

# Study of Nonlinear Distortion in Audio Instruments\*

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Several types of nonlinear distortion in audio instruments are discussed using distortion models. An input pulse is characterized by an asymmetric waveform with zero dc component. Pulse responses corresponding to the *s*-type nonlinearity, clipping and crossover distortion of an amplifier, the transient distortion of a level compressor, and the *s*-type nonlinearity of a loudspeaker are illustrated with their spectra. One result shows that the nonlinearities of an amplifier and a loudspeaker are physically distinguishable.

## 0 INTRODUCTION

In audio an important problem left unsolved is the general correlation of subjective and objective quantities. A more immediate question: Why can we distinguish "amplifier sound" from the sound of a loudspeaker in listening tests? Audible differences in amplifiers lie in their inherent distortions, whatever their forms. The total harmonic distortion of a high-quality amplifier is usually less than 0.1%, while the distortion of a loudspeaker is more than 1%. In spite of this difference in magnitude, we can distinguish "amplifier sound" from the sound of a loudspeaker and point out differences in the quality of amplifiers. This implies that the distortions in amplifiers and loudspeakers differ in properties which cannot be expressed by a total harmonic distortion measurement. A musical sound involves many transient sounds. Waveforms of transient sounds are generally very complicated. A common feature of transient sounds is an asymmetric waveform without a dc component. In this paper several types of nonlinear distortions in audio instruments are discussed using an input pulse which is characterized by an asymmetric waveform with zero dc component [1].

## 1 NONLINEARITY AND PULSE RESPONSE

The waveform of the pulse  $h(t)$  shown in Fig. 1(a) is composed of half-sine curves  $a$  and  $b$ , so that the dc component of the pulse is zero. The spectrum of the

pulse is expressed by

$$\begin{aligned} S(f) &= S_1(f) + S_2(f) \\ &= \frac{A_1/\pi f_1}{1 - (f/f_1)^2} \cos\left(\frac{\pi f}{2f_1}\right) e^{i\pi f/2f_1} \\ &\quad - \frac{A_2/\pi f_2}{1 - (f/f_2)^2} \cos\left(\frac{\pi f}{2f_2}\right) e^{i\pi f(1/f_1+1/2f_2)} \quad (1) \end{aligned}$$

where  $i = \sqrt{-1}$ ,  $f_1 = 1/T_1$ ,  $f_2 = 1/T_2$ , and where  $S_1(f)$  and  $S_2(f)$  denote the spectrum functions of pulses  $a$  and  $b$ , respectively. From  $S(0) = 0$ , we have

$$\frac{A_1}{\pi f_1} = \frac{A_2}{\pi f_2} = N \quad (2)$$

where  $N$  denotes a normalization factor.

As shown in Fig. 1(b), the spectrum function  $\bar{S}(f)$  (normalized by  $N$ ) is characterized by a 6-dB-per-octave slope at low frequencies. Since waveform asymmetry and zero dc component are typical properties of impulsive waves in nature, the pulse of Fig. 1 may be considered one of the simplifications of transient sounds.

### 1.1 *s*-Type Nonlinearity

An *s*-type nonlinearity is assumed to be symmetric with respect to the origin and given in the form

$$Y = A - A^2/2A_c, \quad 0 \leq A \leq A_c \quad (3)$$

where  $Y$  denotes the output magnitude and  $A$  the input

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magnitude,  $A_c$  being a clipping level [see Fig. 2(a)].

The effect of an  $s$ -type nonlinearity on the spectrum is to increase low-frequency components, as shown in Fig. 3.

**1.2 Clipping**

For clipping [Fig. 2(b)] we express the input-output characteristic as

$$Y = \begin{cases} A, & |A| \leq A_c \\ A_c, & |A| > A_c \end{cases} \quad (4)$$

Clipping has effects, as shown in Fig. 4, on both low and high frequencies, in contrast to the soft  $s$ -type nonlinearity.

**1.3 Crossover Distortion**

We assume a crossover distortion characteristic [Fig. 2(c)] of the form

$$|Y| = \frac{A^2}{2A_L}, \quad |A| \leq A_L \quad (5)$$

As shown in Fig. 5, the output waveform is deformed, and the low-frequency component increases with decreasing amplitude of the input pulse.

**1.4 Level Compression**

As an example of dynamic distortion, we calculate the distortion caused by a level compression. For simplification, we assume the pulse to be attenuated uniformly from time  $t = 0$  to  $t = T_1/2$  with the attenuation rate  $r < 1$  [Fig. 6(a)]. The result is shown in Fig. 6(b). Unless the function of a level compressor and expander is ideal, a noise-reduction system causes similar distortion, and

the impression of an impulsive sound becomes somewhat dull or heavy.

