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Lecture 9: Reflection, Transmission and Impedance

1 Boundary conditions at a junction

Suppose we take two taut strings, one thick and one thin and knot them together. What will happen to a wave as it passes through the knot? Or, instead of changing the mass density at the junction, we could change the tension (for example, by tying the string to a ring on a fixed rod which can absorb the longitudinal force from the change in tension). What happens to a sound wave when it passes from air to water? What happens to a light wave when it passes from air to glass? In this lecture, we will answer these questions.

Let's start with the string with varying tension. Say there is a knot at $x=0$ and the tension changes abruptly between $x<0$ and $x>0$. To be concrete, imagine we have a left-moving traveling wave coming in at very early times, hitting the junction around $t=0$ (obviously all parts of the wave can't hit the junction at the same time). We would like to know what the wave looks like at late times. Let us write the amplitude of the wave as $\psi_L(x,t)$ to the left of the knot at $\psi_R(x,t)$ to the right of the knot.

$$\psi(x,t)=\begin{cases}\psi_L(x,t), & x<0\\ \psi_R(x,t), & x\geqslant 0\end{cases} \tag{1}$$

To the left of the knot, the wave must satisfy one wave equation

$$\left[\frac{\partial^2}{\partial t^2}-v_1^2\frac{\partial^2}{\partial x^2}\right]\psi_L(x,t)=0, \quad v_1=\sqrt{\frac{T_1}{\mu_1}} \tag{2}$$

and to the right of the knot, another wave equation must be satisfied

$$\left[\frac{\partial^2}{\partial t^2}-v_2^2\frac{\partial^2}{\partial x^2}\right]\psi_R(x,t)=0, \quad v_2=\sqrt{\frac{T_2}{\mu_2}} \tag{3}$$

Recalling that the Heaviside step function (or theta-function) is defined by $\theta(x)=0$ if $x<0$ and $\theta(x)=1$ for $x\geqslant 0$, we can also write Eq. (1) as

$$\psi(x,t)=\psi_L(x,t)\theta(-x)+\psi_R(x,t)\theta(x) \tag{4}$$

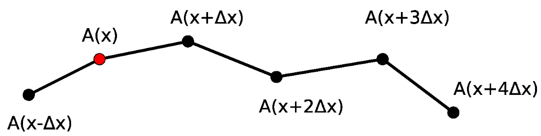
This way of writing $\psi(x,y)$ makes it clear that it is just some function of position and time. We need to determine what the boundary conditions are at the junction, and then find the full solution $\psi(x,t)$ for all times.

Obviously $\psi(x,t)$ should be continuous. So

$$\boxed{\psi_L(0,t)=\psi_R(0,t)} \tag{5}$$

This is one boundary condition at the junction.

Recall from Lecture 6 that a point on the string of mass m gets a force from the parts of the string to the left and to the right:



The force from the part to the left is $T\frac{\Delta\psi}{\Delta x} \approx T\frac{\partial\psi(x,t)}{\partial x}$. This form makes sense, since if the string has no slope, it is flat and there is no force. From the right, the force is $-T\frac{\partial\psi(x,t)}{\partial x}$. The sign has to be opposite so that if there is no difference in slope there is no force (with equal tensions). So if there are different tensions to the right and left, as at $x=0$, we have

$$m\frac{\partial^2\psi(0,t)}{\partial t^2} = T_1\frac{\partial\psi_L(0,t)}{\partial x} - T_2\frac{\partial\psi_R(0,t)}{\partial x} \tag{6}$$

Now m is the mass of an infinitesimal point of string at $x = 0$. But T_1 and T_2 as well as the slopes $\frac{\partial\psi_L(0,t)}{\partial x}$ and $\frac{\partial\psi_L(0,t)}{\partial x}$ are macroscopic quantities. Thus, if the right hand side doesn't vanish, we would find $\frac{\partial^2\psi(0,t)}{\partial t^2} \to \infty$ as $m \to 0$. Equivalently, we can write $m = \mu\Delta x$ then this becomes

$$\mu\Delta x\frac{\partial\psi(0,t)}{\partial t^2} = T_1\frac{\partial\psi_L(0,t)}{\partial x} - T_2\frac{\partial\psi_R(0,t)}{\partial x} \tag{7}$$

Taking $\Delta x \to 0$ we find

$$\boxed{T_1\frac{\partial\psi_L(0,t)}{\partial x} = T_2\frac{\partial\psi_R(0,t)}{\partial x}} \tag{8}$$

So the slope must be discontinuous at the boundary to account for the different tensions.

