

where  $A$  is the current gain of the amplifier and  $\beta$  the feedback ratio (alternatively  $A$  can be the transfer impedance of the amplifier and  $\beta$  the feedback admittance—the product  $A\beta$  still being dimensionless). The result corresponding to (28) is

$$i_1 = (i_s + i_{NS} + i_{NI})R_S/(R_S + Z_i)(1 - A'\beta) \quad \dots\dots(30)$$

where  $A' = AR_S/(R_S + Z_i)$

Now equations (28) and (30) will give us the *closed-loop* signal/noise ratio, that is the signal/noise ratio as modified by the action of the feedback. It is most convenient to compare this with the *open-loop* signal/noise ratio, that is the signal/noise ratio that would be obtained if the feedback parameter  $\beta$  were assumed to be zero. We see from equations (28) and (30) that whether the feedback is of the series or the parallel type, its action is to reduce the effect of the signal, the source noise, and the amplifier noise all in the ratio  $(1 - A'\beta)$ . It is important to remember that this is a complex function of frequency.

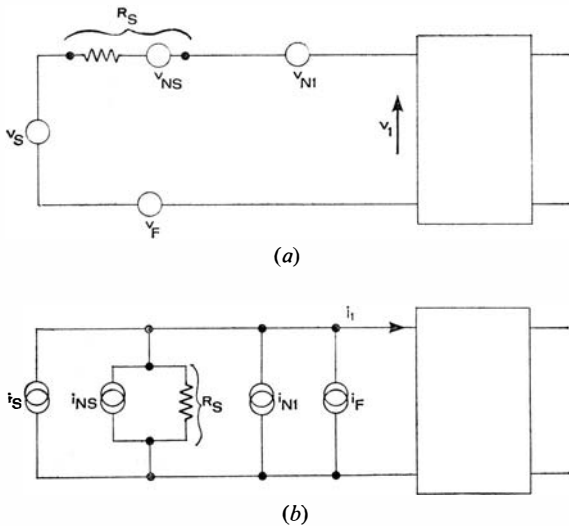


Fig. 7. Equivalent circuit of (a) series-feedback amplifier, (b) parallel-feedback amplifier. The 'black box' represents a noiseless amplifier.

It follows that if we are considering only the noise in a narrow bandwidth around the signal frequency then the closed-loop noise figure is the same as the open-loop noise figure. However, if we are considering the noise over a relatively broad band, the ratio of the closed-loop and the open-loop noise figures depends on the frequency dependence of the factor  $(1 - A'\beta)$ . The form of this frequency dependence also determines the relationship between the closed-loop bandwidth and the open-loop bandwidth, and the effect on the noise figure may be conveniently expressible in these terms; for instance, if (as is frequently the case) the closed-loop bandwidth is

greater than the open-loop bandwidth, then the overall closed-loop noise figure is greater than the overall open-loop noise figure.

We may simplify the discussion by supposing that under closed-loop conditions the amplifier is followed by a filter which 'tailors' the frequency response to be the same as the open-loop response. In this case the closed-loop noise figure is the same as the open-loop noise figure. We may summarize this result by the following statement: *after any changes in the frequency response have been allowed for, the closed-loop noise figure is equal to the open-loop noise figure.*

From a practical point of view, the situations described by Figs. 8 are idealized. In fact, the feedback network will have a finite impedance which appears in series (Fig. 7(a)) or in parallel (Fig. 7(b)) with the input circuit; also it will generate some noise which adds to the equivalent amplifier noise generator. It is the job of the circuit designer to make these effects negligible.

#### 4.2. Practical Application of the Theory

It follows from the result derived in the previous section that when designing a low-noise feedback amplifier, one must design the circuit so that it has the required noise figure before the feedback loop has been closed. Suppose, for example, that we wish to build an amplifier to work from a source resistance of 100 k $\Omega$  and to have an input resistance which is very high compared with this value. By the use of series feedback, the required input resistance can easily be achieved whatever the operating current of the input transistor. In order to decide on the correct value of this current, we must consider what the situation would be in the absence of the feedback. Now as we have seen in Section 3.2, a good noise figure is not obtainable from a bipolar transistor in the common-emitter configuration unless it is operated with its input resistance substantially greater than the source resistance; it follows that in the example quoted, the common-emitter input resistance of the input transistor must be more than 100 k $\Omega$ , and the operating current must be chosen accordingly.

