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Influence of Core Gap in Design of Current-Limiting Transformers

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Abstract—Core gaps are used in the magnetic shunt paths of current-limiting transformers to limit secondary current under short-circuit conditions. It is shown here that for short-circuit current limited to twice normal a current limiting transformer has only 55% of the rating of a noncurrent-limiting transformer of the same physical size. Part of the reduction in rating stems from the space required for magnetic shunts, part from the greater space needed for winding insulation, and part from the increased loss due to the gaps. Reliability of a previously tested gap-loss equation is established, and design examples are given for both stamped laminations and strip-wound cores.

I. INTRODUCTION

Air gaps inserted in magnetic shunt paths of current-limiting transformers cause gap loss which sometimes has been overlooked with consequent overheating and redesign of such transformers. Reliable prediction of gap loss must be available to the transformer designer to avoid this difficulty. Magnetic shunts take up window space. Both the space required by the shunts and the additional loss caused by the air gaps result in a reduction in transformer rating for a given size of core. Gap loss is also an important factor in the design of ac inductors; here too, neglecting gap loss may lead to thermal failure.

II. PREVIOUS WORK

Gap-loss information in this paper is based on data presented at the IEEE 1972 Workshop on Applied Magnetics.[1] These data indicate that losses associated with the relatively large air gaps in cores of current-limiting transformers can be predicted by

$$W_g = G l_g d f B_m^{\ 2} \qquad \text{watts} \tag{1}$$

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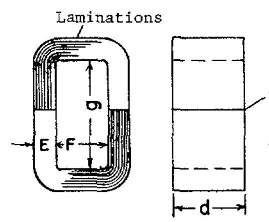

Fig. 1. Wound-core dimensions.

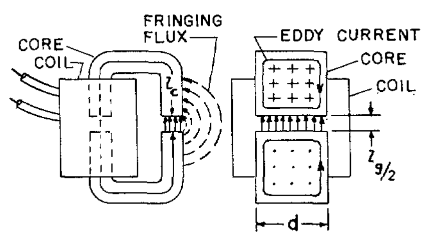

Fig. 2. Coil and core assembly.

where

G = numerical constant, = 0.155×10^{-8}
l_g = core gap, cm
d = core width, cm
f = line frequency, Hz
B_m = maximum core induction, gauss.

Tests described in Ref. 1 were made on wound cores, shaped like the core in Fig. 1, and with air gaps inserted as in Fig. 2. Fringing flux causes gap loss by leaving and reentering the core perpendicular to the plane of laminations as shown in Fig. 2. Use of (1) for various coil arrangements is given in Ref. 1; lack of this information has caused difficulties in the past, with consequent distrust of (1). It is the purpose of this paper to establish the reliability of (1), to extend its use to other kinds of transformer design, and to show the effects of gaps and gap loss on heating, size, and rating of current-limiting transformers.

Except for very small air gaps, the accuracy of (1) is within ± 25%, based on analysis of test data reported in Ref. 1. For

TABLE I
15 kG 400 Hz Gap-Loss Test Results for Several Cores and Coils

Air gap cm	Gap loss watts	Ratio of test/linear	$\lvert x\rvert$	x^2
0.10	5.0	0.42	0.58	0.337
.15	15.0	.80	.20	.04
.28	36.0	.97	.03	.0009
.28	22.6	.64	.36	.13
.28	37.4	.94	.07	.0049
.44	40.6	.74	.26	.0678
.44	61.4	.90	.10	.01
.44	69.0	.80	.20	.04

60 Hz transformers the gap loss fell within the ± 25% deviation from linearity. At 400 Hz the deviations were wider, a particularly poor correlation appearing at the smallest gap tested (0.1 cm). At such small gaps inaccuracies are likely to occur, yet their effects on transformer design are usually negligible; hence this deviation is not sufficient to warrant further widening of the limits for the more accurate and useful large gaps. This will be apparent from Table I, which shows eight measurements at 400 Hz, in which the ratio of actual/linear for low readings and linear/actual for high readings are tabulated, together with the deviation $|x|$ and its square x^2

$$\sum |x| = 1.80 \quad \sum x^2 = 0.6306$$

$$x = 0.225 \quad \overline{x^2} = 0.079$$

$$\sqrt{\overline{x^2}} = 0.281 = \sigma$$

$$h = \frac{1}{\sqrt{\pi}\,x} = \frac{1}{0.225 \times 1.772} = 2.50; \quad h = \frac{1}{\sqrt{2}\,\sigma} = \frac{1}{1.41 \times 0.281} = 2.52. \tag{2}$$