The same methods as those used in the above calculation can be applied to the nonlinearity of a push-pull amplifier, that is, a difference between amplification factors in the positive and negative regions, where the input-output characteristic is a line broken at the origin. In this case the attenuation rate  $r$  is only a few thousandths, but the effect on a sound is detectable.

**1.5 s-Type Nonlinearity of a Loudspeaker**

The  $s$ -type nonlinearity in the amplitude of a vibrating plate is typical of the distortion in a loudspeaker. We assume the sound to be proportional to the velocity of

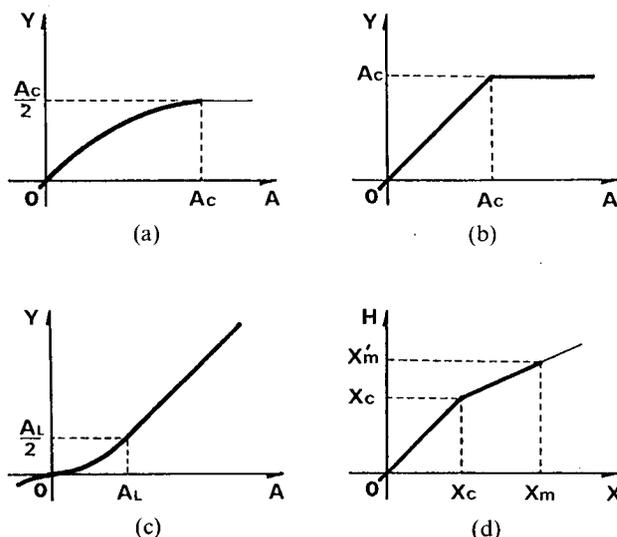


Fig. 2. Nonlinearity models used in calculations. (a)  $S$ -type nonlinearity. (b) Clipping. (c) Crossover distortion. (d)  $S$ -type nonlinearity (loudspeaker).

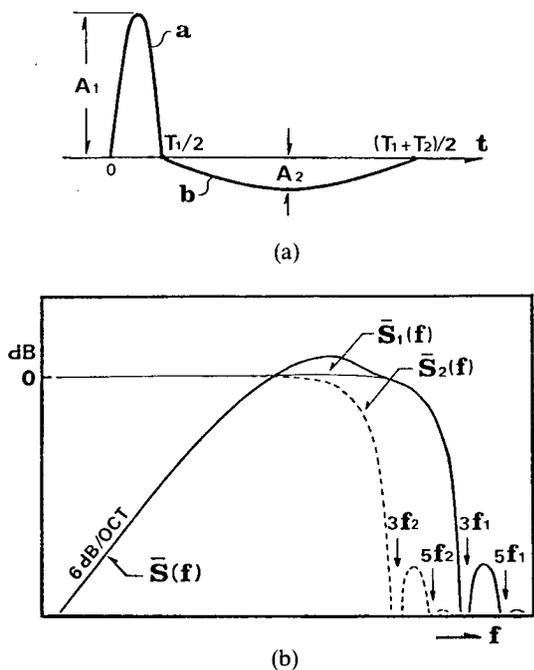


Fig. 1. (a) Waveform of an input pulse  $h(t)$ . (b) Its spectrum  $\bar{S}(f)$ .

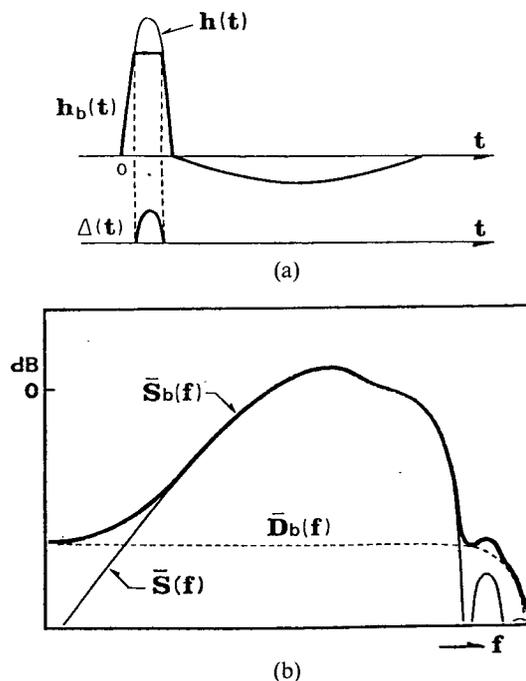


Fig. 3.  $S$ -type nonlinearity. (a) Waveforms of an output pulse  $h_a(t)$  and a distortion  $\Delta t$ . (b) Their spectra  $S_a(f)$  and  $\bar{D}_a(f)$ , respectively.

