Equation (9.31) breaks down when the second-harmonic distortion becomes large, and a more complicated expression, not given here, must be used.

In the case of an exponential horn, the amplitude of the fundamental decreases as the wave travels away from the throat, so that the second-harmonic distortion does not increase linearly with distance. Near the throat it increases about as given by Eq. (9.31), but near the mouth the pressure amplitude of the fundamental is usually so low that very little additional distortion occurs.

The distortion introduced into a sound wave after it has traveled a distance x down an exponential horn for the case of a constant power supplied to unit area of the throat is found as follows:

1. Differentiate both sides of Eq. (9.31) with respect to x, so as to obtain the rate of change in p_2 with x for a constant p_1. Call this equation (9.31*a*).

2. In Eq. (9.31*a*), substitute for p_1 the pressure $p_T e^{-mx/2}$, where p_T is the rms pressure of the fundamental at the throat of the horn in newtons per square meter and m is the flare constant.

3. Then let $p_T = \sqrt{I_T \rho_0 c}$, where I_T is the intensity of the sound at the throat in watts per square meter and $\rho_0 c$ is the characteristic acoustic impedance of air in mks rayls.

4. Integrate both sides of the resulting equation with respect to x. This yields

Per cent second-harmonic distortion

$$= \frac{50(\gamma + 1)}{\gamma} \frac{\sqrt{I_T \rho_0 c}}{P_0} \frac{f}{f_c} [1 - e^{-mx/2}] \quad (9.32)$$

For an infinitely long exponential horn, at normal atmospheric pressure and temperature, the equation for the total distortion introduced into a wave that starts off sinusoidally at the throat is

$$\text{Per cent second-harmonic distortion} = 1.73 \frac{f}{f_c} \sqrt{I_T} \times 10^{-2} \quad (9.33)$$

where f = driving frequency in cycles per second
f_c = cutoff frequency in cycles per second
I_T = intensity in watts per square meter at the throat of the horn

Equation (9.33) is shown plotted in Fig. 9.12. Actually, this equation is nearly correct for finite horns because most of the distortion occurs near the throat.

Equation (9.33) reveals that for minimum distortion the cutoff frequency f_c should be as large as possible, which in turn means as large a flare constant as possible. In other words, the horn should flare out rapidly in order to reduce the intensity rapidly as one travels along the horn toward the mouth.