Close agreement between these values of h is a criterion of reliable data.[2] Statistical analysis indicates that 25% loss limits for (1) provide a 98% confidence level.[3]

As stated by (1), gap loss increases directly as the length of air gap l_g, at least over the range of air gaps tested, which includes practical values used in the design of current-limiting transformers. If the air gap is divided in two as shown in Fig. 2, half of the gap loss as calculated by (1) appears at each half-gap $l_g/2$. This fact reduces the localized heating compared to a gap in one leg only. For very large gaps, good design may require that the gap be subdivided further to avoid overheating coils and other parts.

III. DESIGN EXAMPLES

Many current-limiting transformers are made with cores of stamped laminations and shunts as shown in Fig. 3. The magnetic paths provided by the shunts separate the primary and secondary windings, but at no load virtually the same flux links both windings because the reluctance of the main core path is much lower than that of the air gap. As load is added, secondary ampere-turns force more of the flux into the shunt paths, until at short-circuit the load voltage is zero and the load reflected into the primary is an inductance. Flux fringing across the gap in this case enters the shunt laminations at right angles, all the way across the main core stack; it also enters the main core laminations across the shunt stack. Hence dimension d in (1) is the average of the main and shunt core stacking dimensions. For a square stack of laminations, d is the main lamination tongue width.

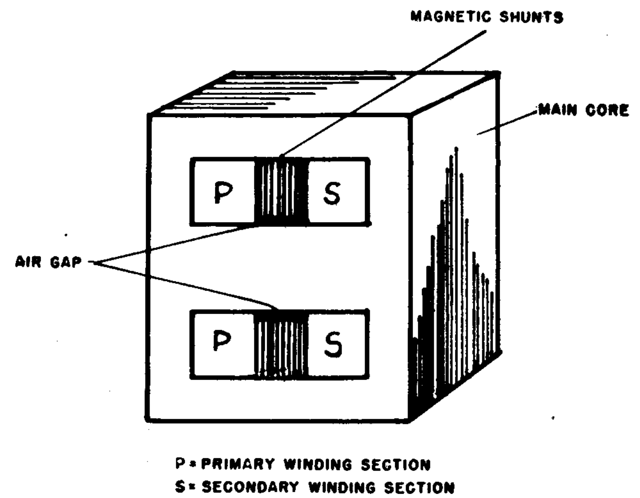

Fig. 3. Current-limiting transformer with stamped lamination core.

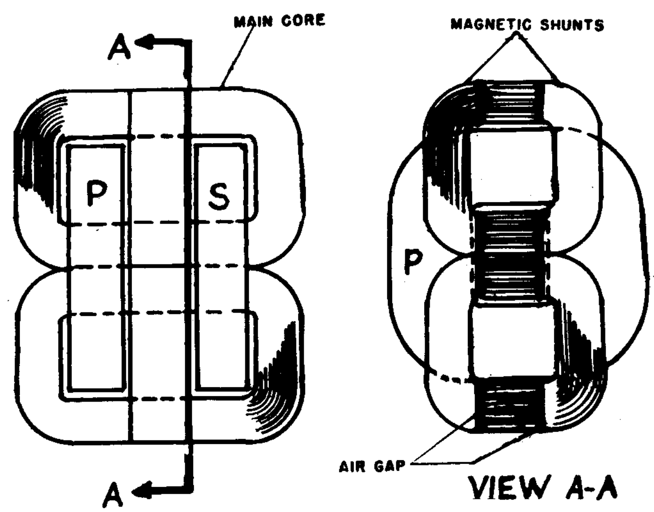

Fig. 4. Current-limiting transformer with wound cores and shunts.

When wound cores are used in current-limiting transformers, the shunts should not be placed as in Fig. 3 because the portion of the core flux passing into the shunts would leave the main core at right angles to the plane of laminations and would cause very high loss. One way of preventing this condition is shown in Fig. 4. The shunts are wound cores, located not in the main core windows, but at the sides, so that the main core flux traverses laminated directions in all cores. Fringing flux then crosses the gaps as in Fig. 2. Because the shunts are longer, they weigh more than do those in Fig. 3, and this partly offsets the normal weight advantage afforded by the grain-oriented steel in wound cores. Equation (1) applies to each pair of shunts except at the innermost gap, and hence is three times the loss calculated from (1), with d = the shunt core width.