an infinite plate, and the nonlinearity is shown in Fig. 2(d), where  $X$  denotes the input force and  $H$  the corresponding displacement. The deviation of the displacement from the ideal is

$$\Delta_H(t) = \begin{cases} QK \sin^2\left(\frac{\pi t}{\tau_1}\right), & \frac{T_1}{2} - \frac{\tau_1}{2} \leq t \leq \frac{T_1}{2} \\ QK \cos^2\left(\frac{\pi t}{\tau_2} - \frac{\pi T_1}{2\tau_2}\right), & \frac{T_1}{2} \leq t \leq \frac{T_2}{2} \end{cases} \quad (6)$$

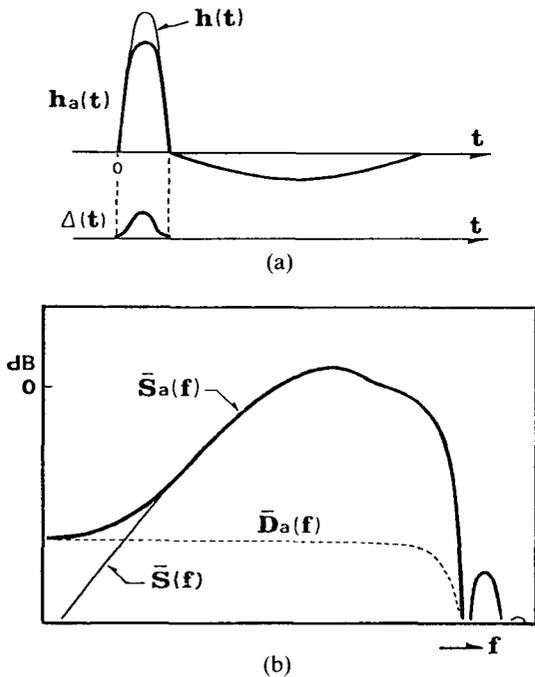


Fig. 4. Clipping. (a) Waveforms of an output pulse  $h_b(t)$  and a distortion  $\Delta t$ . (b) Their spectra  $\bar{S}_b(f)$  and  $\bar{D}_b(f)$ , respectively.

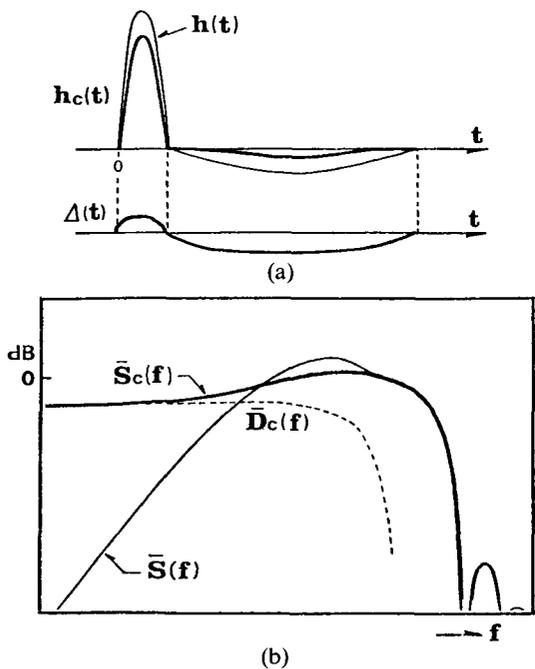


Fig. 5. Crossover distortion. (a) Waveforms of an output pulse  $h_c(t)$  and a distortion  $\Delta(t)$ . (b) Their spectra  $\bar{S}_c(f)$  and  $\bar{D}_c(f)$ , respectively.

where

$$Q = 1 - \frac{X'_m}{X_m}$$

$$\tau_n = T_n - \frac{2T_n}{\pi} \sin^{-1} \sqrt{\frac{X_c}{X_m}}, \quad n = 1 \text{ or } 2$$

with  $X_c$  denoting the maximum limit of a linear response,  $X_m (\geq X_c)$  the maximum amplitude of an input pulse,  $X'_m$  an output corresponding to  $X_m$ , and  $K$  an arbitrary constant. The resulting effect of the nonlinearity of a loudspeaker appears only in the high-frequency region (Fig. 7).

## 2 CONCLUSIONS

A small amount of distortion in an audio system due to nonlinearity gives a subtle change in the reproduced sound quality. Apart from other judgments of a system, we can detect such distortion as a characteristic sound when we listen to music reproduced by the system. This paper has discussed the effect of several types of nonlinearities on the spectrum of a pulse that may be considered a simplification of natural transient sounds. It was shown that amplitude nonlinearities have an effect upon the spectrum at low frequency; in contrast, the displacement nonlinearity of a loudspeaker changes the spectrum at high frequency. The difference implies that one can separate the characteristic sound of an amplifier from that of a loudspeaker in hearing tests. The application of this theory is in a novel measurement of the nonlinearity of amplifiers using a composite pulse [2], where experiments were made to show that the nonlinearities of an amplifier and a loudspeaker are physically distinguishable.

