FFT Test of A/D Converters to Determine the Integral Nonlinearity

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***Abstract*—In this paper, the use of the fast Fourier transform (FFT) test to measure the integral nonlinearity (INL) of analog-to-digital (A/D) converters is examined. The derived INL is a linear combination of Chebyshev polynomials, where the coefficients are the spurious harmonics of the output spectrum. The accuracy of the test is examined theoretically, in simulations and in practical devices, particularly for the critical (and typical) case when sudden jumps are present in the actual INL. The examined methodology appears to be very convenient when the device under test has high resolution (16–20 bits) and a smoothed approximation of the INL is sufficient, as the FFT test is in this case thousands of times faster than the customary histogram test and static nonlinearity test.**

***Index Terms*—Analog-to-digital (A/D) conversion, Chebyshev functions, discrete Fourier transform.**

I. Introduction

The static transfer function of an analog-to-digital converter (ADC) and the associated static error [integral nonlinearity (INL)] are metrological quantities of primary interest when designing or assessing an appliance including such a device. A well-known test method, for this purpose, is the code density or histogram test [1]. The main advantage of this test is its "brute force" nature, which implies the possibility of a very high accuracy: by acquiring a sufficient number of samples, it can measure the ADC static characteristic with (in principle) an uncertainty as small as desired, regardless of the particular shape of the INL. The price to be paid for the accuracy is in terms of *time*, as the necessary number of samples for a given accuracy increases *exponentially* with the number of bits. This drawback makes the histogram test unfeasible with low-speed, high-resolution ADCs (16 bits or more). For this kind of device, it is usually convenient to use the fast Fourier transform (FFT) test, that can be satisfactorily performed using as few as eight thousand samples, regardless of the ADC resolution. One drawback of this test is, on the other hand, that it yields a frequency-domain description of the integral nonlinearity, which cannot be directly employed for assessing the uncertainty of a static measurement, or for linearizing the static characteristic.

Based on these considerations, it comes to attention the idea of deriving the ADC integral nonlinearity from the outcome of the FFT test, as this is in principle possible by exploiting a known theoretical property of the Chebyshev polynomials. There are a few works concerning this issue (see e.g., [2]–[4]), showing that the FFT test can also reconstruct with a very high accuracy the INL, but only when it is a sufficiently smooth curve. There is, therefore, still much to investigate, particularly about the test accuracy in real cases, where usually the INL is *not* a smooth curve and, on the contrary, contains a number of discontinuities. The present work, therefore, developing previous results [4], presents i) a summary of the known theory connecting Chebyshev polynomials and static nonlinearity, ii) some theoretical clarifications related to the specific problem of the static nonlinearity in ADCs, iii) simulation results taking into consideration the problem of discontinuities in the INL, and, finally, iv) experimental results on actual converters with different characteristics.

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II. Known Theory About Static Nonlinearity and Chebyshev Polynomials

A nonlinear static characteristic $y = g(x)$, when stimulated by a sinusoidal input in the form $x(t) = V\cos\omega t + C$, gives a periodic output in the form

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t). \tag{1}$$

A simple relationship exists between the coefficients a_n and the function $g(x)$, if this is a polynomial function. We must use the well-known Chebyshev polynomials of the first kind $C_n(z)$ that, besides meeting the identity $C_n(\cos\theta) = \cos(n\theta)$, are orthogonal with respect to the weighting function $\sqrt{1-x^2}$ [that is, proportional to the reciprocal of the density of the terms $a_n\cos(n\omega t)$]. The orthogonality makes it possible to expand a generic $f(x)$ in the series

$$f(x) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n C_n(x) \tag{2}$$

where the coefficients c_n are given by

$$c_n = \frac{2}{\pi}\int_{-1}^{1} \frac{f(x)C_n(x)}{\sqrt{1-x^2}} dx. \tag{3}$$

Now, it is simple to verify that by substituting $\cos(\omega t) = z$ in (1), after simple algebraic manipulations, we obtain

$$a_n = \frac{2}{\pi}\int_{-1}^{1} \frac{g(Vz+C)C_n(z)}{\sqrt{1-z^2}} dz. \tag{4}$$

It is, therefore, obvious that an expansion of $g(x)$ in the sum of Chebyshev polynomials can be obtained

$$g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n C_n\left(\frac{x-C}{V}\right). \tag{5}$$

