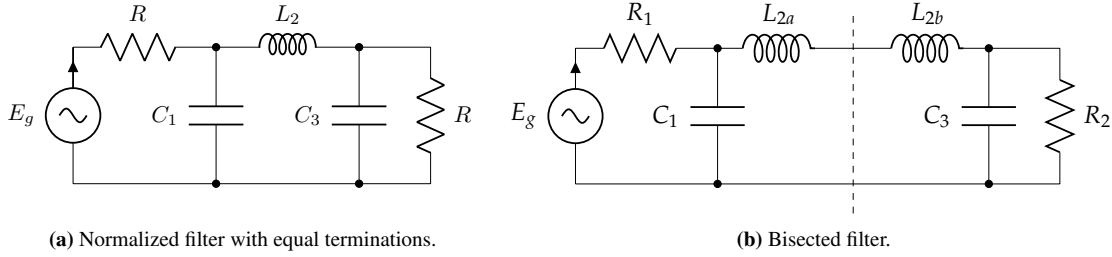


Fig. 4: Using Bartlett's Bisection Theorem.

Curve	f_3 [Hz]	L_E [mH]	V_{AF} [I]
$\beta = 1$, B3	800	1.19	0.644
$\beta = 1$, C3	435	4.86	2.62
$\beta = 2.68$, C3	800	2.65	0.533

Table 1: Component values for the third order responses in figure 5.

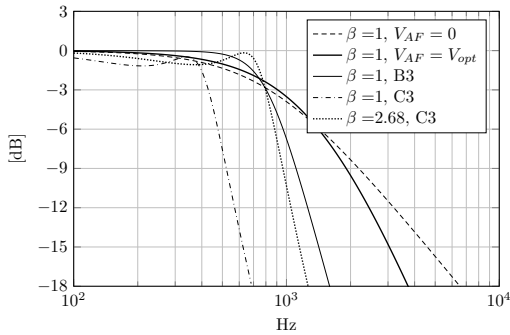


Fig. 5: Examples of third order horn loudspeaker responses, compared to second and first order responses. Data for the third order systems are found in table 1. B3 is third order Butterworth, C3 is third order 1 dB ripple Chebychev.

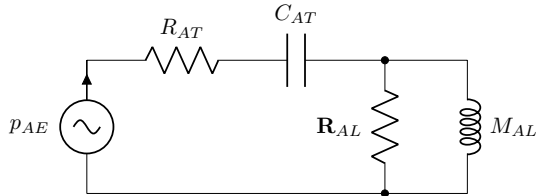


Fig. 6: Low frequency simplified equivalent circuit.

4 Analysis Low-Frequency Range

The simplified equivalent circuit for the low frequency range is shown in figure 6. In this range, it is not

possible to regard the horn as a pure resistance anymore, and an approximation that enables relatively simple expressions to be derived, is to use an infinite horn. It can be shown that the impedance can be expressed as a parallel connection of a mass reactance and a frequency dependent resistance. For a hyperbolic-exponential horn above cutoff, these are [6]

$$M_{AL} = R_{AL} \frac{1}{T \omega_c} \quad (10)$$

$$\mathbf{R}_{AL} = R_{AL} \frac{1}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}, \quad (11)$$

where the bold \mathbf{R}_{AL} indicates the frequency dependence, and $R_{AL} = \rho_0 c / S_t$ as before. ω_c is the cutoff frequency of the horn.

