

# Distortion in Positive- and Negative-Feedback Filters\*

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It is known that the harmonic distortion of an active filter is greater than the distortion of the operational amplifier itself. Positive- and negative-feedback linear filters, namely, Butterworth, Chebyshev, Bessel, and band-pass filters, are analyzed. The distortion multiplication factor  $K_d$  is defined and plotted versus frequency. The maximum values of  $K_d$  found for several filter configurations by means of a computer program are given in a form useful for filter designers.

## 0 INTRODUCTION

The fact that the harmonic distortion in a positive-feedback (PF) active filter is much greater than the distortion in its operational amplifier is not new [1]. The closer the distortion is to the filter cutoff frequency, the larger is its increase. Distortion can be increased by up to two orders of magnitude. Consequently it has been suggested that the unpleasant auditive sensation caused by some filters is due to this fact, and not to the excessive phase rotation usually associated with high-slope filters.

This conclusion was applied to negative-feedback (NF) active filters in a previous publication [2], but the results of this work are not easily applicable to the improvement of filter design, since they involve the solving of complex equations.

Our investigation has a twofold goal. First we want to find simple equations which will enable the designer to quickly estimate the increase of distortion in each filter. Second we try to make the method as widely applicable as possible, so that it holds for filters not included in this paper as well as for filters to be developed in the future.

## 1 DISTORTION MULTIPLICATION FACTOR

The definition of a distortion multiplication factor will help us to estimate quickly the particular behavior of each filter. We will call it  $K_d$  and it will be expressed in decibels for ease of notation and calculus. Then

$$K_d(f) = 20 \log \frac{\text{distortion of active filter at frequency } f}{\text{distortion of operational amplifier with } g = 1} \quad (1)$$

It is evident that  $K_d(f)$  will depend on the frequency (see Fig. 13). In this work, as will be explained later, it is demonstrated that the behavior of  $K_d(f)$  is represented by a bell-shaped curve, which has its maximum value near the filter cutoff frequency  $f_3$ . This holds not only for low-pass or high-pass filters, but also for band-pass filters at the center frequency  $f_0$ .

Then we can concentrate only on the maximum value of  $K_d(f)$ . Thus we are transforming a problem of complex solution into a much simpler problem, which consists in determining the maximum distortion to be obtained with each filter configuration. We know beforehand that for all filters this value will be near  $f_3$  or  $f_0$ . Then

$$K_d = [K_d(f)]_{\max} \quad (2)$$

We have thus developed a method that can be applied easily to filter design. For example, we want to design a filter with a harmonic distortion lower than 0.1%. Looking at the charts of  $K_d$  for each configuration, we see that, for example,  $K_d = 25$  dB. Consequently the operational amplifier will have a distortion with unitary gain below  $-(60 + 25) = -85$  dB (0.0056%).

## 2 $K_d$ FACTOR IN POSITIVE- AND NEGATIVE-FEEDBACK FILTERS

The operational amplifier with which the filter is going to be built is discussed next. When the operational

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amplifier is connected for unitary gain and is driven by an ideal frequency generator  $V_i$ , it delivers to its

output

$$V_o = V_1 + V_2 + V_3 + \dots + V_n$$

where  $V_1$  is the fundamental frequency and  $V_2, V_3, \dots, V_n$  are harmonic distortion components.

The harmonic components will be multiplied by  $K_d$ , thus increasing the distortion measured at the filter output. The equivalent circuit of the operational amplifier with distortion shown in Fig. 1(a) will be used to analyze this problem. The distortion generators  $V_n$  are in series with an ideal distortionless amplifier. However, this circuit can be simplified even further if we take into account that the distortion values of an operational amplifier are generally low. Consequently the second-order products will be negligible, and it will not be necessary to analyze all the generators working at the same time. We will just have to replace  $V_2, V_3, \dots, V_n$  by a single distortion generator  $V_\omega$  whose frequency can be changed so that it replaces any of the harmonics [Fig. 1(b)]. From the viewpoint of circuit theory this implies assuming that the system is linear, and in fact it almost is, since the second-order nonlinearity is negligible. Fig. 2 shows the equivalent circuit of a positive-feedback active filter. This circuit is valid for any type of filter. The passive network  $N$  is defined by two transfer factors  $\alpha$  and  $\beta$ . Then for this circuit, and for  $V_i = 0$ ,