This formula (that can be easily extended to the case of input with nonzero phase $x(t) = V \cos(\omega t + \varphi) + C$ allows one, in principle, to measure exactly a polynomial nonlinearity via a simple FFT test. The trouble is that, of course, the static transfer function of an ADC is not always a polynomial. When $g(x)$ is not a polynomial, (5) is the best approximation of the nonlinear function in the sense that, among all the polynomials of the same degree, it minimizes a weighted sum-squared-error. However, this is not very helpful when one wants to know *how much* the error will be in a practical case, in addition to the fact that one is usually interested in quantities different from the sum squared error, e.g., the maximum absolute error. Therefore, in order to use confidently the Chebyshev polynomials theory in ADC testing, further analysis is certainly needed, starting from some theoretical clarifications about nonlinearity in ADCs.

III. ADC-Related Theoretical Issues

The nonlinear characteristic $g(x)$ of a simple ideal ADC has a very large number of small and equispaced discontinuity points. A satisfactory polynomial approximation of such a nonlinearity would require, of course, an impractically large number of terms. In short, (5) is not directly very useful in *quantization theory*: this is perhaps the reason why it has been seldom tested in the field of ADC testing.

It must be considered, however, that determining the INL of an ADC does not require finding a polynomial approximation of its overall transfer characteristic $g(x)$, which is irreducibly discontinuous: the polynomial shall approximate, instead, a considerably smoother function. This can be made clear by considering the simple nonlinear ADC model of Fig. 1, in which the overall $g(x)$ is seen as the cascade of a nonlinear function $g_s(x)$ and the ideal quantization function $quant(x)$. All the functions $g_s(x)$ such that $|g_s(x) - g(x)| \leq 0.5$ LSB can be equivalently employed in the model; for example, $g_s(x)$ can be a linear piecewise function that connects the points (t_k^{id}, t_k), with t_k the actual and t_k^{id} the ideal threshold levels. Now, the following are easily seen as follows.

- Measuring the INL is equivalent [1] to measuring the function $g_s(x)$, as the actual threshold levels are given by $t_k = g_s^{-1}(t_k^{id})$, where are known *a priori* from the ADC output levels.
- It is reasonable to suppose, at least as a first-order approximation, that a *polynomial* $g_s(x)$ exists such that $quant(g_s(x)) = g(x)$ [this is also justified considering the freedom allowed in choosing $g_s(x)$].

The above statements are illustrated by Fig. 2, that shows the overall nonlinearity $g(x)$ of a simulated ADC and a function $g_s(x)$ such that $quant(g_s(x)) = g(x)$. However, in order to derive the smooth $g_s(x)$ from the output spectrum produced by the whole characteristic $g_x = quant(g_s(x))$, it is necessary that the effect of the $quant(x)$ function on the spectrum be negligible. According to well-known theory, quantization adds to a single-tone signal a very large number of small harmonics, whose power sums up to about $Q^2/12$ [5], [6]. Therefore, the effect of $quant(x)$ is negligible if the ADC resolution is conveniently high and if the nonlinearity $g_s(x)$ can really be approximated by a polynomial of reasonable order. Of course, the FFT test must be performed employing a coherently sampled sine wave, so that the harmonics produced by quantization spread uniformly throughout all the DFT bins. In this case, the underlying polynomial $g_s(x)$ produces a moderate number of large harmonics, while quantization produces a "noise floor" made of a large number of uniformly distributed small harmonics.

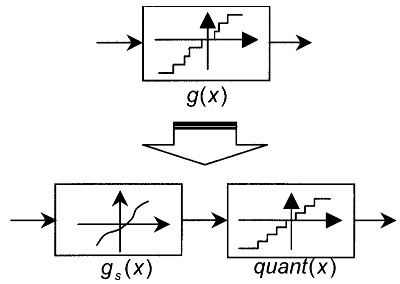

Fig. 1. Decomposition of the discontinuous characteristic $g(x)$ of a nonlinear ADC. The integral nonlinearity is actually given by the smoother nonlinear function $g_s(x)$.