Now we have the boundary conditions. What is the solution?

2 Reflection and transmission

Suppose we have some incoming traveling wave. Before it hits the junction it has the form of a right-moving traveling wave

$$\psi_L(x,t) = \psi_i(x - v_1 t), \quad t < 0 \tag{9}$$

To be clear, $\psi_L(x,\, t)$ is the part of $\psi(x,\, t)$ with $x < 0$. $\psi_i(x)$ is some function describing the wave's shape in this region. It is easy to check that $\psi_L(x,\, t)$ satisfies the wave equation in the $x < 0$ region: $\left[\frac{\partial^2}{\partial t^2} - v_1^2\frac{\partial^2}{\partial x^2}\right]\psi_L(x,\, t) = 0$. The i subscript on $\psi_i(t)$ refers to the **incident wave**. Let $t = 0$ be the time when the first part of the wave hits the knot at $x = 0$.

To be concrete, think of $\psi_i(t)$ as a square wave. For example $\psi_i(z) = 2\text{mm}$ for -1 cm $< z \leqslant$ 0cm and $\psi_i(z) = 0$ otherwise. At $t = 0$, $\psi_L(x,0)$ is zero outside of $-1\,\text{cm} < x < 0$, so it just starts to hit $x = 0$. At earlier times, say $t_1 = -\frac{5\text{cm}}{v_1}$, then $\psi_L(x,t_1)$ is zero outside of $-6\,\text{cm} < x < -5\,\text{cm}$. So as time goes on, it approaches the junction, and hits it just at $t = 0$. So $\psi(x,t) = \psi_L(x,t)\theta(-x)$ is a perfectly good solution of the wave equation for $t < 0$. The real wave doesn't have to be a square wave, it can have any shape.

Actually, it will be extremely helpful to make a cosmetic change and write $\psi_i\left(t - \frac{x}{v_1}\right)$ instead of $\psi_i(x - v_1 t)$. Clearly these functions carry the same information, because we just rescaled the argument. The new form is nicer since at the boundary $x = 0$, ψ_i doesn't depend on v (so Eq. (12) below has a simple form). So let's pretend we wrote $\psi_i\left(t - \frac{x}{v_1}\right)$ from the start of this section (I didn't want to actually write it that way from the start to connect more clearly to what we did before).

Now, after $t = 0$ ψ_L can have left and a right moving components, so we can more generally write

$$\psi_L(x,t) = \psi_i\left(t - \frac{x}{v_1}\right) + \psi_r\left(t + \frac{x}{v_1}\right) \tag{10}$$

where ψ_r is the **reflected wave**. Recall that any wave can be written as a sum of left and right moving waves. So writing ψ_L this way does not involve any assumptions, it is just convenient to solve the wave equation including boundary conditions at the junction.

For $t>0$ there will also be some ψ_R (the part at $x>0$). This part will always be right-moving. We call this the **transmitted wave** and write

$$\psi_R(x,t)=\psi_t\left(t-\frac{x}{v_2}\right) \tag{11}$$

That we can write the wave for $x>0$ in this form follows from the assumption that for $t<0$ then $\psi=0$ for $x>0$. If there were a left-moving component on the right side, then as $t\to-\infty$ it would always be there. Note that the transmitted wave has wave speed v_2, since it is in the string on the right. Note that we are not assuming that the incident, transmitted and reflected waves all have the same shape.

The picture is as follows

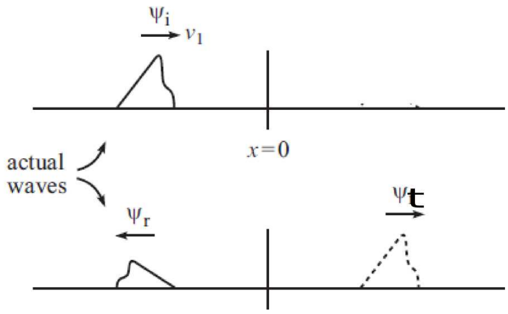


Figure 1. Incident, reflected and transmitted waves.

