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ACOUSTIC LENS, THEIR DESIGN AND APPLICATION

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ABSTRACT

A simple criterion for the design of acoustic lenses is developed. An experimental study of a lens designed to increase the dispersion at 4000 Hz of the far field of a seven inch driver is described and compared to the theoretical predictions.

INTRODUCTION

There are two means by which the sound wave velocity in one or another portion of an acoustic lens may be decreased; first by means of the use of an obstacle array and second through the use of a set of slanted plates. The slant plate lens is simpler to manufacture and is characterized by an index of acoustic refraction, n, which is roughly independent of frequency whereas, the more complex obstacle arrays are typified by indices which characteristically increase with frequency. In this paper, we shall discuss only the slant plate lens, see figure 1, which is found to have an index given by [1]

$$n = l/l_o = \left\{ \cos\theta \right\}^{-1} \qquad \ldots\ldots \quad (1)$$

The separation of the lens plates must be smaller than the wavelength associated with the largest sound frequency of interest and n itself should not be made to exceed about 1.3 if excessive accoustic reflections are to be avoided.

ANALYSIS OF THE SLANT PLATE LENS

In order to make the mathematical analysis of the slant plate lens plus driver system more tractable we shall consider the nature of the sound field eminating from a square source whose surface area is equal to that of the round driver found in typical applications. Equating the areas of the sources guarantees that the axial intensity for each of these sources will be equal.[2]

The expression for the pressure distribution for a square source of side 1.78a, which makes the area of the square source equal to that of a circular source of radius a, turns out to be[4]

$$P= \frac{i\rho c K U_0 a^2}{2r} e^{i(wt-kr)}\left(\frac{SIN[.89Ka\ SIN\alpha]}{.89Ka\ SIN\alpha}\right)\left(\frac{SIN(.89Ka\ SIN\beta)}{.89Ka\ SIN\beta}\right) \quad \ldots \quad (2)$$

Here ρ is the density of air, c' is the speed of sound in air, k is the wave number, u_0 the source velocity amplitude, and w is the angular velocity of the source. Also α is the angular deviation from the y axis in the x y plane, and β is the angular deviation from the y axis in the y z plane. Looking in the y z plane, where $SIN\ \beta = 0$, we find from eq. (2)

$$P= \frac{i\rho c K U_0 a^2}{2r} e^{i(wt-kr)}\left(\frac{SIN[.89Ka\ SIN\alpha]}{.89Ka\ SIN\alpha}\right) \quad \ldots. \quad (3)$$

So that the directivity function

$$\frac{\sin(.89 Ka \sin\alpha)}{.89 Ka \sin\alpha}$$

turns out to be very similar to that for the circular piston given by [3]

$$\frac{2J_1(Ka \sin\theta)}{Ka \sin\theta}$$

A graphical comparison of these two nearly identical functions is shown in figure 2 and is sufficient to justify our replacement of the circular source by the equal area square source for the purpose of analyzing the effect on the sound field of adding a lense.

Parameters characterizing the slant plate lens to be discussed are shown in figure 3 and those parameters which characterize the source are shown in figure 4. The principle action of the lens is simply to increase the effective path length from the source points P (x, y, z) by an amount given approximately by

$$c/b\,(n-1)x \qquad \ldots. \quad (4)$$

This is because the lens is assumed to be placed in the source's near field where spherical divergence does not occur. Expression (4) holds true at these frequencies for which

$$\lambda < \frac{2a^2}{c}$$

For these frequencies, the lens remains in an area where the waves are esentially planar. [3] Since the need for lens induced dispersion is typically greatest at the higher frequencies the usefullness of the expression (4) is assured.

Using the expression (4) in conjunction with the usual expression for the source to field point distance in the absence of the lens gives the effective source to field point distance

$$r' = r - Z \sin\beta + X\left(\sin\alpha - \frac{c}{b}(n-1)\right) \qquad \ldots\ldots (5)$$

where

$$r = (X^2 + Y^2 + Z^2)^{1/2}$$

Thus the incremental contribution to the accoustic pressure at the field point is

$$dp = \frac{i\rho C' K U_0}{2r\pi}\left(e^{i\left(wt - k\left(r - z\sin\beta - x(\sin\alpha - \mathcal{X}\right)\right)}\right) dA \qquad \ldots\ldots (6)$$

where we have defined

$$\mathcal{X} = \frac{c}{b}(n-1)$$

Performing the required integrations gives for the approximate total acoustic pressure field for the source plus lens combination [2]

$$P = \frac{i \rho C K U_0 a^2}{2r} e^{i(wt - kr)} \left(\frac{SIN[.89Ka(SIN\alpha - X)]}{.89Ka(SIN\alpha - X)} \right) \qquad \ldots . \ (7)$$