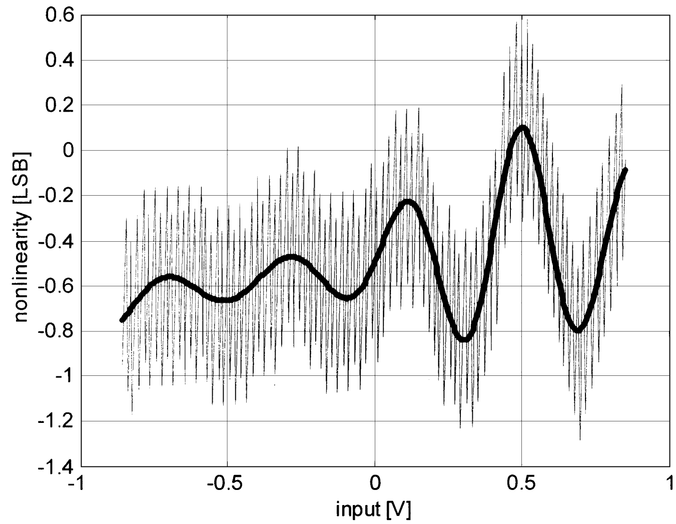

Fig. 2. Comparison between $g_(s)(x) - x$ (thick, smooth line) and the overall nonlinearity $g(x) - x$ (erratic, thin line) in a simulated ADC.

IV. Description of the Algorithm to Derive the INL From the FFT Test

The algorithm for the fast measurement via FFT test of the INL of an ADC [fully represented by $g_s(x)$] can be outlined in three steps.

As a first step, apply a sinusoidal signal with the usual characteristic prescribed by the IEEE Standard [1] for the FFT test, that is

1) the peak-to-peak amplitude must be as close as possible to, but without exceeding, the ends of the full-scale range,

[1]We assume in this paper the standard definition [1] of integral nonlinearity: $inl_k = t_k^{id} - Gt_k - O$, with G and O two constants chosen according to a sensible criterion (e.g., minimize $\max |inl_k|$ or $\sum inl_k^2$).

so that the ADC is fully stimulated without introducing saturation[2];

2) noise and spurious harmonics must have small enough power (below the ideal quantization noise level of the ADC under test).

The parameters of the sampling (signal frequency, sampling frequency and duration of the sampling) must be chosen, as usual, so that the output $y(kT_s)$, $k = 0 : N-1$ has an integer number of periods N_p, prime relative to the number N of samples (*coherent sampling*). The output sine wave $y(kT_s)$ should also have approximately null phase. It is also important that the signal frequency is not too high, compared with the aperture time and the overall dynamic performance of the ADC under test: the test is indeed aimed at measuring the *static* characteristic, avoiding the onset of dynamic effects like hysteresis.

As a second step, evaluate the FFT

$$Y(i) = \sum_{k=0}^{N-1} y(kT_s)e^{-j2\pi ik/N} \quad i = 0 : N-1 \tag{6}$$

and find the indexes $i_n = (nN_p \bmod N)$ with $n = 0, 1, 2, \ldots$, then evaluate the coefficients

$$a_0 = \frac{Y(i_0)}{N}; \tag{7}$$

$$a_n = \frac{2Y(i_n)}{N} \quad n = 1, \ldots N_h. \tag{8}$$

The maximum index N_h of n can be chosen so that all the harmonics $Y(i_n)$ above the noise floor are included. This procedure yields *exactly* the coefficients of the Fourier series expansion (1) of $y(t)$ if the signal *is coherently sampled, is a series of cosines* and *is made of* $N_h < N/2$ *harmonics*. As a third and last step, find the $N_h{}^\circ$ polynomial that approximates $g_s(x)$ using (5).

It must be highlighted that the zero-phase requirement on $y(kT_s)$ is not essential in practice: a possible nonzero phase can be easily nullified with a simple digital "rotation" on the real axis of the complex harmonics given by the DFT formula (6).

A practical problematic issue is that some out-of-phase harmonics (i.e., harmonics with nonzero sine terms, even if the input signal and the fundamental are perfect cosines) can occur in an actual test. While small deviations from the ideal case (when all the harmonics are perfectly in phase) can be ascribed to system noise, the presence of meaningful out-of-phase components is a clear indication that the time-invariant no-memory model of the ADC under test cannot be considered valid. In such a situation, one should lower the sinusoidal signal frequency (and, possibly, the sampling frequency) in order to measure only *the static portion* of the dynamic nonlinear characteristic of the device, which is the goal of the test.

V. Simulations

In order to provide some evidence of the performance of the described test method, it is mandatory first to verify it via computer simulations, which allow choosing freely the static nonlinearity of the device under test. It must be expected, indeed, that the test performance depends heavily on the nature of the function $g(x)$ or, more precisely, on the degree of the smoothest possible polynomial $g_s(x)$ such that $quant(g_s(x)) = g(x)$.