$$V_1 = g\beta(\omega)V_o$$

$$V_o = V_\omega + V_1 = V_\omega + g\beta(\omega)V_o$$

$$V_o = \frac{V_\omega}{1 - g\beta(\omega)}$$

Since

$$K_d(f) = 20 \log \frac{V_o}{V_\omega}$$

then for a positive-feedback filter,

$$K_d(f) = 20 \log \frac{1}{1 - g\beta(\omega)} \quad (3)$$

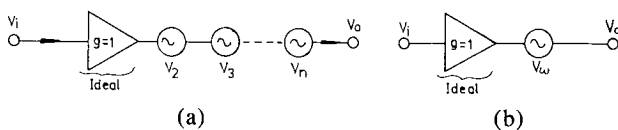


Fig. 1. Equivalent circuit of operational amplifier with distortion.

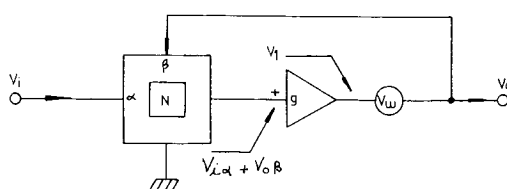


Fig. 2. Positive-feedback active filter.

Eq. (3) will enable us to calculate  $K_d$  according to the passive network transfer  $\beta(\omega)$ . A maximum value of  $K_d(f)$  will be obtained when  $\beta(\omega)$  is real and has a maximum (that is, when it is close to 1).

Let us now analyze the negative-feedback active filter (Fig. 3). Here generator  $V'_\omega$  represents the open-loop distortion of the operational amplifier. Then, for the circuit of Fig. 3,

$$V_1 = -A\beta(\omega)V_o$$

$$V_o = V_1 + V'_\omega = V'_\omega - A\beta(\omega)V_o$$

$$V_o = \frac{V'_\omega}{1 + A\beta(\omega)}$$

If we apply the definition of Eq. (1),

$$K_d(f) = 20 \log \frac{[V_o]_\omega^\beta}{[V_o]_\omega^{\beta=1}}$$

that is, we consider  $\beta = 1$  in order to obtain in the denominator the distortion corresponding to the closed-loop condition ( $g = 1$ ) given by  $V'_\omega/(1 + A)$ . From this viewpoint, and if we want to be accurate, we must say that  $V'_\omega$  is not the open-loop distortion, but the equivalent input distortion as defined by Baxandall [3]. Finally the quotient will be

$$K_d(f) = 20 \log \frac{V'_\omega/[1 + A\beta(\omega)]}{V'_\omega/(1 + A)} = 20 \log \frac{1 + A}{1 + A\beta(\omega)}$$

Taking into account that  $A \gg 1$ , we have, for negative-feedback filters,

$$K_d(f) = 20 \log \frac{1}{\beta(\omega)} \quad (4)$$

Eqs. (3) and (4) will enable us to calculate  $K_d$  for all active filters by just solving the passive network in order to obtain  $\beta$ . Eqs. (3) and (4) refer the filter distortion to the value that corresponds to the amplifier with unitary gain. However, in real operational amplifiers, distortion can have different values depending on which input is selected, positive or negative. This is due to the inherent distortion of the input differential pair, which is, respectively, included in or excluded from the feedback loop. This discrepancy with the the-

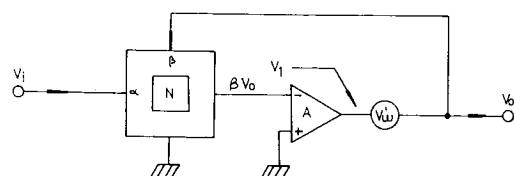


Fig. 3. Negative-feedback active filter.

oretical model can generally be neglected and does not alter noticeably the results obtained from Eqs. (3) and (4).