From these expressions, the radiated power can be calculated as

$$\begin{aligned} P_{AR} &= \frac{P_{AE}^2}{|Z_{AT}|^2} \Re(Z_{AL}) \\ &= \frac{(Bl)^2 e_g^2}{S_d^2 R_E^2} \cdot \frac{R_{AL} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}{\left\{1 - \left(\frac{\omega_c}{\omega}\right)^2 (1 - T^2)\right\} |Z_{AT}|^2} \end{aligned} \quad (12)$$

where

$$\begin{aligned} |Z_{AT}|^2 &= \left(R_{AT} + \frac{R_{AL} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}{1 - \left(\frac{\omega_c}{\omega}\right)^2 (1 - T^2)} \right)^2 \\ &+ \left(R_{AL} \frac{T \frac{\omega_c}{\omega}}{1 - \left(\frac{\omega_c}{\omega}\right)^2 (1 - T^2)} - \frac{1}{\omega C_{AT}} \right)^2. \end{aligned} \quad (13)$$

We can see here that the radiated power would be maximized if the denominator of equation (12) was minimized, and this can be achieved with the technique

commonly known as reactance annulling. Perhaps the earliest reference to this technique is found in Paul B. Flanders' second memorandum on horn theory [7] from 1924, where he writes:

"[...] if possible, it would seem desirable to make receiver [driver] elements so that they would have impedance characteristics nearly equal to the minus reactance surge impedance of the small end of the horn [...] for the required frequency interval."

Flanders' description highlights that reactance annulling is not so much about resonating the horn acoustic mass with the system compliance, but to provide a conjugate impedance match between the driver and the horn, in order to maximize power transfer. The technique was first publicly described by Albert L. Thuras in the patent for the W.E. 555-W compression driver [8], and later expanded on in the patent for the low frequency driver for the Fletcher system [9]. The method was later rediscovered and used by Paul W. Klipsch in his Klipschorn [10], and further developed by Plach and Williams at Jensen Manufacturing Company [11].

If the negative reactance of the total compliance is made equal to that of the positive reactance of the horn, the last term of $|Z_{AT}|^2$ disappears, and the denominator of equation (12) is minimized. It is clear that that this happens when

$$R_{AL} \frac{T \frac{\omega_c}{\omega}}{1 - \left(\frac{\omega_c}{\omega}\right)^2 (1 - T^2)} = \frac{1}{\omega C_{AT}}$$

or, for $\omega = \omega_c$,

$$\omega_c R_{AL} C_{AT} = T. \quad (14)$$

This condition equates the system compliance with the horn reactance at the cutoff frequency. Horns with lower T -values have less reactance above cutoff than the corresponding exponential horn, so that the negative reactance from C_{AT} will overcompensate for the horn reactance. Looking at the total reactance, $X = X_h - X_{CAT}$, figure 7, we see that the reactance can be annulled completely above cutoff for the exponential horn, but that there is still a significant reactive component for horns with $T < 1$. We also note that the reactance of finite horns oscillate around the curves for the infinite horns.

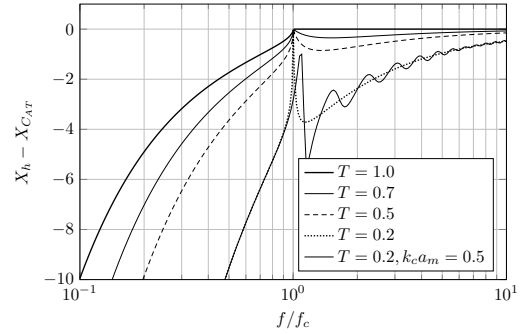


Fig. 7: Total reactance $X_h - X_{CAT}$, normalized. T calculated from equation (14) using $\omega_c = \omega_L$.

As equation (14) shows, low- T horns need very small V_B . This, combined with the low reactance of these horns above cutoff, causes the condition for reactance annulling to actually cause a significant drop in the efficiency above cutoff. Since the real power of reactance annulling lies in providing a conjugate match between driver and horn, it is clear that it works best for horns with T close to 1, and that for low- T horns the rear chamber should be made larger than the equations above predict, in order to match the horn reactance above cutoff.

