Short rectangular pulse (D)

$$y = E\left[k + \frac{2}{\pi}\left(\sin k\pi \cos \theta + \frac{\sin 2k\pi \cos 2\theta}{2} + \ldots + \frac{\sin nk\pi \cos n\theta}{n} + \ldots\right)\right] \quad (12)$$

Half-wave rectifier output (E)

$$y = \frac{E}{\pi}\left(1 + \frac{\pi \cos \theta}{2} + \frac{2 \cos 2\theta}{3} - \frac{2 \cos 4\theta}{15} + \frac{2 \cos 6\theta}{35} - \ldots \ldots (-1)^{n/2+1} \frac{2}{n^2 - 1} \cos n\theta \ldots\right) (n \text{ even}) \quad (13)$$

Full wave rectifier output (F)

$$y = \frac{2E}{\pi}\left(1 + \frac{2 \cos 2\theta}{3} - \frac{2 \cos 4\theta}{15} + \frac{2 \cos 6\theta}{35} - \ldots \ldots (-1)^{n/2+1} \frac{2 \cos n\theta}{n^2 - 1} \ldots\right) (n \text{ even}) \quad (14)$$

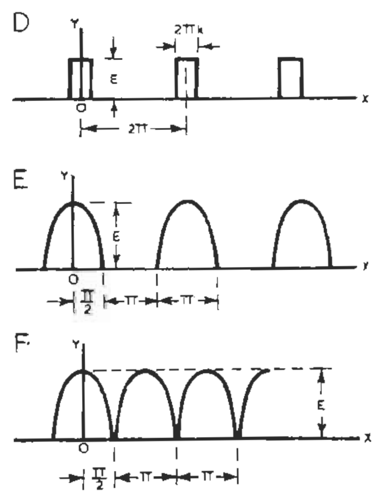



Fig. 6.43

## (ii) Other applications of the Fourier Series

The Fourier Series is particularly useful in that it may be applied to functions having a finite number of discontinuities within the period, such as rectangular and saw-tooth periodic pulses.

The Fourier Series may be put into the exponential form, this being useful when the function lacks any special symmetries.

The Fourier Series may also be applied to non-periodic functions.

For information on these applications, see the list of references—Sect. 9(B).

## (iii) Graphical Harmonic Analysis

Any irregular waveform may be analysed to determine its harmonic content, and the general method is to divide the period along the $X$ axis into a suitable number of divisions (e.g. Fig. 6.44 with 24 ordinates), the accuracy increasing with the number of divisions.

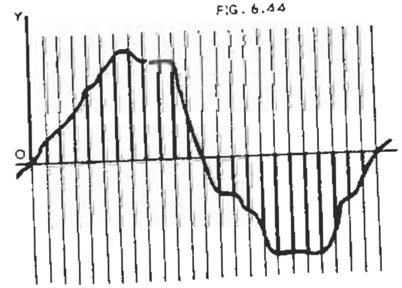
Ordinates are drawn at each point on the $X$ axis and the height of each ordinate is measured. The minimum number of ordinates over the cycle must be at least twice the power of the highest harmonic which it is desired to calculate. Various



methods for carrying out the calculations have been described. Some are based on equal divisions of time (or angle) while others are on equal divisions of voltage.

**In the harmonic analysis of the distortion introduced by valves on resistive loads,** it is possible to make use of certain properties which simplify the calculations:

FIG. 6.44




(1) All such distortion gives a waveform which is symmetrical on either side of the vertical lines (ordinates) at the positive and negative peaks.

(2) It is therefore only necessary to analyse over half the cycle, from one positive peak to the following negative peak, or *vice versa.*

(3) Even harmonic distortion results in positive and negative half cycles of different shape and area, thus causing a steady ("rectified") component.

(4) Odd harmonic distortion results in distorted waveform, but with the positive and negative half cycles similar in shape.

(5) Even harmonics are in phase with the positive fundamental peak, and out of phase with the negative peak, or *vice versa*; they are always maxima when the fundamental is zero.

(6) Odd harmonics are always exactly in phase or 180° out of phase with both positive and negative fundamental peaks, and are zero when the fundamental is zero.

The relative phases of the fundamental ($H_1$) and the harmonics (up to $H_5$) are shown in Fig. 6.45. The fundamental and third, fifth and higher order odd harmonics have zero amplitude at 0°, 180° and 360° on the fundamental scale. The second, fourth, and higher order even harmonics reach their maximum values (either positive or negative) at 0°, 90°, 180°, 270° and 360° on the fundamental scale.

FIG. 6.45