



THE VELOCITY POTENTIAL OF AN  
HYPERBOLIC HORN

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1931

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Submitted in partial fulfillment of the  
requirements for the degree of

DOCTOR OF PHILOSOPHY

from the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1937

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## I Introduction

Since the wave equation in the oblate spheroidal coordinate system is separable, it is possible to calculate the velocity potential of a horn formed by rotating an hyperbola about an axis perpendicular to the line joining its foci.

Consider points lying in a plane through the  $\xi$ -axis making an angle  $\varphi$  with the  $\xi$ -axis and let  $r$  be the distance of a point from the  $\xi$ -axis. The coordinates  $\mu$  and  $\theta$  are defined by the equations

$$r = \frac{\alpha}{2} \cosh \mu \sin \theta$$

$$\xi = \frac{\alpha}{2} \sinh \mu \cos \theta$$

By eliminating  $\theta$ , we obtain

$$\frac{r^2}{\left(\frac{\alpha}{2} \cosh \mu\right)^2} + \frac{\xi^2}{\left(\frac{\alpha}{2} \sinh \mu\right)^2} = 1$$

showing that all points for which  $\mu$  is a constant lie on an ellipse whose foci are at  $r = \pm \alpha/2$  and whose major and minor axes are  $\frac{\alpha}{2} \cosh \mu$  and  $\frac{\alpha}{2} \sinh \mu$  respectively. By eliminating  $\mu$ , we obtain

$$\frac{r^2}{\left(\frac{\alpha}{2} \sin \theta\right)^2} - \frac{\xi^2}{\left(\frac{\alpha}{2} \cos \theta\right)^2} = 1$$

and hence, all points for which  $\theta$  is a constant lie on an hyperbola whose asymptotes make an angle  $\theta$  with the  $\xi$ -axis and whose foci are coincident with those of the ellipse. See Fig.1  
The Cartesian coordinates  $x, y$  and  $z$  of a point whose spheroidal coordinates are  $\mu, \theta$  and  $\varphi$  are therefore given by

$$\xi = \frac{\alpha}{2} \cosh \mu \sin \theta \cos \varphi$$

$$\eta = \frac{\alpha}{2} \cosh \mu \sin \theta \sin \varphi$$

$$\zeta = \frac{\alpha}{2} \sinh \mu \cos \theta$$

The boundary surface of the horn is obtained by putting  $\theta = \theta_0$ , where  $\theta_0$  is one half the angular opening of the horn. The radius of the throat is  $(\alpha/2) \sin \theta_0$ .

Proceeding in the usual manner, we assume that the velocity potential  $V(\mu, \theta, \varphi, t)$  is given by

$$V = \psi(\mu, \theta, \varphi) e^{2\pi i \nu t}$$