Because of this primary observation, the test has been tried on simulated ADCs with three different $g_s(x)$: the first without discontinuities, the second with three large discontinuities (about 1 LSB), and the third with five large discontinuities. Each $g_s(x)$ has been cascaded, in order to produce the overall nonlinearity, with an ideal quantization function of 8, 12, 16, and 20 bits, so covering a wide range of resolutions. It must be noted that the ratio of the INL and the quantization step has been kept the same at all resolutions; in other words, in the simulations the maximum INL was always about 1 LSB, i.e., the magnitude that can be actually observed in real-world converters.

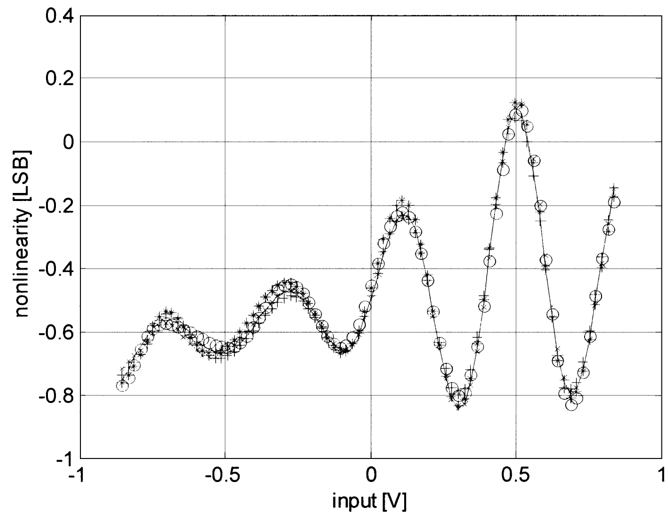

Fig. 3. FFT-test results for the first simulated ADC, reporting the true INL (– – –) and the estimated one for the 8-bit (ooo), 12-bit (xxx), 16-bit (+ + +), and 20-bit ($* * *$) case.

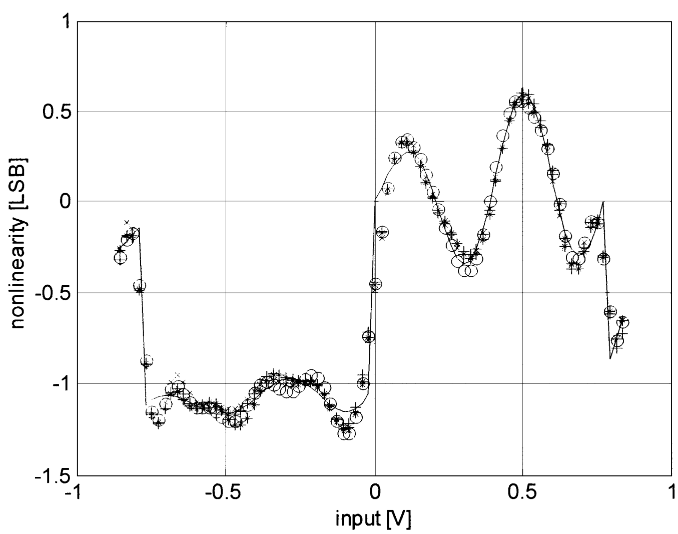

Fig. 4. FFT-test results for the second simulated ADC, reporting the true INL (– – –) and the estimated one for the 8-bit (ooo), 12-bit (xxx), 16-bit (+ + +), and 20-bit ($* * *$) case.

[2]Using a slightly saturating sine wave does not impair the test results much, in principle, regarding the reconstructed static characteristic (which in this case will include, of course, the inherent saturation of the converter). Avoiding saturation, however, allows one to use the same FFT test to also obtain correct measurements of other standard figures of merit (e.g., spurious-free dynamic range, total harmonic distortion, effective bits, etc.)

