

FIRDAC-weights for optimal suppression of far-off jitter?

Suppose you want to play a single-bit sigma-delta modulate, for example a DSD signal, with a FIRDAC, what weights will then optimally suppress jitter? I suspect that the answer is triangular weighting, it is definitely not uniform weighting.

The error in the output signal that you get when a clock edge is not quite at the correct place, depends on the size of the step of the output signal triggered by the clock edge.

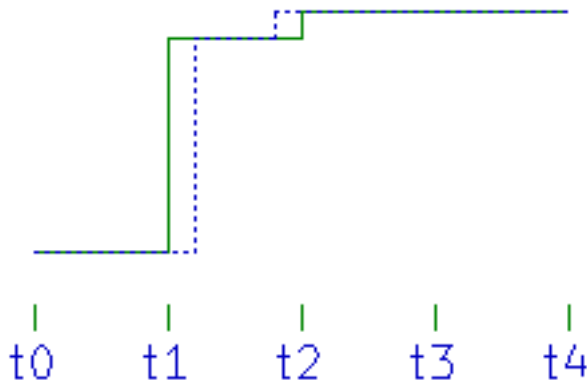


Figure 1: Sketch to show that the effect of a displaced clock edge depends on the size of the step of the output signal

In figure 1, the green line represents the ideal waveform at the FIRDAC output, and the dashed blue line the waveform with jitter (drawn unrealistically large). For a given shift of the clock edge, the area of the deviation from the ideal signal is much larger around t_1 than around t_2 , and around t_2 it is larger than around t_3 . The reason is the size of the step of the output signal. Hence, the trick is to make the steps at the FIRDAC output as small as possible.

The signal going into the FIRDAC is a single-bit sigma-delta modulate, for example a DSD signal. During silence, its spectrum consists of noise shaped quantization noise, that noise looks like approximately white noise with some deep notches in the audio band, as sketched in figure 2. Besides, there is often a big peak at or around half the sample rate $f_s/2$ (which actually gets frequency modulated by the desired signal when there is no silence). Digital signals have repetitive spectra, so you have the same but mirrored between $f_s/2$ and f_s , and the whole thing repeats every f_s .



Figure 2: Shape of a noise shaped quantization noise spectrum sketched from 0 to $f_s/2$. There is usually an additional peak at or near $f_s/2$.

To keep the steps at the FIRDAC output small, you will definitely want to suppress the peak at or near $f_s/2$. Hence, it is desirable that the response of the FIRDAC has a notch at $f_s/2$. It is fairly easy to figure out that a symmetrical FIR filter with even length has such a notch.

Regarding noise, everything gets simpler when we pretend it is white noise, so neglect that the noise in the audio band is suppressed by the noise shaping. The samples of white noise are not correlated. In fact, we will even assume they are independent identically distributed, i.i.d.

The output signal of a FIR filter with impulse response $h[i]$ and input signal $x[n]$ is

$$y[n] = \sum_{i=0}^{N-1} h[i] x[n-i]$$

The difference between two output samples is then

$$y[n] - y[n-1] = \sum_{i=0}^{N-1} h[i] (x[n-i] - x[n-i-1]) = \sum_{i=0}^{N-1} (h[i] - h[i-1]) x[n-i]$$

The extent to which a sample $x[n-i]$ contributes to the step at the output is therefore proportional to $h[i] - h[i-1]$, the difference between the weights of the present and the previous tap. This is logical: when a sample is shifted from one tap to another tap with the exact same weight, the amount it contributes to the output signal stays constant, so it does not contribute to an output step.

One might now be inclined to think that all weights ($h[i]$ s) have to be made equal, so all factors $h[i] - h[i-1]$ are zero. However, there is also an $h[-1]$ and an $h[N]$ in the expression, and $h[i] = 0$ when $i < 0$ or $i > N-1$, because the filter only goes from 0 up to and including $N-1$. One would then have to put all weights at 0. The output signal is then continuously 0 and is indeed independent of clock jitter, but you also don't hear any music anymore.

To avoid such utterly impractical optima, we introduce the constraint

$$\sum_{i=0}^{N-1} h[i] = 1$$

which makes the gain for low frequencies equal to 1.

Uniform weighting

In the case of uniform weighting, we now have:

$$h[i] = 1/N \quad \text{for } 0 \leq i \leq N-1$$

and hence

$$(y[n] - y[n-1])_{\text{uniform}} = \sum_{i=0}^N (h[i] - h[i-1]) x[n-i] = \frac{1}{N} (x[n] - x[n-N])$$

Only the samples of the input signal that just get shifted into or out of the FIR filter contribute to the output step, as was to be expected. After all, the weighting factor for the other samples stays constant.

Assuming independent, identically distributed samples of the input signal, the variance of the step at the output is now:

$$\sigma_{\text{step, uniform}}^2 = \frac{2}{N^2} \sigma_x^2$$

For example, when $N = 16$:

$$\sigma_{\text{step, uniform, 16}}^2 = \frac{2}{16^2} \sigma_x^2 = \frac{1}{128} \sigma_x^2$$

Triangular weighting

With uniform weighting, you get a large $|h[i] - h[i-1]|$ at the start and end of the filter and zero elsewhere. You could also consider making $|h[i] - h[i-1]|$ equally large everywhere (except in the middle, as will be shown shortly). You can then have a smaller $|h[i] - h[i-1]|$ at the start and end of the filter and still meet the constraint $\sum_{i=0}^{N-1} h[i] = 1$. Combining this with the wish for a symmetrical impulse response (and even length), this boils down to triangular weighting.

Example, with $N = 16$:

$$\begin{aligned}h[0] &= h[15] = 1/72 \\h[1] &= h[14] = 2/72 \\h[2] &= h[13] = 3/72 \\h[3] &= h[12] = 4/72 \\h[4] &= h[11] = 5/72 \\h[5] &= h[10] = 6/72 \\h[6] &= h[9] = 7/72 \\h[7] &= h[8] = 8/72\end{aligned}$$

In this case, $|h[i] - h[i - 1]| = 1/72$ for $i = 0...7$ and $i = 9...16$.

Precisely in the middle, at $i = 8$, is $|h[i] - h[i - 1]| = 0$. The same holds for $i < 0$ and $i > 16$.

There are 16 terms with $|h[i] - h[i - 1]| = 1/72$, so 16 samples contribute to the variance of the step at the output. That is,

$$\sigma_{\text{step, triangle, 16}}^2 = \frac{16}{72^2} \sigma_x^2 = \frac{1}{324} \sigma_x^2$$

As $1/324 < 1/128$, this is better than uniform weighting, at least regarding the effect of far-off jitter. I have the strong suspicion it is optimal, but I don't know how to prove that.