Now we impose our boundary conditions. Continuity at $x=0$, Eq. (5) implies

$$\psi_i(t)+\psi_r(t)=\psi_t(t) \tag{12}$$

For the other boundary condition, Eq. (8), we have

$$T_1\frac{\partial\psi_L(0,t)}{\partial x}=T_1\frac{\partial}{\partial x}\left[\psi_i\left(t-\frac{x}{v_1}\right)+\psi_r\left(t+\frac{x}{v_1}\right)\right]_{x=0}=\frac{T_1}{v_1}[-\psi_i'(t)+\psi_r'(t)] \tag{13}$$

and

$$T_2\frac{\partial\psi_R(0,t)}{\partial x}=T_2\frac{\partial}{\partial x}\left[\psi_t\left(t-\frac{x}{v_2}\right)\right]_{x=0}=-\frac{T_2}{v_2}\psi_t'(t) \tag{14}$$

Thus,

$$\frac{T_1}{v_1}[-\psi_i'(t)+\psi_r'(t)]=-\frac{T_2}{v_2}\psi_t'(t) \tag{15}$$

In other words

$$\frac{d}{dt}\left[-\frac{T_1}{v_1}\psi_i(t)+\frac{T_1}{v_1}\psi_r(t)+\frac{T_2}{v_2}\psi_t(t)\right]=0 \tag{16}$$

Since a function whose derivative vanishes must be constant, we then have

$$\frac{T_1}{v_1}[-\psi_i(t)+\psi_r(t)]=-\frac{T_2}{v_2}\psi_t(t)+\text{const} \tag{17}$$

If the constant were nonzero, it would mean that the wave on the righthand side, ψ_t has a net displacement at all times. There is nothing particularly interesting in such a displacement, so we set the integration constant to zero.

Substituting Eq. (12) into Eq. (17) we get

$$\frac{T_1}{v_1}[-\psi_i(t)+\psi_r(t)]=-\frac{T_2}{v_2}[\psi_i(t)+\psi_r(t)] \tag{18}$$

or

$$\left(\frac{T_1}{v_1}+\frac{T_2}{v_2}\right)\psi_r=\left(\frac{T_1}{v_1}-\frac{T_2}{v_2}\right)\psi_i(t) \tag{19}$$

which implies

$$\psi_r = \frac{\frac{T_1}{v_1} - \frac{T_2}{v_2}}{\frac{T_1}{v_1} + \frac{T_2}{v_2}} \psi_i \tag{20}$$

We have found that the reflective wave has exactly the same shape as the incident wave, but with a different overall magnitude. By Eq. (12) the transmitted wave also has the same shape. The relevant amplitudes are the main useful formulas coming out of this analysis.

Defining

$$Z_1 = \frac{T_1}{v_1}, \quad Z_2 = \frac{T_2}{v_2} \tag{21}$$

we have

$$\boxed{\psi_r = \frac{Z_1 - Z_2}{Z_1 + Z_2} \psi_i} \tag{22}$$

Substituting back in to Eq. (12) we get

$$\boxed{\psi_t = \frac{2Z_1}{Z_1 + Z_2} \psi_i} \tag{23}$$

Sometimes this solution is written as

$$\psi_r = R\psi_i, \quad \psi_t = T\psi_i \tag{24}$$

where

$$R = \frac{Z_1 - Z_2}{Z_1 + Z_2} \tag{25}$$

is the **reflection coefficient** and

$$T = \frac{2Z_1}{Z_1 + Z_2} \tag{26}$$

is the **transmission coefficient**.

Z is known as an **impedance**. In this case it's tension over velocity, but more generally

Impedance is force divided by velocity

That is, impedance tells you how much force is required to impart a certain velocity. Impedance is a property of a medium. In this case, the two strings have different tensions and different velocities. Using $v = \sqrt{\frac{T}{\mu}}$ we can write

$$Z = \frac{T}{v} = \sqrt{T\mu} \tag{27}$$

Note that as $Z_1 = Z_2$ there is no reflection and complete transmission. If we want no reflection we need to **match impedances**. For example, if we want to impedance-match across two strings with different mass densities μ_1 and μ_2 we can choose the tensions to be $T_2 = \frac{\mu_1}{\mu_2} T_1$ so that

$$Z_2 = \sqrt{T_2\mu_2} = \sqrt{\frac{\mu_1}{\mu_2} T_1 \mu_2} = \sqrt{T_1\mu_1} = Z_1 \tag{28}$$

Thus the impedances can agree in strings of different mass density.