This example helps to make clear a fallacy which has often led to unsatisfactory circuit and system design. This is the idea that the application of series feedback to an amplifier, because it increases the input resistance, can enable the amplifier to give a satisfactory noise figure from a higher source resistance than before.

Another error which has often led to difficulties is the failure to realize that when a series feedback resistor is inserted in the emitter lead of the input transistor, this resistor is effectively in series with the signal source and its value must be added to the series noise resistance of the amplifier.

Similarly, any resistor used for parallel feedback has the effect of reducing the parallel noise resistance of the amplifier. These considerations do not, of course, affect the general result that, for a given frequency response, the closed-loop noise figure is equal to the open-loop noise figure; but in calculating the open-loop conditions we must take care to include the effects of all the components of the feedback network, even although the latter is assumed to be inoperative.

4.3. CE, CC and CB Connections

It is often said that there are three basic ways of using a bipolar transistor in a linear circuit, these being the common-emitter (CE), common-base (CB), and common-collector (CC) configurations. Now although this approach is often useful it can also be very misleading, and has certainly led to a plethora of time-wasting algebraic work. Generally speaking, we should regard the basic amplifying action of the transistor as being with the input voltage applied between base and emitter, and the output current generated in the emitter-collector circuit; this approach is clearly brought out by the well-known hybrid- $\pi$  equivalent circuit. The CB and CC configurations then appear as feedback modifications of the basic action: in normal feedback terminology, the CB arrangement is one of parallel current feedback, and the CC (emitter-follower) one of series voltage feedback.<sup>8</sup>

the mistake of assuming that, because the CB arrangement has a much lower input resistance than the CE arrangement, it can be used in conjunction with a much lower source resistance without detriment to the noise figure; it should be clear from Fig. 8 that this is not correct.

The case of the CC (emitter-follower) amplifier is not so simple, because of the fact that the output is the voltage across the emitter resistor itself, rather than the output current of the transistor. When a detailed calculation is done, we find the result illustrated in Fig. 9.

Since the feedback is of the series type, the signal source is most appropriately represented as a voltage

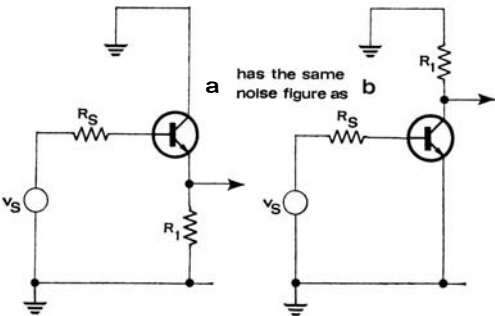


Fig. 9. CC noise figure.

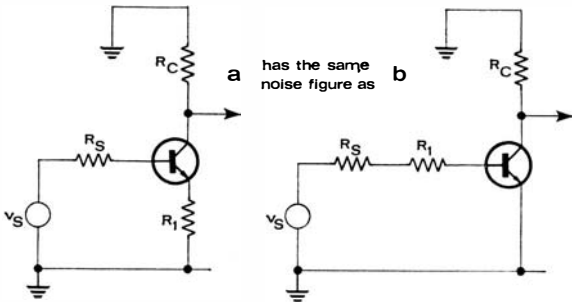


Fig. 10. Noise figure of CE amplifier with emitter feedback.

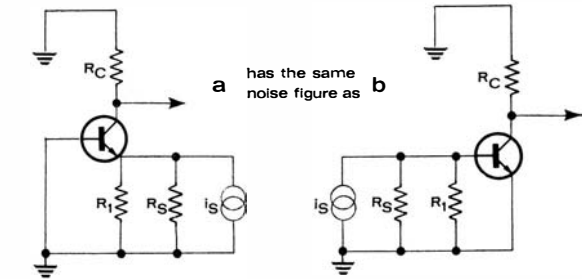


Fig. 8. CB noise figure.