This result, the principally important one of this paper, is similar to the pressure field for the source alone except for the presence of the term $X = \frac{c}{b}(n-1)$. The effect of the lens therefore is to shift the graph of the directivity function along the $KA\,SIN\,\alpha$ axis by an amount KaX as shown in figure 5.

APPLICATIONS

The condition that the acoustic pressure should stand at 70% of its value on axis (the point where the field intensity has decreased from its maximum value by 3db) requires that

$$Ka(SIN\alpha - X) = 1.6 \qquad \ldots \ldots \ (8)$$

As an example, suppose we wish to increase to 60^0 the 35^0 coverage angle (defined as twice the angular distance from the axis to the 3db down points on either side) which is characteristic of the response at 4000 Hz of a 7 cm radius cone type loudspeaker. We have then

$\frac{C'}{\lambda} = \frac{W}{2K} = 4000\,HZ \qquad a = .07\,m \qquad \beta(DESIRED) = \frac{60^\circ}{2} = 30^\circ$

so that, according to eq (8) we can put

$$(73.27\,m^{-1})(.07\,m)(\sin 30^\circ - X) = 1.6$$

this leads to

$$X = .19 = \frac{c}{b}(n-1)$$

a set of acceptable values for the parameters a, b, and c satisfying the requirements n $<$ 1.3 and $c < \frac{2a^2}{\lambda}$ are thus seem to be

$$a = b = c = .07\,m$$

$$\theta = \cos^{-1}\left(\frac{1}{1.19}\right) = 33^\circ$$

EXPERIMENTAL

A lens of these dimensions was constructed in order to compare the predictions of this sample calculation. These results are shown in figure 6. It should be mentioned that the laboratory in which these measurements were taken posed special problems, in that it was not anachoic. Attempts were made to quite the noise and decrease the unwanted reflections by lining the walls in the corner of the room with sound deadening material and hanging numerous heavy drapes to enclose this corner.

The source and lens were placed in the corner of the room and measurements were taken with a Simpson sound pressure level meter at various points away from the corner and at a distance of 3 meters from the source. While the data was difficult to assemble it appears to bear out the validity of the approximations made.

ACKNOWLEDGEMENTS

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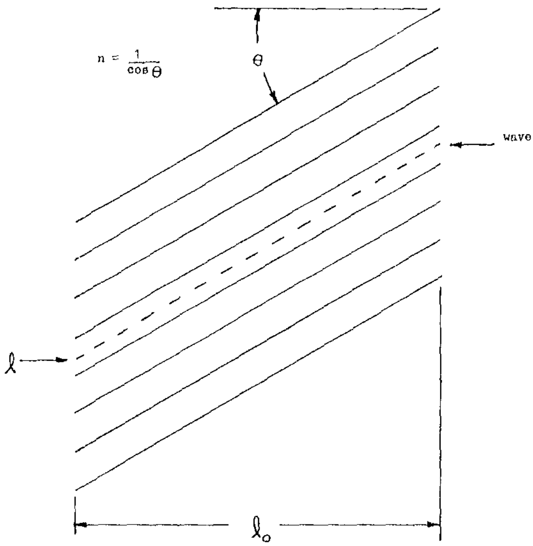

Figure 1 - Slant plate lense structure for refracting sound waves.

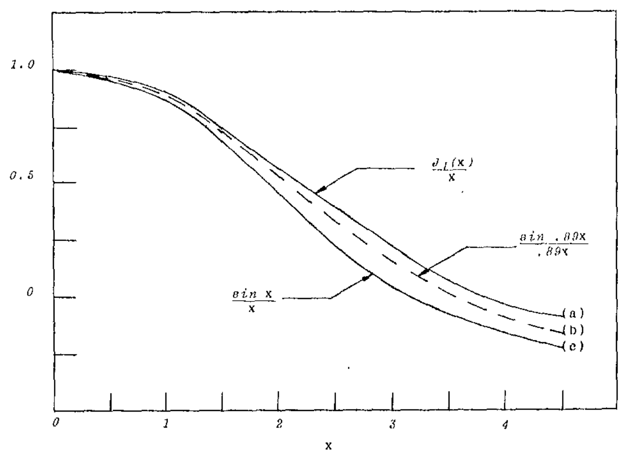

Fig. 2 A comparison of directivity functions for a) circular source, b) square source corrected for area, c) square source uncorrected