3  $K_d$  VALUES FOR ORDINARY FILTERS

Since an analytic solution of the passive network beta transfer is quite complex, we decided to use a computer program, the CNAP, supplied by Hewlett-Packard. Thus the charts for Figs. 4 and 6 were obtained. The program was also used to determine whether the maximum value of  $K_d(f)$  occurred exactly at the cutoff frequency. With this important information we went on to solve the network analytically, but just for the  $K_d$  frequency. Likewise we were able to obtain simple equations to calculate  $K_d$  values that will enable the designer to make do without charts or computer programs. In Figs. 4 and 6 the computed exact  $K_d$  values are also included.

Fig. 4 shows the popular third-order Chebyshev filter. For a filter with a 2-dB ripple the maximum  $K_d(f)$  value is 38.4 dB, which coincides with the value obtained by Billam [1] by means of experimental methods. It must be noted that this filter multiplies 83 times the distortion of the amplifier used.

Fig. 5 shows, as a calculation example, the five-node network used to solve the filter in Fig. 4. This network gives the  $\beta$  transfer in node 4. The generator placed between nodes 4 and 5 has the function of subtracting 1 V from the voltage in node 4. The output obtained is thus  $\beta(\omega) - 1$ . This output enables the computer to print the values of  $20 \log [\beta(\omega) - 1]$ , which will be the same as the  $K_d(f)$  value given by Eq. (3), except for the sign. As regards negative-feedback filters, it is not necessary to use the generator since the  $\beta$  transfer can be obtained directly.

In the charts of Figs. 6–9 the resistors and capacitors were standardized in accordance with the  $Q$  factor of the filter. This fact is very important, because the designer always knows beforehand the  $Q$  for which the filter must be designed. Inversely, if the filter has already been calculated by means of charts or computer programs, the designer will be able to find the  $Q$  of each section of the filter, and from it the  $K_d$  factor, since the former is dependent on the relationship of capacitors

or resistors. For this purpose Figs. 6–9 include, beside the charts, the equations that enable the designer to calculate the filters and their  $Q$ .

We shall now see the analytic expression that provides  $K_d$  as a function of  $Q$  for a low-pass filter with positive feedback (Fig. 9). By solving the network with a computer, we learned that the maximum value for  $K_d(f)$  occurs for a frequency identical to the cutoff frequency  $f_3$ . Thus we will calculate the  $\beta$  value for that frequency (designated  $\omega_0$  in Fig. 9).

From the low-pass circuit in Fig. 9 we can build the  $\beta$  network of Fig. 10. In order to solve the latter, we will take the following into account. Standardizing  $\omega_0 = 1$ , we have  $R\sqrt{C_1C_2} = 1$ . Then

$$C_2 = \frac{1}{R^2C_1}$$
 (5)

and

$$4Q^2 = \frac{C_2}{C_1}$$

Hence

$$C_2 = 4Q^2C_1$$
 (6)

Equating Eqs. (5) and (6),

$$2QRC_1 = 1$$
 (7)

Solving  $1/C_1$  and  $1/C_2$  from Eqs. (6) and (7),

$$\frac{1}{C_1} = 2QR, \quad \frac{1}{C_2} = \frac{R}{2Q}$$
 (8)

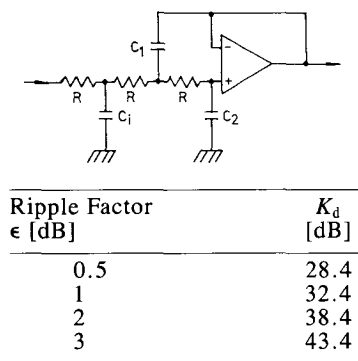


Fig. 4. Third-order positive-feedback Chebyshev filter.