The application of the theory of this section to the CB case is perfectly simple. Figure 8(a) shows the a.c. equivalent circuit of a CB-connected transistor, with an emitter resistor  $R_1$  and operating from a signal source of resistance  $R_s$ , shown here in its current-generator equivalent form. The corresponding open-loop arrangement is shown in Fig. 8(b) and, by the principle developed in Section 4.1, we see that Figs. 8(a) and 8(b) will both give the same noise figure at any given frequency, assuming of course that the transistor is operating under the same d.c. conditions in the two cases. Some designers have made

generator. The figure shows that in calculating the noise figure of a CC stage we must regard the resistor  $R_1$  as being merely shunted across the output and not as being an additional resistance in series with the input circuit.

This is in contrast with the situation shown in Fig. 10, which is a CE amplifier with emitter feedback. Here the feedback is of the normal series current type, and the emitter resistor appears in series with the input circuit.

It is easy to make an experimental demonstration of the essential difference between the CC stage and

the CE stage with emitter feedback. The circuit of Fig. 10(a) may be set up with  $R_C = R_1$ , and driven from an a.f. signal source of very low resistance. With typical circuit values, say  $I_C = 1$  mA and resistors of several kilohms, there is a spectacular difference between the signal/noise ratio measured at the emitter and at the collector, although of course the magnitude of the signal gain is unity at each point.

#### 4.4. Use of Negative Feedback to Improve Noise Figure

In one sense, as we have seen, it is impossible to improve the noise figure of an amplifier by applying negative feedback; this statement is true when we are comparing the closed-loop situation with the open-loop situation for an amplifier whose circuit remains otherwise unchanged. On the other hand, we may compare one amplifier which provides certain performance parameters (e.g. gain, input impedance, output impedance, power output, efficiency, bandwidth) without the use of negative feedback, with another amplifier which utilizes the principles of negative feedback to obtain the same performance parameters. On this basis of comparison, it is often possible to obtain a great improvement in noise figure by the use of negative feedback.

A simple example of this arises in the case where one requires an amplifier to have a very low input resistance, say  $1\ \Omega$ , but to give a good noise figure from a comparatively high source resistance, say  $1\text{ k}\Omega$ . One could obtain the required input resistance by connecting a  $1\ \Omega$  resistor across the input terminals of a non-feedback transistor amplifier; but the effect on the noise figure would be disastrous. By the use of the parallel-feedback (operational-amplifier) technique we can obtain the required input resistance without any adverse effect on the noise figure.

Another example arises when we are dealing with power output stages, which may have a very poor noise figure. By adding a low-noise preamplifier, as discussed in Section 2.6, we may improve the noise figure; on the other hand, this preamplifier may increase the overall gain to an unacceptably high value. The use of negative feedback will enable the gain to be brought back to the required value without change in the overall noise figure.

Yet another case of practical importance is that in which we are required to provide a low-noise amplifier of widely variable gain. An output attenuator is usually not acceptable because of its effect on the dynamic range; and an input attenuator will ruin the noise figure. It is often possible to obtain a variable gain with constant noise figure by providing a negative-feedback network with variable feedback ratio.

Basically, the virtue of negative feedback is that it enables the input resistance, and the gain, of the

system to be adjusted to the required value without detriment to the noise figure.

## 5. Practical Circuit Design

### 5.1. Worst-case Design of the Input Stage

The discussion in the previous sections has in principle given enough information to enable low-noise a.f. amplifiers to be designed. However, in practice the designer is not usually in possession of all the required parameters, and in this section we shall consider some experimental short-cuts which will prove useful.

The basic problem is the lack of reliable information about flicker-noise parameters, although in a later paper we shall be presenting the results of a large number of flicker-noise measurements on devices submitted to us by manufacturers as low-noise devices.

From a practical engineering point of view, we want to design the input circuit in such a way that we are sure that the component values are correct. If the circuit then does not give the required noise figure, we shall know that the fault is in the input device itself rather than in its operating conditions; the selection of a suitable input device, if only by a process of trial-and-error, is then a straightforward matter.