Note that the transmission coefficient is greater than 1 if $Z_1 < Z_2$. That means the amplitude increases when a wave travels from a medium of lower impedance to a medium of higher impedance. This is an important fact. We'll discuss a consequence in Section 7.1 below.

3 Phase flipping

What happens when a wave hits a medium of higher impedance, such as when the tension or mass density of the second string is very large? Then $Z_2 > Z_1$ and so, $R = \frac{Z_1 - Z_2}{Z_1 + Z_2} < 0$. Thus, if $\psi_i > 0$ then $\psi_r < 0$. That is, the wave flips its sign. This happens in particular if the wave hits a wall, which is like $\mu = \infty$.

On the other hand if a wave passes to a less dense string then $Z_2 < Z_1$ and there is no sign flip. This can happen if $Z_2 = 0$, for example, if the second string is massless or tensionless – as in an open boundary condition.

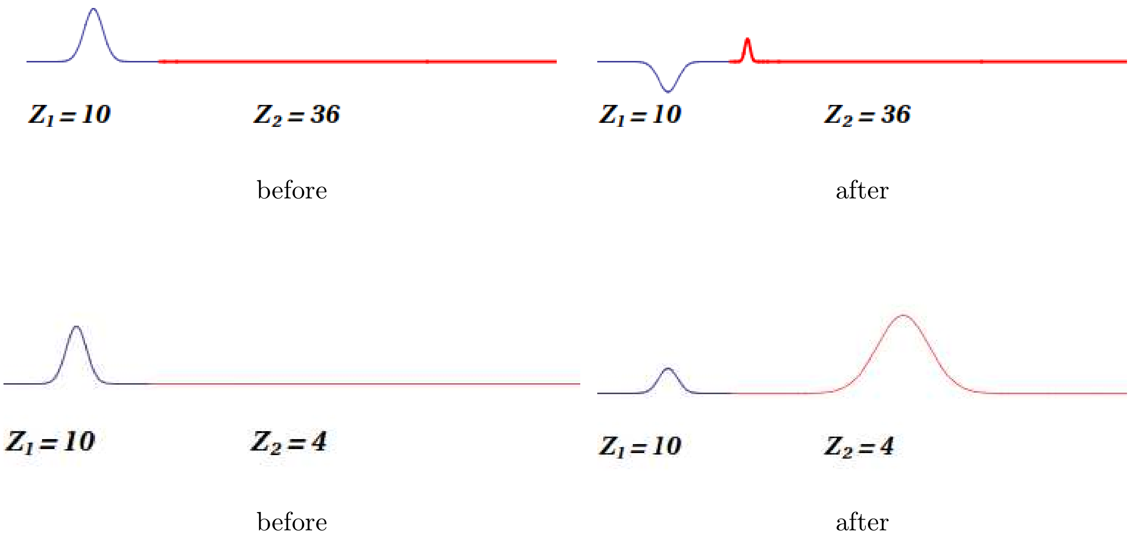


Figure 2. Phase shift of reflected pulses on a string. Top has pulse going from lower to higher impedance. Bottom has pulse going from higher to lower impedance.

This phase flipping has important consequences due to interference between the reflected pulse and other incoming pulses. There will be constructive interference if the phases are the same, but destructive interference if they are opposite. We will return to interference after discussing light.

4 Impedance for masses

To get intuition for impedance, it is helpful to go back to a more familiar system: masses. Suppose we collide a block of mass m with a larger block of mass M

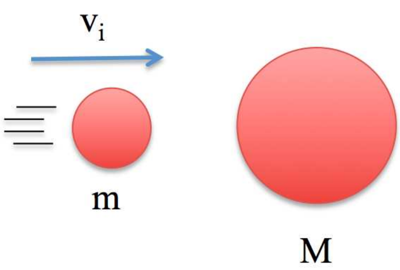


Figure 3. Mass m starts with velocity v_i, with M at rest.