It is not particularly difficult to find the optimum system compliance: the average reactance in a specified frequency range above cutoff should be zero. The average reactance over the range ω_c to $n_x \omega_c$ is

$$\begin{aligned} \bar{X} &= \frac{1}{(n_x - 1) \omega_c} \int_{\omega_c}^{n_x \omega_c} \{X_h - X_{CAT}\} d\omega = \\ &= \frac{R_{AL}}{(n_x - 1) \omega_c} \int_{\omega_c}^{n_x \omega_c} \left\{ \frac{T \omega_c}{\omega \left(1 - (1 - T^2) \frac{\omega_c^2}{\omega^2}\right)} - \frac{1}{\omega RC} \right\} d\omega \\ &= \frac{1}{(n_x - 1) \omega_c} \left\{ \frac{T \omega_c}{2} \ln \frac{|n_x^2 + T^2 - 1|}{T^2} - \frac{1}{RC} \ln n_x \right\}. \quad (15) \end{aligned}$$

where $RC = R_{AL} C_{AT}$. To achieve reactance annulling over the desired range, \bar{X} must be zero. Rearranging equation (15), we get the optimal value for C_{AT} for

maximum power output in the range above cutoff:

$$C_{AT} = \frac{1}{R_{AL} T \omega_c} \cdot \frac{2 \ln n_x}{\ln \left(\frac{|n_x^2 + T^2 - 1|}{T^2} \right)} = \frac{1}{R_{AL} \omega_c} \cdot f_{n_x}(T). \quad (16)$$

$f_{n_x}(T)$ is plotted in figure 8 for $n_x = 2$. It can be seen that in the range $0.7 \leq T \leq 1.1$, the value is very close to unity. Unfortunately, equation (16) cannot be solved directly for T , and either graphical or numerical methods must be used. It is in general interesting to note that the dependence on T for reactance annulling is opposite of that predicted by equation (14), i.e. larger compliance for lower values of T .

The effect of using this new equation is quite dramatic for low- T horns, as shown in figure 9. For the $T = 0.2$ horn, efficiency is improved by about 6 dB in the octave above cutoff.

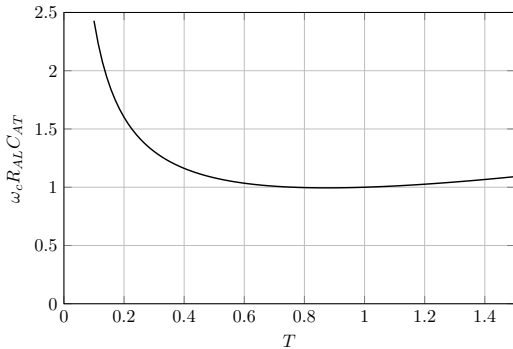
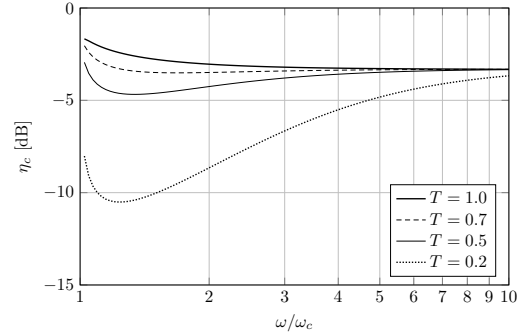
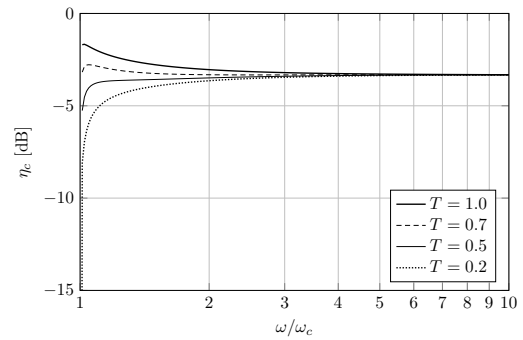


Fig. 8: Optimal C_{AT} to maximize power in the octave above ω_c ; $n_x = 2$.