We shall assume that the required upper-limit noise figure  $F_1$  has been specified—this will often be in the neighbourhood of 1–3 dB, because such a noise figure is likely to be indistinguishable from unity (0 dB). The first case to be considered is that in which the source resistance  $R_s$  is specified, and we do not wish to use a transformer. As a first choice for an input device we consider a bipolar transistor, the assumption being that it is a modern device designed for low-noise work at audio frequencies, having a direct-current gain of over 100 at collector currents down to  $1\ \mu\text{A}$  or less. As far as flicker noise is concerned, the situation is that we cannot accurately predict the value of  $\beta'_0$  (defined by equation (20)) as a function of frequency and collector current for any individual transistor; however, we know that generally the flicker-noise characteristic frequency  $\omega_F$  falls as  $I_C$  is reduced, so that provided we have chosen a device in which the d.c. gain  $\beta_0$  is practically independent of  $I_C$  we shall obtain the least flicker noise by operating the transistor at as low a value of  $I_C$  as possible. In practice, the choice of  $I_C$  is normally made on the basis of equation (22); although this is only correct if conditions (21) are satisfied, it is always sufficiently accurate to be used for worst-case design. Equation (22) shows that even if  $\beta'_0$  were infinite a lower limit to  $I_C$  would be imposed by the requirement that

$$(r_b + r_e/2) < R_s(F_1 - 1) \quad \dots\dots(31)$$

The actual optimum value of  $I_C$  cannot be determined accurately; however, it is reasonable to assume that  $\beta'_0$  does not vary very rapidly with  $I_C$  so that, for a given value of  $\beta'_0$ , the transistor will be operating from its optimum source resistance when the second and third terms in equation (22) are equal. Now the worst acceptable value for  $\beta'_0$  is that which gives the required noise figure  $F_1$  when the optimum source resistance is used; accordingly we obtain the condition

$$(r_b + r_e/2) = R_s(F_1 - 1)/2 \quad \dots\dots(32)$$

as the basis for calculating the operating current of the input transistor.

If we are dealing with broad-band amplifiers, the effective value of  $\beta'_0$  is a suitable average over the required frequency range; the same principles apply to the choice of operating current.

In applying equation (32), we put in typical values of  $r_b$  for the transistor type to be used, if this has already been decided on; otherwise we put in a value of  $100\ \Omega$ , which is about the lowest value that can be reliably obtained from present-day low-noise devices. If equation (32) then gives  $r_b \ll r_e/2$ , we may use a device with a higher value of  $r_b$  without substantially affecting the noise figure. On the other hand, it may be impossible to satisfy equation (32) with  $r_b = 100\ \Omega$  and a positive value of  $r_e$ , in which case we use the parallel-transistor technique described in Section 2.5. In fact, it is generally undesirable to operate a transistor under conditions where  $r_e < r_b$ , because such conditions make unduly high demands on the value of  $\beta'_0$  if a good noise figure is to be achieved; in practical terms, this means that input transistors should not normally be run at collector currents greater than about  $250\ \mu\text{A}$ . Thus if equation (32) leads us to the requirement that  $r_e < r_b$ , we should consider the use of  $n$  transistors in parallel, applying equation (32) with  $R_s$  replaced by  $nR_s$ .

We now come to the case where an input transformer is to be used. In principle this means that we have a free choice of our value of effective source resistance, although in practice there will be an upper limit to the usable secondary impedance imposed by winding capacitance and also by mechanical considerations. In practice, for a noise figure of 1 dB it is not necessary to use a secondary impedance greater than  $10\ \text{k}\Omega$ .

So far we have not considered the use of j.f.e.t. as an input device. This is because most of the currently available j.f.e.t.s have high values of  $R_{Nv}$  at low audio frequencies so that satisfactory noise-matching is only obtained from rather large source resistances. However, the time may come when low-cost j.f.e.t.s are available with values of  $R_{Nv}$  of a few hundred ohms throughout the audio-frequency range,

and they will then be the natural first choice as input devices.

## 5.2. The Feedback Resistor

Any modern circuit design is likely to incorporate a high degree of negative feedback. This will involve the inclusion of a resistor either in series or in parallel with the input circuit, and we must now discuss the way in which the results of Section 5.1 may be modified to allow for the effect of this feedback resistor.

The effect on the input circuit of a series feedback resistor  $R_{FS}$  can very simply be included in the discussion of Section 5.1 by replacing the base resistance  $r_b$  by the total effective ohmic series resistance ( $r_b + R_{FS}$ ). In the case where we have a parallel feedback resistor  $R_{FP}$ , we must include in equation (22) an additional term ( $R_s/R_{FP}$ ); generally speaking, we should choose  $R_{FP}$  so that the contribution of this term to the noise figure is less than the contributions of the terms dependent on transistor characteristics. This implies that

$$R_{FP} > 3R_s/(F_1 - 1)$$

but this criterion may, in practical cases, not be sufficiently rigorous. When the required value of  $R_{FP}$  has been established, the operating current of the first stage may be determined from equation (32) with the quantity  $(F_1 - 1)$  replaced by  $(F_1 - 1 - R_s/R_{FP})$ . This modification places more stringent requirements on the value of  $\beta'_0$ , for which reason it may be desirable to satisfy the above condition with as large a margin as is practicable.

