

# Interpolating between the data for a termination resistance of 10 and single-sided termination in Zverev's filter handbook

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Anatol I. Zverev's filter handbook, *Handbook of filter synthesis*, Wiley, New York, 1967, contains lists of normalized component values for many types of standard low-pass filter. The values are shown for different ratios of input and output termination resistances. Figure 1 shows an example.

At first sight, there appears to be a remarkable discontinuity between the values for a resistance ratio of ten and single-sided termination. For the 0.05° linear phase, sixth-order case, the numbers are:

$R_s$	$C_1$	$L_2$	$C_3$	$L_4$	$C_5$	$L_6$
2.5	0.0901	1.383	0.2969	2.1698	0.4277	4.5579
3.3333	0.0669	1.834	0.2217	2.8849	0.3182	5.822
5	0.0441	2.7343	0.1472	4.3129	0.2103	8.3408
10	0.0218	5.4312	0.0732	8.5924	0.1041	15.8769
$\infty$	1.505	1.0306	0.8554	0.7283	0.5389	0.2147
$1/R_s$	$L_1$	$C_2$	$L_3$	$C_4$	$L_5$	$C_6$

The output termination resistance is always 1.

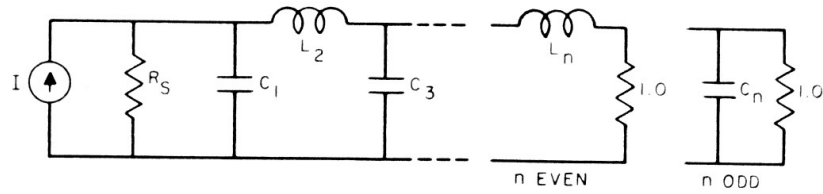
Using the upper header and looking at the rows  $R_s = 2.5$  up to and including  $R_s = 10$ , the capacitances are seen to decrease with increasing  $R_s$ . They are roughly inversely proportional to  $R_s$  when  $R_s$  is well above 1. The inductances are roughly proportional to  $R_s$ . The row for a ratio approaching infinity is totally different - fortunately, as otherwise the capacitances would all be exceedingly small (possibly even 0) and the inductances exceedingly large (possibly even  $\rightarrow \infty$ ).

Looking a bit more carefully, you see that when you read the values for a ratio of ten from left to right and those for an infinite ratio from right to left, there is always approximately a factor of ten between them.  $C_1$  for a ratio of ten is close to 1/10 of  $L_6$  for an infinite ratio,  $L_2$  for a ratio of ten is close to 10 times  $C_5$  for an infinite ratio,  $C_3$  for a ratio of ten is close to 1/10 of  $L_4$  for an infinite ratio,  $L_4$  for a ratio of ten is close to 10 times  $C_3$  for an infinite ratio,  $C_5$  for a ratio of ten is close to 1/10 of  $L_2$  for an infinite ratio,  $L_6$  for a ratio of ten is close to 10 times  $C_1$  for an infinite ratio.