Leach comments that for an exponential horn, it is not possible to achieve reactance annulling for $\omega_c = \omega_L$, and that a Hypex horn is required for this [12]. While it is possible to specify a lower ω_L to achieve reactance annulling at the horn cutoff frequency, Leach argues that this is not an optimal solution, as the driver/rear chamber then has a larger bandwidth than the horn can transmit, and that the efficiency will also be reduced. However, as we have seen above, low T -values will result in low efficiency above cutoff, and when using Leach's system design method, one may easily end up with low T -values.



(a) With C_{AT} from equation (14).



(b) With C_{AT} from equation (16).

Fig. 9: Efficiency above cutoff. $\beta = 1$, $R_{AM} = 0.0357 R_{AL}$, M_{AD} neglected.

5 Finite Horns

Finite horns typically have fairly large ripples in the throat impedance. By adjusting β , the variation in power output can be minimized. The procedure is described in detail by A. L. Thuras for the low frequency loudspeaker for the Fletcher system [9, 13]. Assuming that the horn is properly reactance annulled,

$$P_{AR} = p_{AE}^2 \frac{R_{AL}}{(R_{AT} + R_{AL})^2}.$$

It follows therefore that for minimum variation in output power, the variation in the factor $R_{AL}/(R_{AT} + R_{AL})^2$ should be as small as possible. By differentiating it with respect to R_{AL} and equating the result to zero, we find that $R_{AT} = R_{AL}$.

This ratio will however depend on the horn in question. In order to minimize the variation in output power, we must set

$$\frac{R_{AL_{max}}}{(R_{AT} + R_{AL_{max}})^2} = \frac{R_{AL_{min}}}{(R_{AT} + R_{AL_{min}})^2}. \quad (17)$$

Solving for R_{AT} , we find that $R_{AT} = \sqrt{R_{ALmax} R_{ALmin}}$, or

$$\beta = \frac{1}{\sqrt{\zeta_{max} \zeta_{min}}}, \quad (18)$$

where ζ_{max} and ζ_{min} describe the deviation from the asymptotic throat resistance R_{AL} , and are defined so that $R_{ALmax} = \zeta_{max} R_{AL}$, and similar for ζ_{min} .

6 Large Signal Analysis

At low frequencies, power output is limited by the maximum cone excursion, and at high frequencies by the heat dissipated in the voice coil. Due to the high efficiency of horns, maximum power input can be significantly higher than the rated power of the driver. The maximum power input is therefore [2]

$$P_{E,in} = \frac{P_{E,max}}{1 - \eta_c}. \quad (19)$$

One should, however, keep in mind that at maximum power, the voice coil resistance can easily be two times or more the rated R_E , lowering efficiency.

If maximum power output is desired, the system should be designed so that maximum displacement limited power output at the lowest frequency of interest coincides with the maximum thermally limited power output. The first of these is frequency dependent, and is [2, 14]

$$P_{AR,X} = \frac{X_{max}^2 \omega_L^2}{2} \cdot \frac{\rho_0 c}{S_t} S_d^2, \quad (20)$$

and increases with the compression ratio.

The second condition only depends on efficiency

$$P_{AR,E} = P_{E,max} \frac{\eta_c}{1 - \eta_c}. \quad (21)$$

Neglecting driver and rear chamber losses, i.e. $R_{AM} = 0$, The factor $\eta_c/(1 - \eta_c)$ in equation (21) reduces to R_{AE}/R_{AL} , and the equation becomes

$$P_{AR,E} = P_{E,max} \frac{R_{AE} S_d}{\rho_0 c} \left(\frac{S_d}{S_t} \right)^{-1}, \quad (22)$$

which decreases with the compression ratio. Equating these expressions, we find that the condition for maximum output as

$$\frac{S_d}{S_t} = \frac{\sqrt{2P_{E,max} R_{AE}}}{\rho_0 c X_{max} \omega_L} = \sqrt{\frac{2P_E}{R_E}} \frac{Bl}{\rho_0 c S_d X_{max} \omega_L}. \quad (23)$$

Note that the quantity under the square root is the peak input current.