## 5.3. Worst-case Design of the Second Stage

One of the commonest errors in low-noise circuit design is the failure to ensure that the second stage does not contribute to the total flicker noise. In this section we shall make a worst-case analysis which will lead to a rule-of-thumb to aid the circuit designer. The worst case can be set up by assuming that the source resistance is zero, so that the input transistor is generating less noise than it ever will in a practical situation. With this assumption, we shall attempt to find a criterion which ensures that the noise contribution of the second stage is negligible.

If the input transistor is being operated under low-noise conditions, the effect of the base resistance  $r_b$  can be neglected in this discussion. From equations (23) we see that the noise in the input transistor can therefore be represented by a voltage generator with mean-square value  $2r_{e1}kT\Delta f$ , where  $r_{e1}$  is the emitter resistance of the input transistor. Now the transfer admittance (mutual conductance) of this transistor is approximately  $1/r_{e1}$ , so the noise current generated by the input transistor in the CE or CB configuration

has a mean-square value  $2kT\Delta f/r_{e1}$ . Now, as we shall see shortly, the noise contribution of the second transistor is normally dominated by the effect of the equivalent noise-current generator. Equations (23) show that the mean-square value of this generator is  $2kT\Delta f/\beta'_{o2}r_{e2}$ , where the subscript 2 refers to the values for the second transistor. If this is not to contribute substantially to the overall noise, we must establish the condition that

$$2kT\Delta f/\beta'_{o2}r_{e2} \ll 2kT\Delta f/r_{e1}$$

which is equivalent to the condition

$$\beta'_{o2}r_{e2} \gg r_{e1} \quad \text{.....(33)}$$

To obtain the corresponding condition for the case of a CC (emitter-follower) input stage, we observe that if  $R_S$  is zero the output of the first transistor is equivalent to a voltage generator of mean-square value  $2r_{e1}kT\Delta f$  in series with a resistance  $r_{e1}$ . This is equivalent to a current generator of mean-square value  $2kT\Delta f/r_{e1}$  in parallel with  $r_{e1}$ , and it follows that condition (33) is also applicable to the CC configuration.

Although equation (33) has been derived on the assumption of a somewhat extreme worst-case, it provides a very convenient design basis for the instrumentation or communications engineer. At worst, it will result in a circuit which contains one more transistor than is strictly necessary; but transistors are cheap, and design and development effort is expensive.

It is interesting to notice that equation (33) cannot possibly be satisfied either by the traditional 'Darlington pair' configuration, in which the emitter current of the input transistor is equal to the base current of the second transistor, or by the analogous complementary design in which the collector current of the input transistor is equal to the base current of the second transistor (see Fig. 11). This is because the collector current in the second transistor is greater than that in the first transistor by a factor of  $\beta_{o2}$ , so that  $r_{e2} = r_{e1}/\beta_{o2}$ . Thus with a zero source resistance, even in the absence of flicker noise, the second transistor makes a noise contribution equal to that in the first transistor; in practice the situation will probably be made very much worse by the effect of flicker noise. The direct-coupled stages shown in Fig. 11 can be greatly improved by shunting the input of the second transistor with a resistor to enable it to run with a lower value of collector current, appropriate changes being made in the remainder of the circuit.

Now even a reasonably low-noise transistor may show a value of  $\beta'_o$  approaching unity at the lower end of the audio-frequency range; this would be

true, for example, at 20 Hz if  $\beta_o$  were 100 and the flicker-noise characteristic frequency  $\omega_F$  were 2 kHz. Therefore, if we want to be reasonably sure, even under the worst conditions (zero source resistance) and in the lowest part of the audio-frequency range, that the second transistor does not contribute to the overall noise, we make use of condition (33) in the form

$$r_{e2} = r_{e1} \quad \text{.....(34)}$$

This condition is equivalent to the statement that the d.c. collector current in the second transistor should be equal to that in the first transistor.