Regardless of the kind of core used, magnetic shunts occupy space and result in a larger transformer. Gap loss must be dissipated safely, and this too increases size. Primary winding V-A exceeds secondary V-A by more than the transformer losses because the reflected load is partly inductive. For all these reasons, a given size of transformer has much lower rating when it is used for current limiting than for ordinary use.

Assume that a transformer (primary 230 V, 60 Hz; secondary 11 V @ 60 A normal load) is required to limit short-circuit current to twice normal. This may be achieved with designs

TABLE II
Comparison of 660 V-A Current-Limiting Transformer Designs

Kind of core	Overall dim. cm.	Total wt. kg.	Losses - watts winding	core	shunts	gap	Dim. d cm	Gap(l_g) cm.	Effy. %
Fig. 3	17.8x17x12.7	15.4	21.4	17.5	1.0	16	5.10	0.28	92
Fig. 4	25x16x12.4	14.1	16.8	16.8	1.8	15	2.06	0.35	93

TABLE III
Value of Term G in (1) for Various Configurations

Core	Coils*	Gap	Dim. d	Value of G
Wound (Fig.1)	1 as in Fig.2	All in outer leg	Core strip width	0.155×10^{-8}
Wound (Fig.1)	1 as in Fig.2	$l_g/2$ in each leg	Core strip width	0.078×10^{-8}
Wound (Fig.1)	2 coils, one on each leg	$l_g/2$ in each leg	Core strip width	0.039×10^{-8}
Stamped (Fig.3)	as in Fig. 3	l_g under each stack of shunts	Tongue width **	0.155×10^{-8}
Wound (Fig.4)	as in Fig. 4	$l_g/2$ between each shunt and main core	Shunt strip width	0.465×10^{-8}

* with normal coil clearances to core.
** for a square core stack

listed in Table II. The winding temperature rise for these designs is 55°C. To give an idea of the reduction in rating, a transformer of the same size in ordinary (non-current limiting) use is rated at 1200 V-A and has 96% efficiency.

If a separate inductor is used in series with the primary instead of magnetic shunts, the transformer size is reduced. For the wound-core design of Fig. 4, weight changes to

transformer	7.65 kg
inductor	6.00 kg
total	13.65 kg.

The weight of the pair of units is slightly less than the weight of the Fig. 4 transformer in Table II. The ratio of weights could vary somewhat from these figures for any specific application. The cost of the current-limiting transformer would be lower than that of the pair because of the smaller number of parts.

IV. SUMMARY OF RESULTS

1) Reliability of (1) is established within ±25% and with a confidence level of 98% over the range of gaps tested (up to $l_g/l_c = 0.04$).

2) Equation (1) applies to transformers with stamped lamination cores as well as to those with wound cores.

3) Core gaps in current-limiting transformers cause an increase in size of 82% compared to non-current-limiting transformers. If a separate inductor is used instead of magnetic shunts, the total weight is nearly the same as for a current-limiting transformer but the total cost is higher.

4) Values of constant G in (1) for several coil-core arrangements are listed in Table III.

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Magnetically Controlled Thyristor Chopper for Seikan Undersea Tunnel Battery Locomotive

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Abstract—We have developed thyristor chopper equipment which is controlled by magnetic phase shifters as the speed control equipment of a battery locomotive for the Seikan Undersea Tunnel. The main feature of this equipment is adoption of a balanced half-wave type magnetic phase shifter, having high reliability together with many functions, in a simple structure to the thyristor gate control. Special consideration was given to the smooth control characteristics at around the minimum and maximum points of the chopper output voltage. This construction of equipment received special consideration regarding its ability to withstand severe working conditions. Satisfactory results were obtained in various performance and reliability tests, and locomotives supplied with this equipment are already operating in excellent condition.

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I. INTRODUCTION

Presently under construction is the Seikan Undersea Tunnel which connects Japan's main island of Honshu with Hokkaido to the north (1).

The total length of this tunnel is approximately 45 km (34 miles); even its undersea portion is approximately 25 km (16 miles). This is the world's longest tunnel with which there is no comparison. In 1971 the main tunnel work was started, and is scheduled to be completed in 1979 – six years later.