# LOW PASS ELEMENT VALUES

LINEAR PHASE WITH  
EQUIRIPPLE ERROR

PHASE ERROR =  $0.05^\circ$



n	$R_s$	$C_1$	$L_2$	$C_3$	$L_4$	$C_5$	$L_6$	$C_7$
5	1.0000	0.2751	0.6541	0.8892	1.1034	2.2873		
	0.9000	0.3031	0.5868	0.9841	0.9904	2.4589		
	0.8000	0.3380	0.5197	1.1026	0.8774	2.6733		
	0.7000	0.3827	0.4529	1.2548	0.7648	2.9484		
	0.6000	0.4420	0.3865	1.4575	0.6526	3.3144		
	0.5000	0.5248	0.3204	1.7408	0.5410	3.8254		
	0.4000	0.6486	0.2549	2.1651	0.4302	4.5896		
	0.3000	0.8544	0.1899	2.8713	0.3205	5.8595		
	0.2000	1.2649	0.1257	4.2817	0.2120	8.3922		
	0.1000	2.4940	0.0624	8.5082	0.1051	15.9739		
	INF.	1.5144	1.0407	0.8447	0.6177	0.2456		
6	1.0000	0.2374	0.5662	0.7578	0.8760	1.1163	2.2448	
	1.1111	0.2120	0.5272	0.6799	0.9726	0.9977	2.4214	
	1.2500	0.1870	0.7032	0.6023	1.0931	0.8807	2.6396	
	1.4286	0.1622	0.8008	0.5253	1.2475	0.7652	2.9174	
	1.6667	0.1373	0.9306	0.4487	1.4530	0.6512	3.2849	
	2.0000	0.1139	1.1118	0.3725	1.7401	0.5387	3.7958	
	2.5000	0.0901	1.3830	0.2969	2.1698	0.4277	4.5579	
	3.3333	0.0669	1.8340	0.2217	2.8849	0.3182	5.8220	
	5.0000	0.0441	2.7343	0.1472	4.3129	0.2103	8.3408	
	10.0000	0.0218	5.4312	0.0732	8.5924	0.1041	15.8769	
	INF.	1.5050	1.0306	0.8554	0.7283	0.5389	0.2147	
7	1.0000	0.2085	0.4999	0.6653	0.7521	0.8749	1.0671	2.2845
	0.9000	0.2302	0.4488	0.7374	0.6768	0.9687	0.9580	2.4538
	0.8000	0.2573	0.3978	0.8274	0.6013	1.0861	0.8489	2.6655
	0.7000	0.2919	0.3470	0.9431	0.5258	1.2369	0.7400	2.9375
	0.6000	0.3380	0.2964	1.0972	0.4503	1.4381	0.6314	3.2996
	0.5000	0.4023	0.2461	1.3127	0.3749	1.7196	0.5235	3.8051
	0.4000	0.4986	0.1960	1.6356	0.2995	2.1416	0.4163	4.5613
	0.3000	0.6585	0.1463	2.1734	0.2242	2.8445	0.3101	5.8180
	0.2000	0.9778	0.0970	3.2480	0.1492	4.2496	0.2052	8.3246
	0.1000	1.9340	0.0482	6.4698	0.0744	8.4623	0.1017	15.8281
	INF.	1.4988	1.0071	0.8422	0.7421	0.6441	0.4791	0.1911
n	$1/R_s$	$L_1$	$C_2$	$L_3$	$C_4$	$L_5$	$C_6$	$L_7$

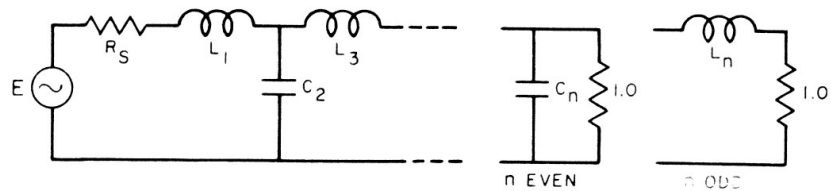


Figure 1: Example table from Zverev's filter handbook

This remarkable relation can be explained by looking at the lower header and using reciprocity. The lower headers in Zverev's tables apply to the dual filter network: resistances are replaced by conductances, inductances become capacitances, capacitances become inductances and the transfer stays the same. Due to reciprocity, the source and the load can be swapped without changing the transfer. When you first scale up the impedance of the ratio of ten filter and then swap it, you end up with something close to the infinite termination resistance ratio filter, see figure 2.