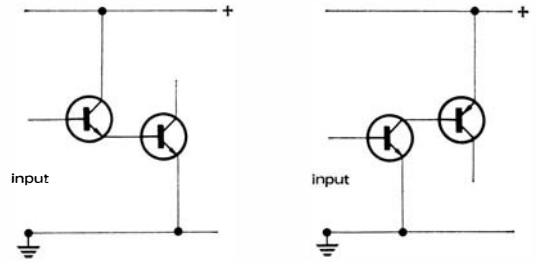


Fig. 11. Examples of noisy circuit design.

As a last step, we must arrange that the voltage gain of the first stage is sufficiently high to ensure that we have satisfied the requirement, mentioned above, that the voltage-generator noise in the second transistor should be negligible. First we assume that the first stage has the CE or CB configuration, with collector load resistance  $R_C$ —this being assumed to include the effect of the input resistance of the second transistor. As we have seen, the noise current generated by the input transistor has a mean-square value  $2kT\Delta f/r_{e1}$ , and this provides a noise voltage across  $R_C$  with mean-square value  $2kT\Delta fR_C^2/r_{e1}$ . Now the voltage-generator noise in the second transistor has a mean-square value  $2r_{e2}kT\Delta f$ , so our required condition is

$$R_C^2 \gg r_{e1}r_{e2} \quad \text{.....(35)}$$

This condition is easily satisfied. The lowest value we are likely to use for  $R_C$  arises in a direct-coupled complementary circuit where the direct voltage drop across  $R_C$  is only about 600 mV. Assuming for example that the d.c. collector current in the first transistor is 10  $\mu$ A, then  $r_{e1}$  will be 2.5 k $\Omega$  and  $R_C$  will be 60 k $\Omega$ . It is clear that (35) will be satisfied even if the second transistor has the same d.c. collector current as the first so that  $r_{e2} = r_{e1}$ .

For a CC (emitter-follower) input stage the situation is not so favourable. Since the voltage gain is unity, the condition equivalent to (35) is

$$r_{e1} \gg r_{e2} \quad \text{.....(36)}$$

which can be formally obtained from (35) by putting  $R_C = r_{e1}$  but which is really self-evident. Obviously, we cannot satisfy both condition (34) and condition (36); for this reason the CC configuration is inherently more noisy for an input stage than the CE or the CB configurations. In some practical cases the difference is so slight as to be negligible, and the CC stage may be preferred for other reasons.

#### 5.4. General Comments on Circuit Design

There are many textbooks on transistor circuit design, but it is an unfortunate fact that on the whole they are highly unsatisfactory.

We must avoid being misled by the idea of a 'norm' of amplifier design consisting of a string of separate single-transistor or two-transistor stages with capacitor coupling and with feedback (if any) provided by individual emitter resistors. The amplifier should be designed as a whole, with direct coupling between the transistors unless there is some good reason to the contrary, and with a high degree of d.c. and a.c. feedback taken around each direct-coupled group. As a matter of course, silicon transistors will be used throughout. Apart from the obvious economies in components, this approach enables a greater number of transistors to be included in the feedback loop so that the noise contribution of the feedback resistor can be reduced to the lowest possible level.

We must also avoid the assumption that noise considerations are a sort of 'extra' which only needs to be taken into account under exceptional circumstances. As a matter of general engineering workmanship, the good designer ensures that, whatever the specification, his circuits do not show an *unnecessarily* high noise level; and a prospective purchaser who has

to choose between two otherwise identical pieces of equipment will always choose the one showing the lower noise level. It is by no means unusual to find a factor of 100 or more between the noise levels shown by the product of a 'noise-conscious' designer and the product of a designer who follows typical textbook principles.

#### 6. Acknowledgment

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cases where  $V_2 = 0.2\text{ V d.c.}$ ,  $0.5\text{ V d.c.}$  and  $V_1$ . The lowest trace shows the zero level of the output. The circuit works well as a divider for input voltages from  $0.01$  to  $1.5\text{ V}$ .

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## OPTIMUM DESIGN OF LOW-NOISE AMPLIFIERS

The letter outlines theoretical and experimental work on the optimum design of low-noise transistor amplifiers.