For an infinite exponential horn, the same procedure, and assuming reactance annulling is used so that $C_{AT} = S_t/\rho_0 c \omega_c$, gives the condition for maximum output as

$$\frac{S_d}{S_t} = \sqrt{\frac{2P_E}{R_E (\omega_L^2 - \omega_c^2)}} \frac{Bl}{\rho_0 c S_d X_{max}}, \quad (24)$$

i.e. there is a factor $\sqrt{\omega_L^2 - \omega_c^2}$ replacing ω_L . It shows us that the closer to the cutoff frequency of the horn we put the lower frequency of maximum power, the higher compression ratio we need, since the radiation resistance falls away. The equation for power output under this condition stays the same.

For finite horns, one has to take into account the peaks and dips in the throat resistance. Below the first peak in the throat impedance, the resistance falls away quickly, and trying to obtain high power output in this range requires a high compression ratio, as shown above for the infinite horn, with resultant loss of efficiency and output power at higher frequencies, see equation (22). It may be possible to find analytical expressions for this, but the expressions are likely to be too complex to be useful.

7 Parameter Sensitivity

In [4], Keele did an analysis of the sensitivity of vented box alignments to variations in driver and box parameters. The sensitivity function of a system function $M(w)$ to the parameter x is defined as

$$\begin{aligned} S_x^M(w) &= \frac{dM(w)/M(w)}{dx/x} \\ &= \frac{x}{M(w)} \cdot \frac{\partial M(w)}{\partial x} \approx \frac{\Delta M(w) \text{ in } \%}{\Delta x \text{ in } \%}. \end{aligned} \quad (25)$$

A sensitivity value of +1 indicates that, say, a 5% increase in x results in a 5% increase in M . Keele analysed the vented box alignments by analytically deriving the sensitivity function for several parameters. With finite horns this can easily become complex, and we will restrict ourselves to a numerical study using the approximate form in equation (25). According to Keele, this is acceptable for a variation of $\pm 15\%$. For this study, perturbations of 15% will be made in each direction, and the average sensitivity will be plotted.