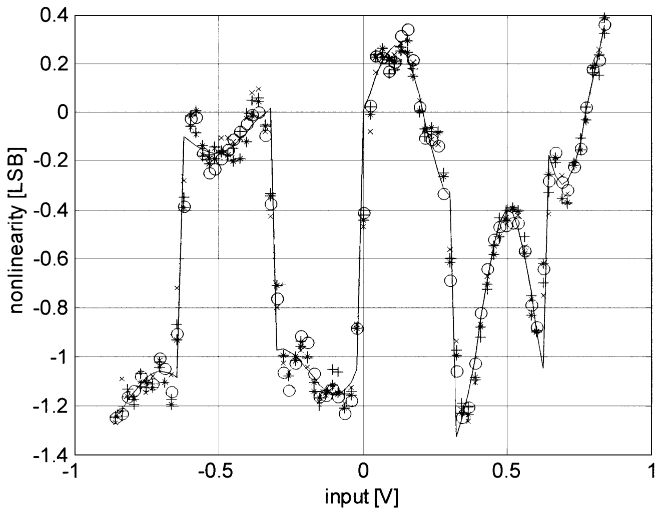

Fig. 5. FFT-test results for the third simulated ADC, reporting the true INL $(- - -)$ and the estimated one for the 8-bit (ooo), 12-bit (xxx), 16-bit $(+ + +)$, and 20-bit $(* * *)$ case.

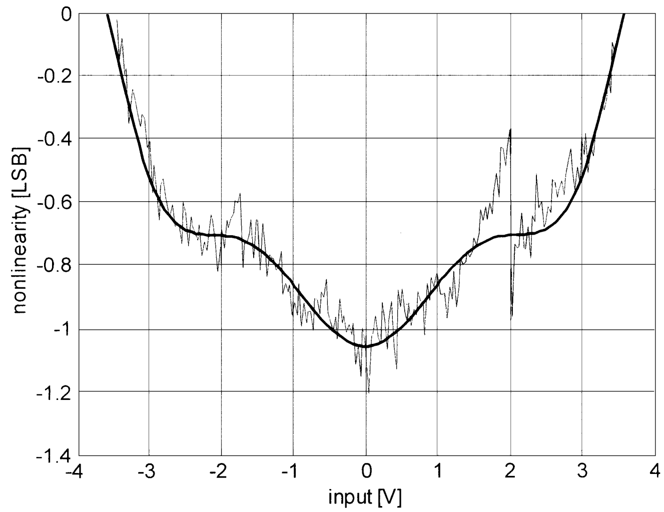

Fig. 6. Comparison between the results of the 8,192-point FFT-test (thick, smooth line) and results of the histogram test (thin, erratic line) for an actual 8-bit ADC.

Figs. 3, 4, and 5 show the results of the simulated tests, comparing the true $g_s(x)$ $(- - -)$ with the estimation made via the 8192-point FFT test, in the case of 8-bit (ooo), 12-bit (xxx), 16-bit $(+ + +)$, and 20-bit $(* * *)$ ADC. It is obviously convenient to use an integer power of two number of samples to optimize the computational performance of the FFT algorithm. In all the simulations, only the harmonics above the noise floor level have been considered.

In the first case, the test resulted in an 18th-degree polynomial. It is evident that the obtained estimate is very accurate: the maximum error in measuring $g_s(x)$ is indeed about 0.05 LSB. This strikingly good performance is clearly due to the smoothness of $g_s(x)$. In order to really understand the advantage of the FFT method for this case, one should think of how many samples would be needed for the same accuracy, if a standard histogram test were employed to test a 20-bit ADC.

The simulations relevant to the second and the third case show that the test is more critical when dealing with discontinuities in the static characteristic. Of course, a higher number of terms are needed to approximate satisfactorily the INL: the method yields a 27th-degree polynomial in the second case with three large discontinuities and a 33rd-degree polynomial in the third case with five large discontinuities. More important, from these simulations comes evidence that larger measurement errors are present near the discontinuities. This is a clear consequence of the representation of $g_s(x)$ as a truncated series of orthogonal functions: as expected from theory, the reconstructed function passes by the midpoints of the discontinuities of the true INL and near these jumps a clear "ripple" can be observed. The ripple, in particular, is more apparent in the last case, when the jumps are closer each other. It can be seen, however, that the maximum measurement error in the continuous pieces of the INL is about 0.1–0.15 LSB, which can be considered acceptable, especially if one considers that the measurement has been achieved with very few samples compared to the ADC resolution.

Summing up, the simulation results indicate that the FFT test with about 8,000 samples is able to reconstruct very accurately the INL of an ADC if this function is smooth and without discontinuities, while small errors occur near the jumps of the INL if this is a discontinuous function. It is intuitive that the method cannot be used to detect very narrow pulses or oscillations in the INL function, but the cases reported here show that continuous portions of reasonable size in the INL are well approximated by the polynomial derived via the FFT test.

In real-world ADCs, however, the INL can have very different and strange shapes and it is sensible to ask for a demonstration of the test in some practical cases. In the next section, experimental verifications of the method are therefore reported.