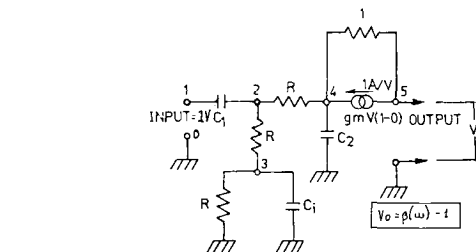
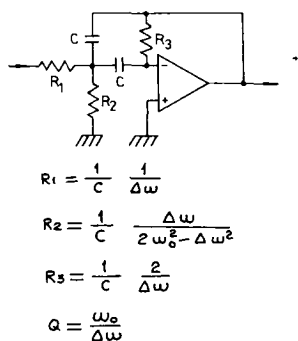


Fig. 5. Network of third-order Chebyshev filter, used for CNAP computer program.



Exact computed values

Q	K <sub>d</sub> [dB]
1	9.5
3	25.6
6	37.3
10	46.1
20	58.1

Exact expression:  
 $K_d = 20 \log(1 + 2Q^2)$

Fig. 6. Negative-feedback band-pass filter.

We now calculate  $V_1$  (Fig. 10):

$$V_1 = \frac{RZ_{RC1}/(R + Z_{RC1})}{X_{C2} + RZ_{RC1}/(R + Z_{RC1})}$$

But  $Z_{RC1} = R - j/C_1$  because  $\omega_0 = 1$ , and

$$V_1 = \frac{R(R - j/C_1)}{-j/C_2(R + R - j/C_1) + R(R - j/C_1)}$$

Replacing by Eq. (8) and solving,

$$V_1 = \frac{Q - j2Q^2}{-j(2Q^2 + 1)} \quad (9)$$

and

$$\begin{aligned} \beta(\omega_0) &= V_1 \frac{X_{C1}}{R + X_{C1}} = V_1 \frac{-j/C_1}{R - j/C_1} \\ &= V_1 \frac{-j2Q}{1 - j2Q} \end{aligned}$$

Replacing  $V_1$  by its value from Eq. (9) and solving,

$$\beta(\omega_0) = \frac{2Q^2}{2Q^2 + 1}$$

Then

$$\begin{aligned} K_d &= 20 \log \frac{1}{1 - \beta(\omega_0)} \\ &= 20 \log \frac{1}{1 - 2Q^2/(2Q^2 + 1)} \end{aligned}$$

and hence,

$$K_d = 20 \log(2Q^2 + 1) \quad (10)$$

Eq. (10) allows us to calculate directly the value of  $K_d$  as a function of  $Q$  for the low-pass and high-pass filters in Fig. 9.

In our second example we analyze a negative-feed-

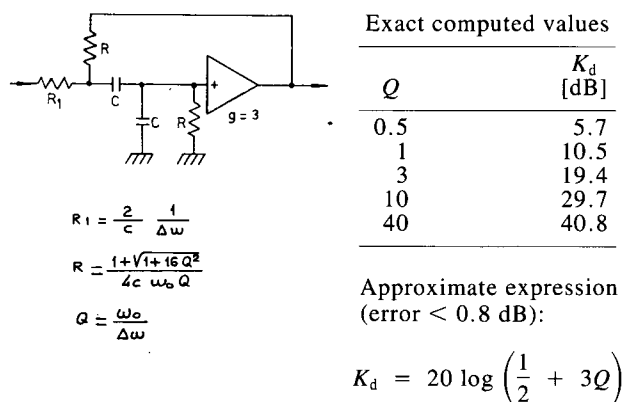


Fig. 7. Positive-feedback band-pass filter.

back low-pass filter. The filter in Fig. 8 has its maximum value of  $K_d(f)$  very close to  $f_3$ , but for lower values of  $Q$ ,  $K_d(f)$  is slightly displaced. For higher values of  $Q$  it is almost coincident with  $f_3$ . As the previously mentioned drift is very small, we use an approximate expression of  $K_d$  for design purposes only. Thus we

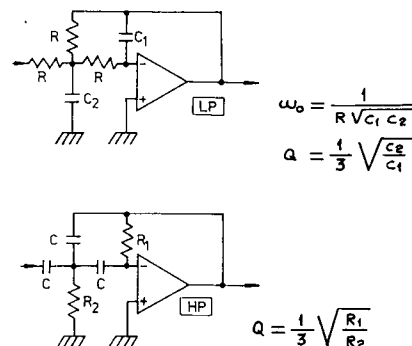


Fig. 8. Negative-feedback low-pass and high-pass filters.

