$Z_0 = \rho c/\pi r^2$ by an amount δZ, these quantities being given in terms of the bend parameter B by

$$\frac{\delta v}{c} = -\frac{\delta Z}{Z_0} = \left(\frac{2I}{\pi B}\right)^{1/2} - 1, \tag{8.73}$$

where

$$I = \int_0^{\pi/2} \cos\theta \ln\left(\frac{1 + B\cos\theta}{1 - B\cos\theta}\right) d\theta. \tag{8.74}$$

Figure 8.20 shows this relation in graphical form for values of the bend parameter between zero, a straight tube, and almost 1, which is the geometrical limit. The practical limit for bent tubing in a brass instrument is about $R \approx 0.8$.

The analysis leading to these equations, or the similar equations of Nederveen, is not rigorous, so that it is clearly necessary to check the conclusions against experiment. This was done by Keefe and Benade (1983) for tubing with about the maximum curvature found in brass instruments ($B = 0.728$). For this tubing the theory predicts $\delta v/c = -\delta Z/Z_0 = 8.9\%$. The measured values were significantly smaller than this, however, with $\delta v/c \approx 4.7\%$ and $\delta Z/Z_0 \approx -6.3\%$. The discrepancy between these two values, and between both of them and the theory, is puzzling. Keefe and Benade ascribe it to the neglect of viscosity in the calculation, but it is not clear that there is any shear except at the walls, so it may be some other aspect of the approximations made in the derivation that is to blame. Despite this, the theory does give at least a semiquantitative account of the effect of bending.

From this discussion it is clear that bending a tube shortens its acoustic length and lowers its impedance, and that the corrections may be large enough to require consideration in applications such as design of the valve tubing in trumpets and the larger bends in tubas and saxophones. As pointed out by Nederveen, insertion of a curved section into a cylindrical bore creates least acoustic missmatch if the cross-section of the curved part is reduced slightly from that of the bore, to match the impedances. In the case of a conical bore, the length of the curved section should also be increased slightly, with corresponding decrease in cone angle, to allow for the increased phase velocity.