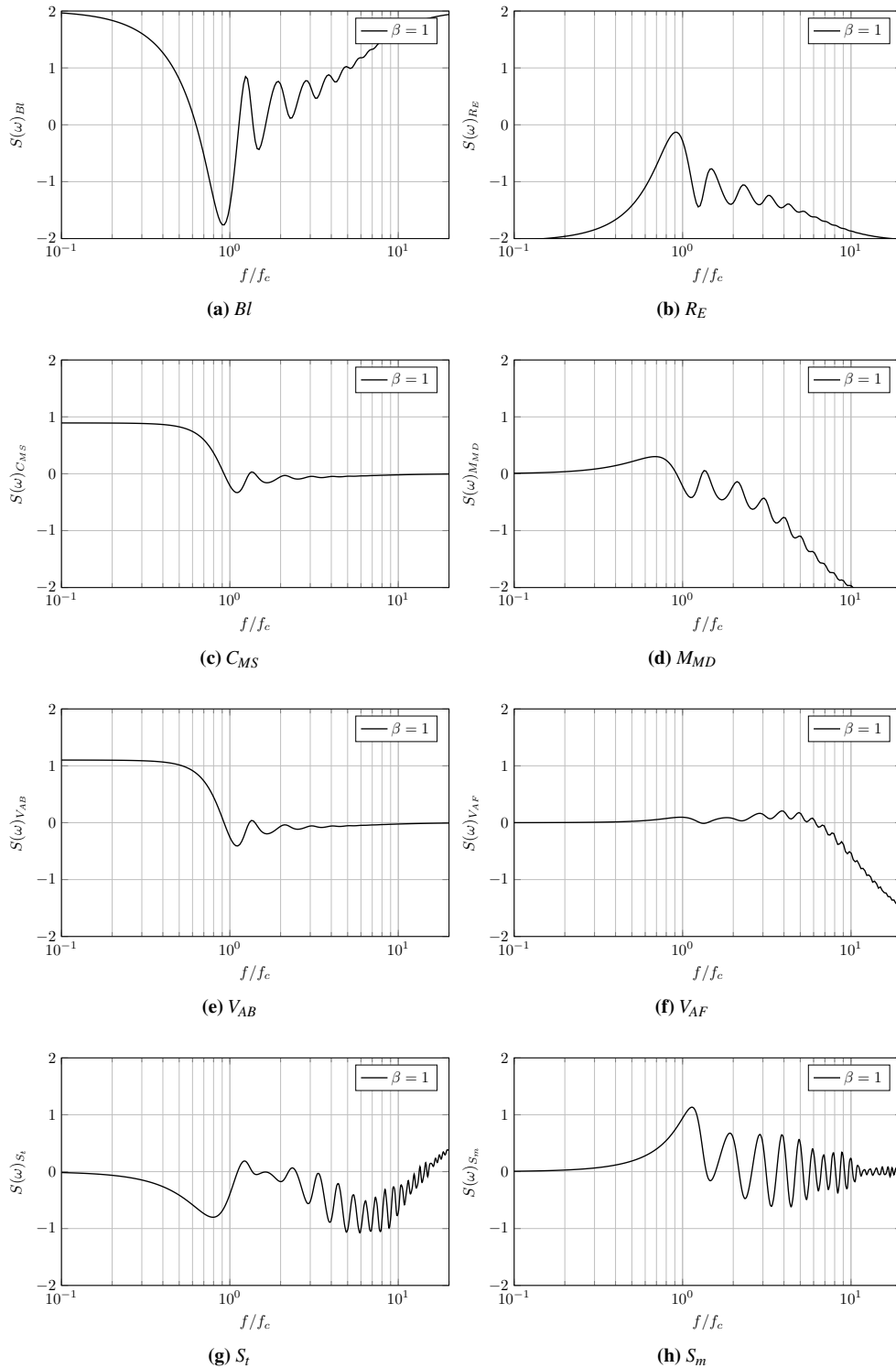


Fig. 10: Sensitivity of a front loaded horn loudspeaker to variation in driver and system parameters.

Figure 10 sums up the results for a horn using a driver with $Bl = 18\text{Tm}$, $R_E = 6\Omega$, $S_d = 350\text{cm}^2$, $C_{MS} = 2 \cdot 10^{-4}\text{m/N}$, $M_{MD} = 64.5\text{g}$ and $R_{MS} = 2\text{Ns/m}$. L_E is neglected. For the horn, $f_c = 45\text{Hz}$, $k_c a_m = 0.7$ and $T = 1$. The system uses $\beta = 1$, $V_{AB} = 28.45\text{l}$ and $V_{AF} = 1.24\text{l}$.

In judging these results, it appears that the driver parameters Bl , R_E and M_{MD} are the most important for horn loudspeaker performance. The sensitivity to variations in R_E is unfortunate, as R_E will vary with voice coil temperature. If the system is to be used at high levels this should be taken into account in the design phase.

Sensitivity to variations in C_{MS} and V_B is large only close to, and below, f_c , and at higher frequency they have little effect. M_{MD} has some effect in the midband, but the sensitivity increases towards, and beyond, the mass corner frequency. Sensitivity to variations in V_{AF} is confined to frequencies above the upper corner frequency. R_{MS} had so little effect on the results that the curve has been omitted. Sensitivity to horn throat and mouth areas is medium in the pass band. At higher frequencies the throat area influences the mass corner frequency. The sensitivity to mouth area changes is greatest around cutoff, as one would expect.

8 System Design With Driver

The basic procedures of horn system design have been described by Small [2], Keele [1] and Leach [3]. Leach's method is in general sound, and will not be repeated. One should, however, remember the issue with reactance annulling mentioned above.