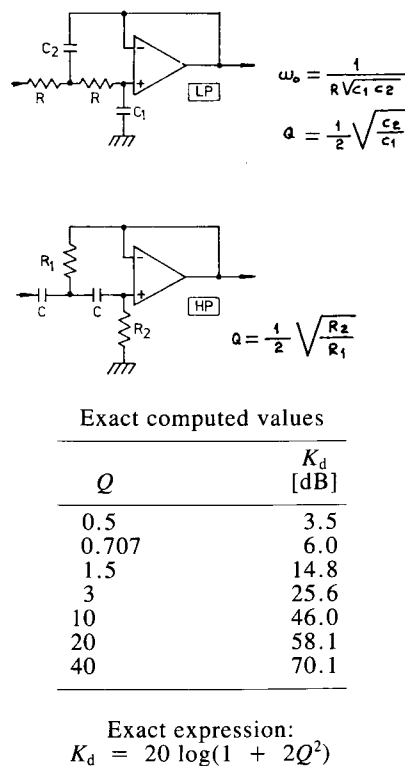


Fig. 9. Positive-feedback low-pass and high-pass filters.

consider that the  $\omega_0$  value of Fig. 8 was the one that gives us the maximum value of  $K_d(f)$ . Fig. 11 shows the  $\beta$  network to be analyzed. Similarly to our previous example, we find

$$\frac{1}{C_1} = 3QR, \quad \frac{1}{C_2} = \frac{R}{3Q} \quad (11)$$

We calculate  $Z_A$  and  $Z_B$  and substitute Eq. (11),

$$Z_A = R \frac{1 - j3Q}{2 - j3Q}, \quad Z_B = R \frac{1}{1 + j3Q}$$

So

$$V_1 = \frac{Z_B}{Z_A + Z_B}$$

Replacing and solving,

$$V_1 = \frac{2 - j3Q}{3 + 9Q^2 - j3Q}$$

Also,

$$\beta(\omega_0) = V_1 + V_R$$

and

$$V_R = (1 - V_1) \frac{R}{R - j/C_1}$$

Replacing and solving further,

$$\beta(\omega_0) = \frac{3}{3 + 9Q^2 - j3Q}$$

Then

$$K_d = 20 \log \left| \frac{1}{\beta(\omega_0)} \right|$$

$$= 20 \log [(1 + 3Q^2)^2 + Q^2]^{1/2}$$

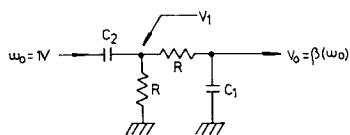


Fig. 10.  $\beta$  network for low-pass filter with positive feedback.

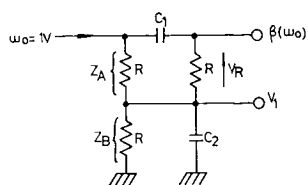


Fig. 11.  $\beta$  network for low-pass filter with negative feedback.

and hence,

$$K_d \approx 20 \log (1 + 3Q^2) \quad (12)$$

The value given by Eq. (12) is only approximate. However, if we compare it with the exact values of the chart in Fig. 8, we see that its maximum error is below 0.3 dB. Consequently it can be considered accurate for practical purposes. It is possible to find analytic expressions for the remaining types of active filters following the above procedures.

In Figs. 6–9 the designer will find the exact or an approximate expression that will permit calculating  $K_d$  directly. Note that among the positive- and negative-feedback realizations of low-pass or high-pass filters (Figs. 8 and 9) the differences in  $K_d$  values for the same  $Q$  are in fact small. Conversely, in second-order pass-band filters the differences are very important, since the  $K_d$  value for the positive-feedback configuration is much lower than that for the negative-feedback configuration. It is obvious that we have not dealt with all the types of active filters. However, the method we have described holds for any kind of filter, either by the solution of the  $\beta$  network by computer, or by analytical calculus for frequencies  $f_3$  or  $f_0$ .

