

Thus $\Theta_{b,c}$ means that a is zero, b is equal to the fifth characteristic value, and c has a value corresponding to the second harmonic. Then by making use in the usual way of the orthogonal properties of the functions involved, it may be shown that

$$B_{abc} = \frac{\omega}{2\pi} \frac{\int_0^{a_0} \int_{-\pi/2}^{\pi/2} u(\theta, \varphi, t) \sin \theta \Theta_{abc} e^{-2\pi i a \varphi} e^{-i\omega_c t} d\theta d\varphi dt}{\int_0^{a_0} \Theta_{abc}^2 \sin \theta d\theta}$$

The function V is then expressed in the form

$$V = \sum_{abc} A_{abc} M_{abc}(\omega) \Theta_{abc}(\theta) e^{2\pi i a \varphi} e^{i\omega_c t}$$

When $\mu=0$ we must have by condition (3) that

$$u(\theta, \varphi, t) = \sum_{abc} A_{abc} M'_{abc}(0) \Theta_{abc} e^{2\pi i a \varphi} e^{i\omega_c t}$$

Thus to satisfy the boundary condition at the diaphragm we put

$$A_{abc} = \frac{B_{abc}}{M'_{abc}(0)}$$

For the simple case in which the diaphragm is assumed to move like a piston, the velocity at the diaphragm is independent of position and

$$u = v_0 e^{i\omega t}$$

where v_0 is the maximum velocity. For this case

$$B = \frac{\omega v_0}{2\pi} \frac{\int_0^{a_0} \sin \theta \Theta_{abc} d\theta \int_{-\pi/2}^{\pi/2} e^{i(\omega - \omega_c)t} dt \cdot \int_{-\pi/2}^{\pi/2} e^{-2\pi i a \varphi} d\varphi}{\int_0^{a_0} \Theta_{abc}^2 \sin \theta d\theta}$$

Thus B is zero unless $a=0$ and $\omega=\omega_c$. We shall drop the subscripts a and c , B_n being the coefficient when $b=b_c$, $a=0$, and c has only the one value obtained from the frequency at which the piston is being driven. Hence

$$B_n = \frac{v_0 \int_0^{a_0} \sin \theta \Theta_n d\theta}{\int_0^{a_0} \Theta_n^2 \sin \theta d\theta}$$

Or in terms of W and z

$$B_n = \frac{v_0 \int_{-1}^1 W_n(\cos \theta) d\theta}{\int_{-1}^1 W_n^2(\cos \theta) d\theta}$$

and the velocity potential is given by

$$V = e^{i\omega t} \sum_{n=0}^{\infty} A_n W_n(\cos \theta) F_n(\sinh \mu)$$