A horn system can be designed based on several requirements. Leach's analysis is based on specifying the the upper and lower corner frequencies of the system, while as we have seen above, specifying a system by a given value for β may be required, and a method for this will be developed here. In the case of design for minimum ripple, some knowledge of the horn performance is required. At least an approximate simulation of the throat impedance is useful.

In order to design a system with a driver, we need to know the driver parameters f_s , V_{AS} , Q_{ES} and Q_{MS} . We also need to specify the lower corner frequency f_L and β .

If a one wants to specify the frequency range based on a -3 dB frequency from a third order low pass filter, determine β from equation (9).

From the relation

$$(Q_{EC}^{-1} + Q_{MC}^{-1})^{-1} = \frac{1}{\omega_0 R_{AT} C_{AT}}, \quad (26)$$

and using the relation that $Q_{EC} = \frac{\omega_0}{\omega_s} Q_{ES}$, $Q_{MC} = \frac{\omega_0}{\omega_s} Q_{MS}$, and consequently $(Q_{EC}^{-1} + Q_{MC}^{-1})^{-1} = \frac{\omega_0}{\omega_s} Q_{TS}$, we find that

$$R_{AT} = \frac{\omega_s}{\omega_0^2 Q_{TS} C_{AT}}. \quad (27)$$

Combining this equation with

$$C_{AT} = \frac{1}{R_{AT}(\beta + 1)\omega_L} \quad (28)$$

makes it possible to calculate the system resonance frequency without knowing ω_H .

1. If we are designing a driver for a given horn with a specified cutoff and T -value, the value of ω_L required for reactance annulling is given as

$$\omega_L = \frac{\beta \omega_c}{(\beta + 1) f_{n_x}(T)} \quad (29)$$

using $f_{n_x}(T)$ from equation (16).

2. From $\omega_0^2 = \omega_L \omega_H$, calculate the system resonance frequency

$$\omega_0 = \sqrt{\frac{\omega_L \omega_s (1 + \beta)}{Q_{TS}}}. \quad (30)$$

3. Calculate the upper corner frequency as

$$\omega_H = \frac{\omega_s (1 + \beta)}{Q_{TS}}. \quad (31)$$

4. Calculate the system Q , $Q_{TC} = \frac{\omega_0}{\omega_L + \omega_H}$.

5. Find the compliance ratio and rear volume as

$$\alpha = \left(\frac{\omega_0}{\omega_s}\right)^2 - 1 = \frac{\omega_L (1 + \beta)}{\omega_s Q_{TS}} - 1, \quad (32)$$

$$V_B = V_{AS}/\alpha. \quad (33)$$

6. Calculate the throat area as

$$S_t = \frac{\omega_0 Q_{TC} V_{AS} \left(1 + \frac{1}{\beta}\right)}{(\alpha + 1)c} \quad (34)$$

7. Calculate $Q_{EC} = \sqrt{\alpha + 1} Q_{ES} = \frac{\omega_0}{\omega_3} Q_{ES}$ and $Q_{LC} = (Q_{TC}^{-1} - Q_{EC}^{-1} - Q_{MC}^{-1})^{-1}$, and find η_c from equation (2).
8. If not aiming for a third order HF response, calculate V_{AF} and ω_3 using the equations given by Leach.

9 Conclusion

The analysis of a front loaded horn loudspeaker as presented by Keele [1], Small [2] and Leach [3], has been extended. It has been shown that the high frequency rolloff can be designed to conform to a third-order slope of known properties by proper adjustment of electrical inductance and front chamber volume.

Shortcomings of the traditional method of reactance annulling have been demonstrated, and an alternative method, that improves performance in the octave above horn cutoff, has been shown.

A method to find the optimum compression ratio for a horn with large variations in throat impedance have been presented.

Since several of the above methods require a certain value for the impedance matching parameter β , a system design method has been presented that takes this into account.

Finally, large signal analysis and parameter sensitivity analysis have been performed.

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