#### 4 HOW TO CALCULATE $Q_{\max}$ FOR BUTTERWORTH, CHEBYSHEV, AND BESSEL FILTERS

When we design a filter of higher than second order, we build it with two or more second-order sections. Each of these sections will have its own distortion due to the  $Q$  with which it operates.

As the distortion increases by  $K_d$ , and since  $K_d$  is proportional to  $Q^2$ , the overall filter distortion will be almost the same as the distortion in the section of higher  $Q$ . This is why the designer should find the  $Q$  that corresponds to each pair of conjugated complex poles. The maximum value (corresponding to a specific filter section) will then give the overall distortion. The value of  $Q$  for a pair of poles  $\alpha \pm j\beta$  is given by

$$Q = \frac{\sqrt{\alpha^2 + \beta^2}}{2\alpha} \quad (13)$$

As regards the Butterworth polynomials, this value can be calculated easily, since the poles are given by [4]

$$S_{2V+1} = -\sin \frac{(2V+1)\pi}{2n} + j \cos \frac{(2V+1)\pi}{2n}$$

where  $V = 0, 1, 2, \dots, n-1$ ,  $n$  being the filter order.

If we now apply Eq. (13),

$$Q = \frac{1}{2 \sin(2V+1)\pi/2n}$$

For  $V = 0$  we have

$$Q_{\max} = \frac{1}{2 \sin(\pi/2n)} \quad (14)$$

Then we can find the  $Q_{\max}$  for an  $n$ th order Butterworth filter by means of Eq. (14). For a simple, second-order filter,  $Q = 0.707$ , and for  $n = 10$ ,  $Q_{\max} = 3.196$ . If we use a positive-feedback configuration similar to the one in Fig. 9, then  $K_d = 6$  dB in the first example and 26.6 dB in the second. This implies that if the same distortion is desired in both filters, then in the second example ( $n = 10$ ) it is necessary to have an amplifier with ten times less distortion than in the first example.

For Chebyshev filters the analytic expression is more complex. So it will be convenient to obtain the root values from a polynomial chart [4]. In order to have a comparative reference, with a ripple of 1 dB and  $n = 4$ , we have  $Q_{\max} = 3.56$ . On the other end, with a ripple of 3 dB and  $n = 10$ ,  $Q_{\max} = 35.85$ . In this case Eq. (13) is used to calculate the  $Q$  values.

Bessel filters can also be calculated according to the roots found in the charts, but the  $Q$  values are so low that they do not cause any noticeable increase in distortion. For instance, for  $n = 10$ , we have  $Q_{\max} = 1.42$  in a Bessel filter.

Eq. (13) will be useful to calculate any other type of filter, provided the roots of the approximate polynomial are known.

## 5 EXPERIMENTAL MEASUREMENT OF $K_d(f)$

The following method can be used if we want to design a new kind of active filter with  $K_d$  unknown. Instead of calculating the  $K_d$  value, it is sometimes faster to measure it directly. For this purpose the simple circuit layouts in Fig. 12 can be used. For positive feedback Fig. 12(a) is used, based on the analysis circuit in Fig. 2. This method allows the designer to find  $K_d(f)$  by selecting  $V_1 = 10$  mV. It is also possible to plot the  $K_d(f)$  curve by means of a standard frequency-response plotter (on paper or cathode-ray tube). Fig. 13 shows the  $K_d(f)$  curve made by such a plotter, connected as shown in Fig. 12(a).