Say m has velocity v_i. To find out the velocity of M we solve Newton's laws, or more easily, use conservation of momentum and energy. The initial momentum and energy are

$$p_i = m v_i, \quad E_i = \frac{1}{2} m v_i{}^2 \tag{29}$$

After the collisions, m bounces off M and goes back the way it came with "reflected velocity" v_r and M moves off to the right with "transmitted velocity" v_t:

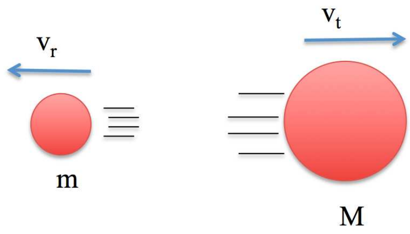


Figure 4. After the collision, m has the reflected velocity v_r and M the transmitted velocity v_t.

The final momentum and energy are

$$p_f = -v_r m + v_t M, \quad E_f = \frac{1}{2} m v_r^2 + \frac{1}{2} M v_t^2 \tag{30}$$

Conservation of momentum implies

$$v_t = \frac{m}{M}(v_i + v_r) \tag{31}$$

then conservation of energy implies

$$\frac{1}{2} m v_i{}^2 = \frac{1}{2} m v_r + \frac{1}{2} M \left[\frac{m}{M}(v_i + v_r)\right]^2 \tag{32}$$

After a little more algebra we find

$$v_r = \frac{M - m}{M + m} v_i, \quad v_t = \frac{2m}{m + M} v_i \tag{33}$$

These equation have exactly the same form as Eqs. (22) and (23) with $Z_1 = m$ and $Z_2 = M$. Thus for masses, **impedance is mass**. This makes sense – the bigger the mass, the less force you can impart with a given velocity.

Let's take a concrete example. Suppose $m = 1$, $M = 3$ and the incoming velocity is v. Then the final velocity of M is

$$v_t = \frac{2m}{m + M} v = \frac{2(1)}{1 + 3} v = \frac{1}{2} v \tag{34}$$

Thus the mass M gets half the velocity of m. Now say we put a mass $m_2 = 2$ in between them. When m bangs into m_2 it gives it a velocity

$$v_2 = \frac{2m}{m + m_2} v = \frac{2(1)}{1 + 2} v = \frac{2}{3} v \tag{35}$$

Then m_2 bangs into M and gives it a velocity

$$v_t = \frac{2m_2}{m_2 + M} v_2 = \frac{2(2)}{2 + 3}\left(\frac{2}{3} v\right) = \frac{4}{5}\frac{2}{3} v = \frac{8}{15} v = 0.533 v \tag{36}$$

Thus M goes faster. Thus inserting a mass between the two masses helps impedance match. Similarly inserting lots of masses can make the impedance matching very efficient.

5 Complex impedance

It is sometimes useful to generalize impedance to complex numbers. For example, suppose we have a driven oscillator satisfying

$$m\ddot{x} + kx = F_0 e^{i\omega t} \tag{37}$$

First consider the case where $k \approx 0$. Then $m\ddot{x} = F_0 e^{i\omega t}$. Integrating this gives

$$\dot{x} = \frac{F_0}{i\omega m} e^{i\omega t} \tag{38}$$

Then

$$Z_m = \frac{\text{force}}{\text{velocity}} = \frac{F_0 e^{i\omega t}}{\frac{F_0}{i\omega m} e^{i\omega t}} = i\omega m \tag{39}$$

Thus at fixed driving frequency ω, $Z_m \propto m$ as with the masses.

In the other case, when $m \approx 0$, $kx = F_0 e^{i\omega t}$ and so

$$\dot{x} = i\omega \frac{F_0}{k} e^{i\omega t} \tag{40}$$

Then

$$Z_k = \frac{\text{force}}{\text{velocity}} = \frac{F_0 e^{i\omega t}}{i\omega \frac{F_0}{k} e^{i\omega t}} = -i\frac{k}{\omega} \tag{41}$$

The impedance of the whose system is the sum of the impedances

$$Z_{\text{total}} = Z_m + Z_k = i\left(\omega m - \frac{k}{\omega}\right) \tag{42}$$

So at high frequencies, the mass term dominates. This is called **mass-dominated impedance**. Physically, when the driver is going very fast, the mass has no time to react: a lot of force at high frequency has little effect on velocity. At low frequencies, the k term dominates. For slow motion, how much velocity the mass gets for a given force depends very much on how stiff the spring is. This is called **stiffness dominated impedance**.