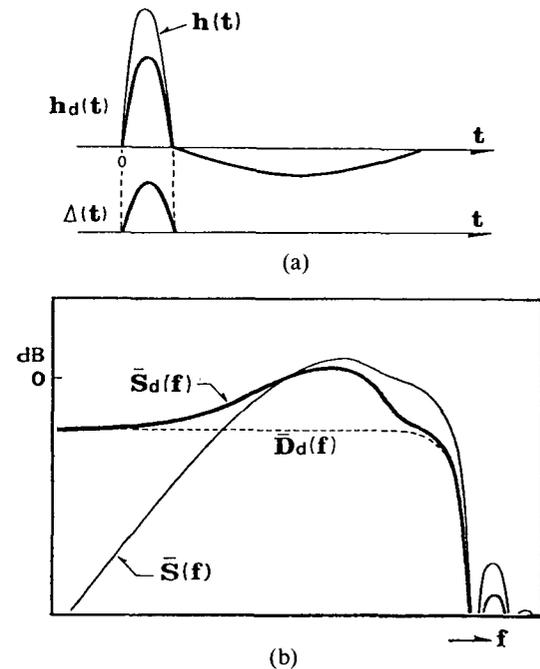


Fig. 6. Level compression. (a) Waveforms of an output pulse  $h_d(t)$  and a distortion  $\Delta(t)$ . (b) Their spectra  $\bar{S}_d(f)$  and  $\bar{D}_d(f)$ , respectively.

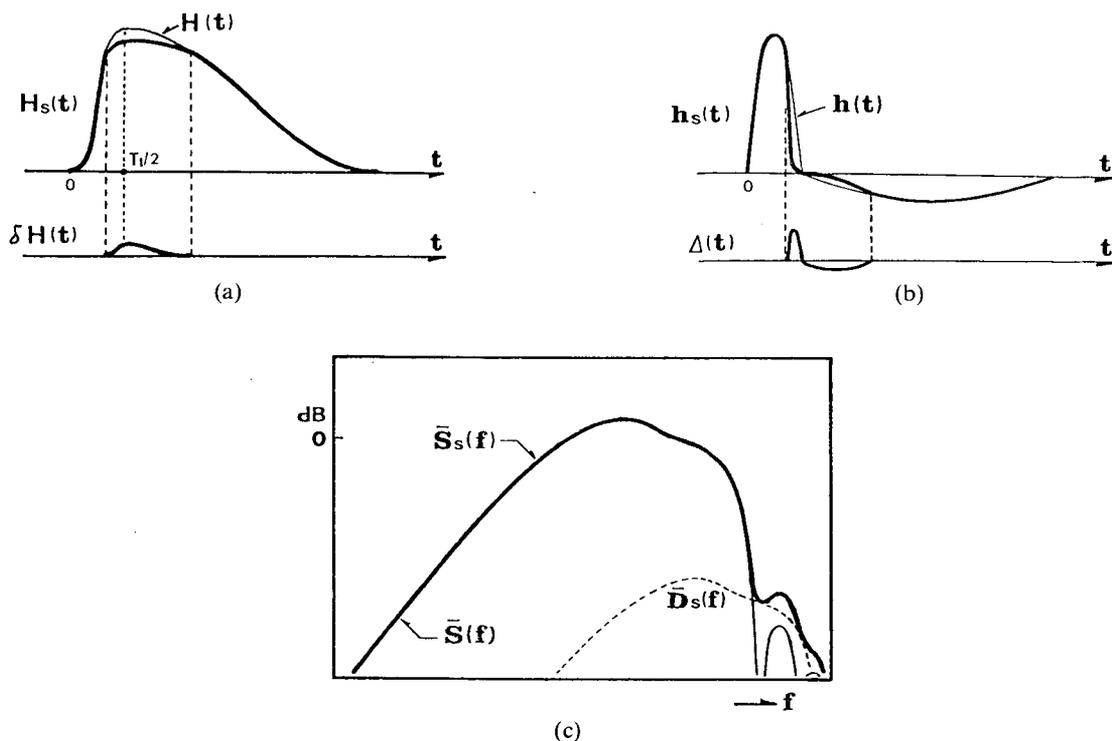


Fig. 7. S-type nonlinearity of a loudspeaker. (a) Displacement of a loudspeaker  $H_s(t)$  and a deviation  $\Delta_H(t)$ . (b) Waveforms of an output sound pressure  $h_s(t)$  and a distortion  $\Delta(t)$ . (c) Spectra  $\bar{S}_s(f)$  of  $h_s(t)$  and  $\bar{D}_s(f)$  of  $\Delta(t)$ .

### 3 REFERENCES

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 [2] Y. Hirata, M. Ueki, T. Kasuga, and T. Kitamura,

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Mr. Hirata's biography was published in the May issue.