For negative-feedback filters the configuration shown in Fig. 12(b) is used. This circuit enables the designer to obtain the value of  $1/\beta(\omega)$  directly. Here it is also

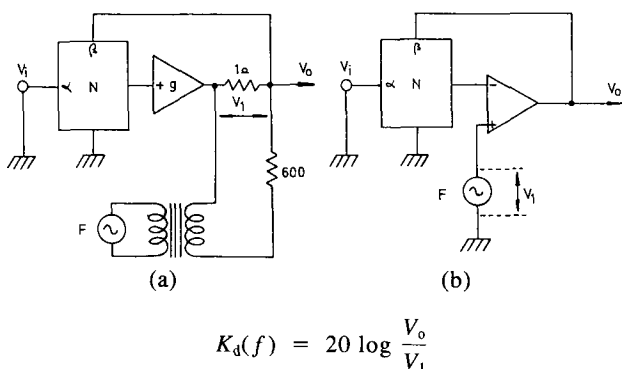


Fig. 12. Analog experimental method used to plot  $K_d(f)$ . (a) Positive feedback. (b) Negative feedback.

possible to plot the output. Fig. 14 shows the superimposed curves of  $K_d(f)$  for the same filter in the positive- and negative-feedback configurations.

## 6 CONCLUSION

The concept of the distortion multiplication factor has been analyzed, and a simple definition is given of its wide application. We have derived equations that enable us to calculate  $K_d$  as a function of  $Q$ , either accurately or with sufficient approximation. We have demonstrated how to calculate  $Q_{\max}$  so as to build a filter. The last two concepts enable us to predict the distortion that will be obtained from any type of filter with the most common circuit configurations. Finally we have introduced three ways of finding the  $K_d$  value of an active filter. The first method is based on the resolution of a network, by means of a computer; the second is analytical, using Eqs. (3) and (4); and the third is experimental, based on a very simple measurement (Fig. 12).

## 7 REFERENCES

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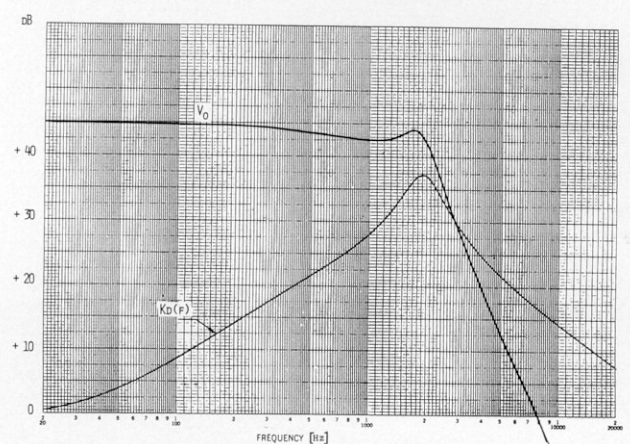


Fig. 13. Plot of  $K_d(f)$  for third-order Chebyshev filter.  $F_C = 2$  kHz;  $\epsilon = 2$  dB.

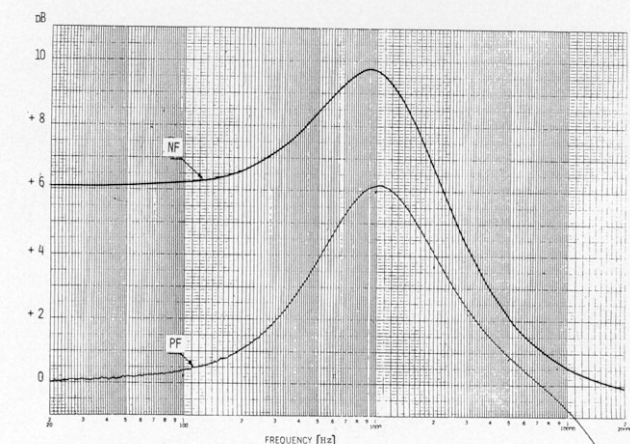


Fig. 14. Plot of  $K_d(f)$  for second-order positive- and negative-feedback Butterworth filters.  $F_C = 1$  kHz.

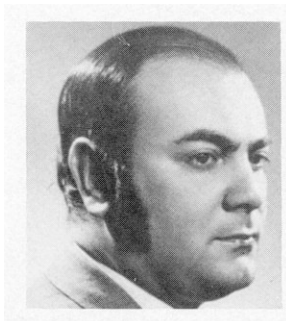
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