Note that $Z_{\text{total}} = 0$ when $\omega = \sqrt{\frac{k}{m}}$, that is, no resonances. At the resonant frequency, nothing impedes the motion of the oscillator: a small force gives a huge velocity.

With complex impedances you can add a damping term.

$$\gamma\dot{x} = F_0 e^{i\omega t} \quad \Rightarrow \quad \dot{x} = \frac{F_0}{\gamma} e^{i\omega t} \tag{43}$$

Thus,

$$Z_\gamma = \frac{F}{v} = \gamma \tag{44}$$

This makes perfect sense: damping impedes the transfer of energy from the driver to the oscillator.

With all 3 terms,

$$Z_{\text{total}} = \gamma + i\left(\omega m - \frac{k}{\omega}\right) \tag{45}$$

Now the impedance is always nonzero, for any frequency.

6 Circuits (optional)

An important use of complex impedances is in circuits. Recall that the equation of motion for an LRC circuit is just like a damped harmonic oscillator. For a resistive circuit:

$$V = IR = \dot{Q}R \tag{46}$$

where Q is the charge, I is the current, R is the resistance and V is the voltage. For a capacitor

$$V = \frac{Q}{C} \tag{47}$$

For an inductor

$$V = L\dot{I} = L\ddot{Q} \tag{48}$$

Putting everything together, the total voltage is

$$V_{\text{tot}} = L\ddot{Q} + \frac{Q}{C} + \dot{Q}R \tag{49}$$

This is the direct analog of

$$F = m\ddot{x} + kx + \gamma\dot{x} \tag{50}$$

Instead of driving the mass with an external force $F = F_0 e^{i\omega t}$, we drive the circuit with an external voltage $V = V_0 e^{i\omega t}$. That is we find the simple correspondence

mass/spring	F	x	$\dot{x}$	$\ddot{x}$	γ	k	m	$Z = \frac{F}{\dot{x}}$
circuit	V	Q	$I = \dot{Q}$	$\dot{I} = \ddot{Q}$	R	$\frac{1}{C}$	L	$Z = \frac{V}{I}$

(51)

Thus instead of being $Z = \frac{F}{\dot{x}}$, impedance for a circuit is

$$Z = \frac{V}{\dot{Q}} = \frac{V}{I} \tag{52}$$

A resistor has

$$Z_R = \frac{V}{I} = R \tag{53}$$

A capacitor has

$$Z_C = \frac{V}{I} = \frac{1}{i\omega C} \tag{54}$$

and an inductor has

$$Z_L = i\omega L \tag{55}$$

Impedance of an AC circuit plays the role that resistance does for a DC circuit. We can add impedances in series or in parallel just like we do for resistance. Impedance has the units of resistance, that is Ohms. In practice, impedances are more easily measured than calculated.

Matching the impedances of two different wave carrying media is of critical importance in electrical engineering. Say one wishes to drive an antenna, such as the wifi antenna on your router. The maximum power we can couple into the antenna occurs when the impedances of the power supply and antenna are equal in magnitude. This is pretty important in high power applications, where can waves which are reflected from your antenna can come back and destroy your amplifying equipment. It's also critical if you are a receiver. All modern radios have impedance matching circuits in them. This is because antennas are resonant devices, and as we just saw, tuning away from resonances causes some impedance. Thus you would need to match your radio input impedance to your antenna as you pick up different wavelengths.

7 Impedance for other stuff

For air, we recall $v = \sqrt{\frac{B}{\rho}}$ with $B = \gamma p = \rho v^2$ the bulk modulus and v the speed of sound in the gas. Then

$$Z_0 = \frac{B}{v} = \rho v \tag{56}$$

$Z_0 = \rho v$ is called the **specific impedance**. Z_0 is a property of the medium. For example, in air

$$\rho = 1.2\frac{kg}{m^3}, \quad v = 343\frac{m}{s} \quad \Rightarrow Z_{\text{air}} = 420\frac{\text{kg}}{m^2 s} = 420\frac{\text{Pa}\cdot s}{m} \tag{57}$$

for water

$$\rho = 1000\frac{kg}{m^3}, \quad v = 1480\frac{m}{s} \quad \Rightarrow Z_{\text{water}} = 1.48 \times 10^6 \frac{\text{kg}}{m^2 s} = 1.48 \times 10^6 \frac{\text{Pa} \cdot s}{m} \tag{58}$$

Thus if you try to yell at someone under water, you find that the amount reflected is

$$R = \frac{Z_{\text{air}} - Z_{\text{water}}}{Z_{\text{air}} + Z_{\text{water}}} = -0.9994 \tag{59}$$

So almost all of the sound is reflected (and there is a phase flip).