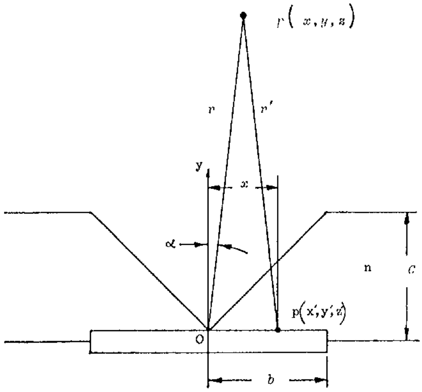

Fig. 3 Coordinate configuration for the lens modification of the radiation pattern

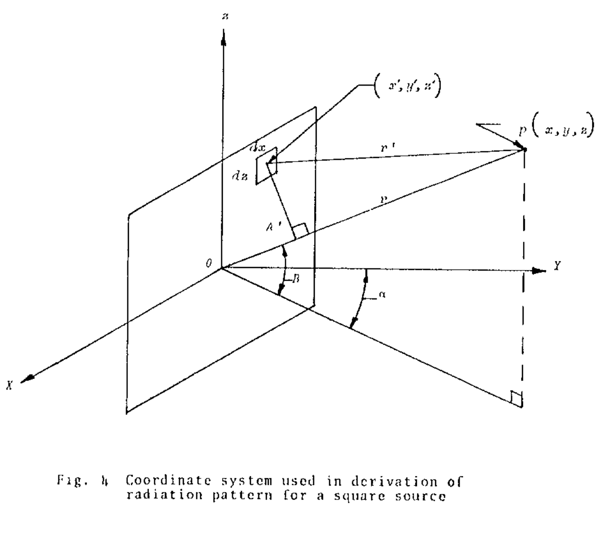

Fig. 4 Coordinate system used in derivation of radiation pattern for a square source

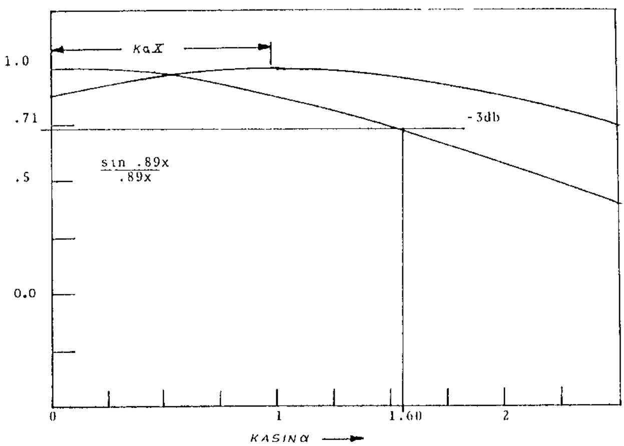

Fig. 5 : Graph of Directivity Function for Area Corrected Square Source

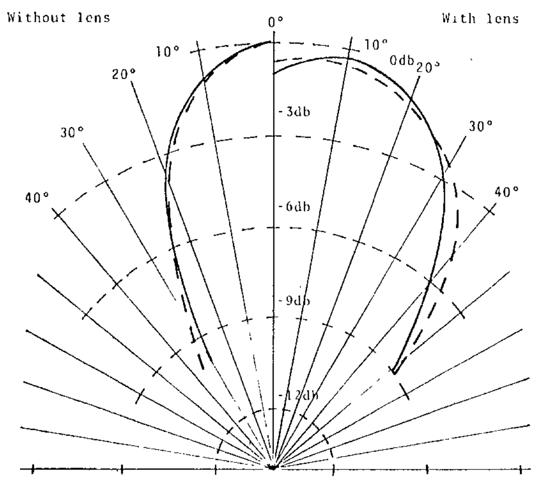

Fig. 6 : Polar responses of a 7 cm. loudspeaker at 4000 Hz with and without a lens. Theoretical values are plotted (solid lines) along with measured values (dotted lines).