reasonable compass for playing and transport. This bending is significant in instruments such as the bassoon and saxophone, and extreme in the case of brass instruments. It is therefore important to know what effect, if any, it has on the acoustic properties of the horn.

To an obvious first approximation we should expect to measure the effective length of the horn along a curved axis passing through the centroid of its cross-section at every point. As expected, this is a good approximation for cases in which the bend radius R is large compared with the tube radius r, as defined in Fig. 8.20, but for more extreme bends a careful analysis is required. The problem has been considered by Nederveen (1969) and by Keefe and Benade (1983), both of whom come to similar conclusions. If we define a parameter $B = r/R$ to measure the severity of the bend, then the analysis of Keefe and Benade shows that the wave velocity in the duct is increased above the normal sound velocity c by an amount δv and the characteristic impedance of the duct is decreased from its normal value $Z_0 = \rho c/\pi r^2$ by an amount δZ, these quantities being given in terms of the bend parameter B by

$$\frac{\delta v}{c} = -\frac{\delta Z}{Z_0} = \left(\frac{2I}{\pi B}\right)^{1/2} - 1, \tag{8.73}$$

where

$$I = \int_0^{\pi/2} \cos\theta \ln\left(\frac{1 + B\cos\theta}{1 - B\cos\theta}\right) d\theta. \tag{8.74}$$

Figure 8.20 shows this relation in graphical form for values of the bend