If the wavelength of the sound waves is smaller than the size of the cavity holding the waves (for example in a pipe) then one must account for this finite size in the impedance. For air in a finite size cavity, the relevant quantity is not the specific impedance (which is a property of the gas itself), but the impedance per area

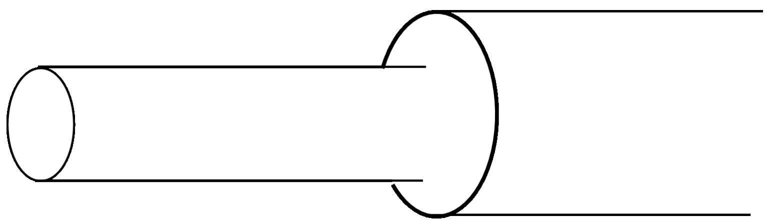
$$Z = \frac{Z_0}{A} = \frac{B}{v \cdot A} = \frac{B}{\text{volume flow rate}} = \frac{\rho v}{A}, \quad \lambda > \sqrt{A} \tag{60}$$

This is relevant when $\lambda > \sqrt{A}$ where λ is the wavelength of the sound wave and A is the cross sectional area of the pipe.

For air of the same density, the impedance is effectively $\frac{1}{\text{area}}$. Thus the reflection coefficient going between pipes of different radii is

$$R = \frac{\frac{1}{A_1} - \frac{1}{A_2}}{\frac{1}{A_1} + \frac{1}{A_2}} = \frac{A_2 - A_1}{A_1 + A_2} \tag{61}$$

So a situation like this will have bad impedance matching:



On the other hand, a megaphone is designed to impedance match much better:

Now you know why megaphones are shaped this way!

7.1 Solids

For liquids or solids, impedance is also $Z = \rho v$. The nice thing about a formula like this is that both ρ (density of the solid) and v (speed of sound in the solid) are easy to measure, in contrast to the bulk modulus (what is that?) and the pressure (what is pressure for a solid?). For example,

material	density (kg/m^3)	speed of sound (m/s)	specific impedance (MPa·s/m)
brick	2,200	4,200	9.4
concrete	1,100	3,500	3.8
steel	7,900	6,100	48
water	1,000	1,400	1.4
wood	630	3,600	2.3
rubber	1,100	100	0.11
rock	2,600	6,000	16
diamond	3,500	12,000	42
dirt	1,500	100	0.15

Table 1. Properties of various liquids and solids. 1 MPa = 10^6 Pascals = $10^6 \frac{\text{kg}}{m\,s^2}$.

It's good to have a little intuition for speeds of sounds and densities, which you can get from this table. For example, sound goes very fast in diamond. That's because diamond is very hard and rigid, so the the atoms move back to their equilibrium very quickly as the wave passed through (spring constant is high). Steel is also hard and has a fast sound speed. Rubber and dirt are soft, so waves propagate slowly through them. Dirt is denser than concrete, but sound goes much slower since it is not rigid.

Regarding the impedance, because impedance is $\rho \cdot v$, soft stuff generally has small ρ and small v, so it has much lower impedance. The highest impedances are for the hardest substances: steel and diamond, the lowest for the softest stuff, water and dirt.

As an application, recall from Eq. (26) that when the impedance goes down $T > 1$ and the amplitude increases. Now, consider an earthquake as it travels from rock (Z_1=16 MPa·s/m) into dirt or landfill (Z_2=16 MPa·s/m). Then $T = \frac{2Z_1}{Z_1 + Z_2} = 1.98$. So the amplitude of the shaking will double in amplitude! That's why you shouldn't build houses on landfill in an earthquake zone.