VI. Experimental Results

Fig. 6 presents the results relevant to the first experimental test, performed on an actual 8-bit ADC. The "true" nonlinearity was derived performing a histogram test with a highly pure sine wave (the same subsequently utilized for the FFT test) and with some thousands of samples per each code bin. The figure shows clearly that the true $g_s(x)$ presents many fast variations, one of which is particularly large ($\approx$ 0.5 LSB at the input level of about 2 V). The FFT test yields an 8th-degree polynomial approximation of this curve (eight is the maximum harmonic order above the threshold), which is clearly very close to the true and passes at the middle points of its discontinuities.

Fig. 7 shows the results relevant to the second test, performed on an actual 12-bit ADC. Also in this case the accurate $g_s(x)$, yielded by the histogram test, has many sudden variations, too fast for a polynomial approximation (some of them are larger than 0.5 LSB). The 19-degree polynomial yielded by the FFT test with only 8,192 samples is, however, a very good smooth approximation of the true $g_s(x)$ and again it passes at the middle points of the jumps.

The results obtained on actual converters, therefore, confirm that the FFT test should not be considered at all reliable for obtaining an estimate of the *maximum static* INL, as this test is practically "blind" to very fast variations in the nonlinearity.

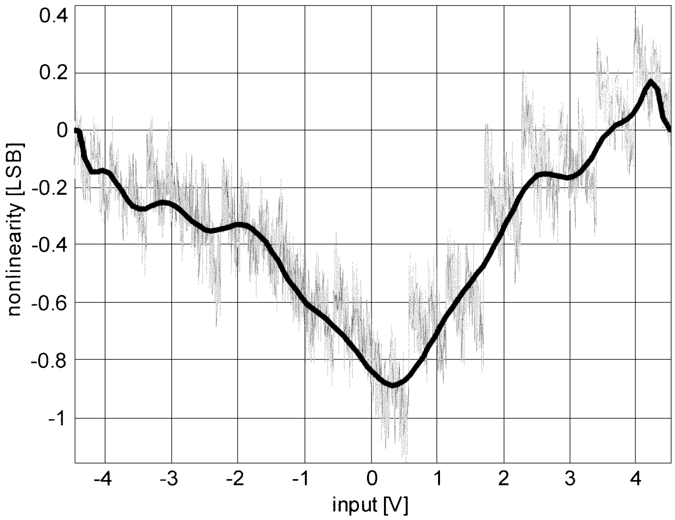

Fig. 7. Comparison between the results of the 8,192-point FFT-test (thick, smooth line) and results of the histogram test (thin, erratic line) for an actual 12-bit ADC.

The test is instead very efficient in determining the smooth part of the INL, which is responsible for the harmonic distortion (the "noisy" part of the INL contributes mainly to the quantization noise floor).

VII. Conclusion

From the illustrated theory, simulations and experiments it is clear that the FFT test is incomparably faster than the histogram test in measuring the integral nonlinearity—especially for high-resolution ADCs—but it has a drawback that must always be kept in mind, i.e., it gives only the *best polynomial approximation of a given degree* to the INL. Since in actual converters the INL is usually very erratic on a microscopic scale (i.e., it is in the strict sense a very-high-order nonlinearity), the results yielded by the FFT should never be used to assess, for example, the maximum static INL usually reported in ADC data-sheets and in instrument specifications. The strong point of the test is that a few thousands of samples (8,192 in the presented results) are sufficient to measure the characteristic with small errors, *regardless of the ADC resolution*. It is easy to check experimentally [4] that four to eight thousand samples are sufficient if the ADC under test is not too noisy: more samples do not improve meaningfully the test accuracy, while fewer samples make the results less repeatable.

The FFT test for measuring the INL can be, nonetheless, very useful for a number of different uses. First, it can be very accurate when the ADC has a smooth nonlinear characteristic. This circumstance is especially likely to occur when resolution-enhancement techniques like *dithering* are used, because these techniques usually "smear" the static characteristic. Second, the measured polynomial approximation of the INL can be useful for many different uses, for example to assess different (but not less important) figures of merit like the *mean-squared* INL (which is related to the *distortion power* introduced by the ADC). Even more interesting is the possibility of using the FFT-derived polynomial nonlinearity to implement a fast and computationally inexpensive *linearization algorithm*, in order to minimize the distortion and maximize the spurious-free dynamic range. Further investigations are needed to develop these issues and, possibly, make the FFT analysis more useful in the field of ADC and instrumentation testing.

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