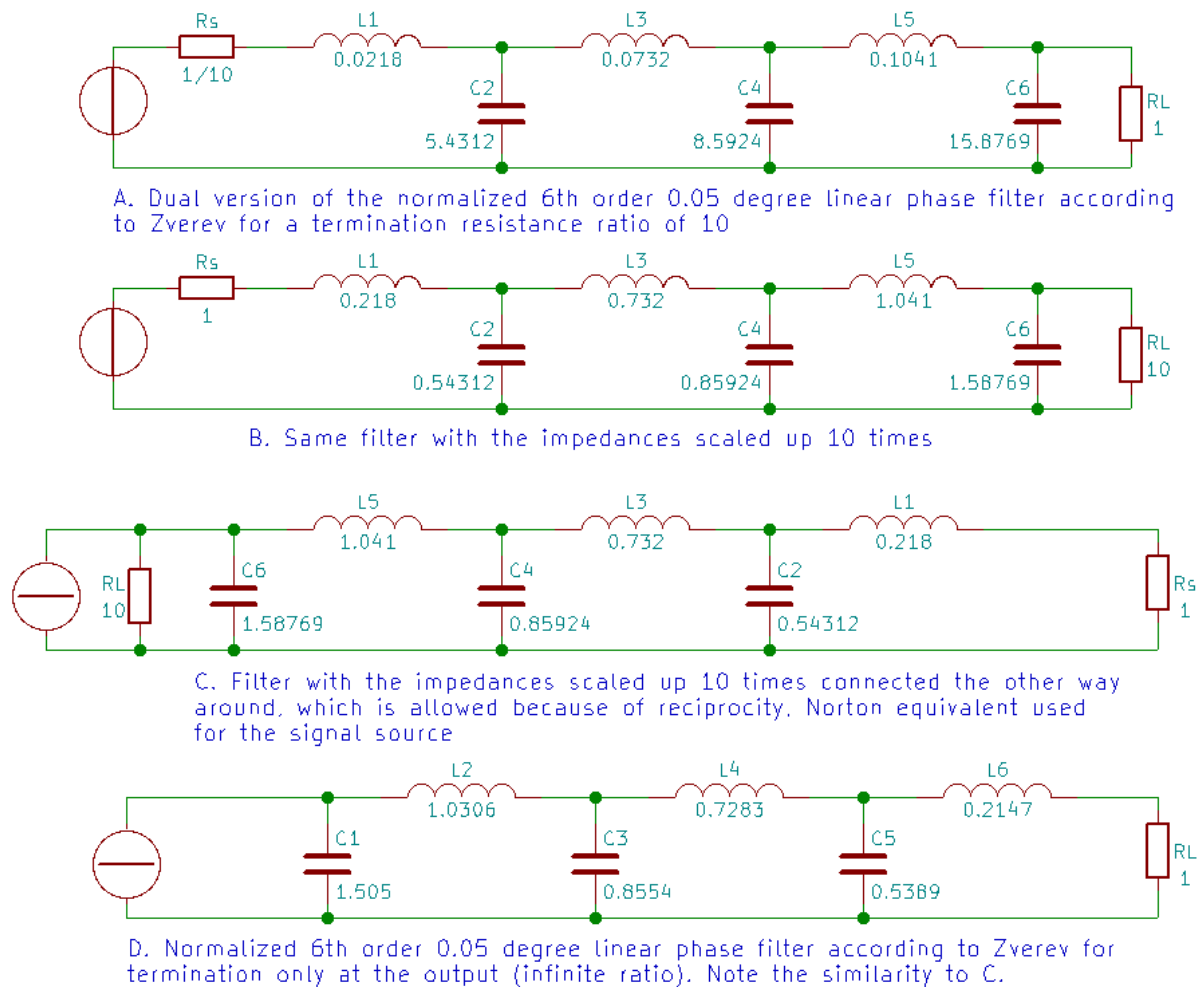


Figure 2: Network transformations on the ratio of ten network that turn it into something close to the infinite ratio network

All even-order filters in Zverev's tables of low-pass element values can be treated this way.

All in all, for even-order filters driven from a resistance of more than 10 times the load resistance, this leads to this procedure:

- Swap the order of the component values in the table for  $R_s = 10$
- Divide the capacitances by 10 and multiply the inductances by 10
- Calculate the reciprocal of the ratio between the source and the load resistances
- Interpolate linearly between the  $R_s = 10$  (so reciprocal = 0.1) and  $R_s \rightarrow \infty$  (so reciprocal  $\rightarrow 0$ ) cases
- Scale up the impedance to the correct termination resistance. For example, for a 600 ohm

termination resistance, all inductances and resistances go up by a factor of 600 and all capacitances are divided by a factor of 600

F. Scale to the correct frequency. Zverev's values are for 1 rad/s, so divide all inductances and capacitances by  $2\pi f_c$ , where  $f_c$  is the desired cut-off frequency.

The procedure for odd-order filters is similar but slightly different because Zverev only tabulates  $R_s$  values smaller than or equal to 1 and infinity for those, and because you have to swap the normal rather than the dual network. That is, when you take the values for  $R_s = 0.1$ , swap the network and scale up all impedances by ten, you are again at something that is similar though not equal to the network for  $R_s \rightarrow \infty$ . Hence,

A. Swap the order of the component values in the table for  $R_s = 0.1$

B. Divide the capacitances by 10 and multiply the inductances and resistances by 10. You have now converted it to a source resistance of 10.

C. Calculate the reciprocal of the ratio between the source and the load resistances

D. Interpolate linearly between the  $R_s = 10$  (so reciprocal = 0.1) and  $R_s \rightarrow \infty$  (so reciprocal  $\rightarrow 0$ ) cases

E. Scale up the impedance to the correct termination resistance. For example, for a 600 ohm termination resistance, all inductances and resistances go up by a factor of 600 and all capacitances are divided by a factor of 600

F. Scale to the correct frequency. Zverev's values are for 1 rad/s, so divide all inductances and capacitances by  $2\pi f_c$ , where  $f_c$  is the desired cut-off frequency.