The fundamental sources of noise in a bipolar transistor have been analysed by van der Ziel,<sup>1</sup> from whose equations the following expression emerges for the low-frequency noise figure  $F$  in terms of the source impedance  $R_S$  (assumed real), and the emitter resistance  $r_e$ , the base resistance  $r_b$  and the low-frequency common-emitter current gain  $\beta_0$  of the transistor:

$$F = 1 + \frac{(r_b + \frac{1}{2}r_e)}{R_S} + \frac{(R_S + r_b + r_e)^2}{2\beta_0 r_e R_S} \quad (1)$$

In this equation, the effect of collector-leakage current is ignored and the signal frequency is assumed high enough to ensure that  $1/f$  noise is negligible, but well below the frequency  $f_T/\sqrt{\beta_0}$  at which the noise begins to increase because of high-frequency effects in the transistor.<sup>2</sup>

If we further assume that we are using a modern low-noise silicon planar transistor, and that the direct collector current  $I_C$  does not exceed  $1\text{ mA}$ , we may assume that  $\beta_0 \gg 1$  and also that  $\beta_0 r_e \gg r_b$ ; so that, to a good approximation, we can rewrite eqn. 1 in the form

$$F = 1 + \frac{(r_b + \frac{1}{2}r_e)}{R_S} + \frac{R_S}{2\beta_0 r_e} \quad (2)$$

Finally, we express eqn. 2 in terms of noise voltage and current generators, which are most conveniently evaluated in terms of a series noise resistance  $R_{Nv}$  and a parallel noise resistance  $R_{Ni}$ , with correlation coefficient  $\gamma$ ; expressed in terms of these parameters, the noise figure  $F$ , the optimum source resistance  $(R_S)_{opt}$  and the corresponding minimum-noise figure  $F_{min}$  are given by the relations<sup>3</sup>

$$\left. \begin{aligned} F &= 1 + \frac{R_{Nv}}{R_S} + \frac{R_S}{R_{Ni}} + 2\gamma\sqrt{\frac{R_{Nv}}{R_{Ni}}} \\ (R_S)_{opt} &= \sqrt{(R_{Nv}R_{Ni})} \\ F_{min} &= 1 + 2(1 + \gamma)\sqrt{\frac{R_{Nv}}{R_{Ni}}} \end{aligned} \right\} \quad (3)$$

Equating coefficients of  $R_S$  between eqns. 2 and 3, we arrive at the conclusion

$$R_{Nv} = (r_b + \frac{1}{2}r_e); R_{Ni} = 2\beta_0 r_e; \gamma = 0 \quad (4)$$

In Fig. 1, these two noise resistances are plotted against  $I_C$  in a straight-line approximation for an 'ideal' transistor in which  $\beta_0 = 100$ , independent of  $I_C$ , and  $r_b = 100\Omega$ . The dotted line shows  $(R_S)_{opt}$ , which is simply the geometric mean of the two noise resistances; furthermore, the quantity  $(F_{min} - 1)$ , given according to eqns. 3 and 4 by  $2\sqrt{(R_{Nv}/R_{Ni})}$ , is indicated at any value of  $I_C$  by the separation between the dotted line and one of the solid lines.

The range of  $I_C$  in which  $\frac{1}{2}r_e \gg r_b$ , so that the two noise resistances are substantially proportional to each other, may be called the low-noise régime, where  $F_{min} = 1 + 1/\sqrt{\beta_0}$ . In the region of higher  $I_C$ , where the series noise resistance becomes dominated by the base resistance,  $F_{min}$  increases with  $I_C$ .

We have carried out a large number of measurements on the noise generators in silicon planar transistors at frequencies between  $10\text{ Hz}$  and  $20\text{ kHz}$ , which will be discussed fully in a

later publication. The general result is that, in the specified frequency range, the observations can usually be fitted to curves of the form in Fig. 1, provided that  $r_b$  and  $\beta_0$  are treated as adjustable parameters. We have no evidence that the limiting value of  $R_{Nv}$  is usually equal to the effective ohmic resistance between the base terminal and the emitter-base junction, although it obviously cannot be less than this, and we have not found it to be lower than  $50\Omega$  for any transistor.

Our results are in agreement with the well known fact that there is not much difficulty in obtaining from bipolar transistors a noise figure very close to unity at audio frequencies for source resistances in the range  $200\text{--}2\text{ k}\Omega$ . The difficulty has arisen with source resistances less than  $2\text{ k}\Omega$ ,

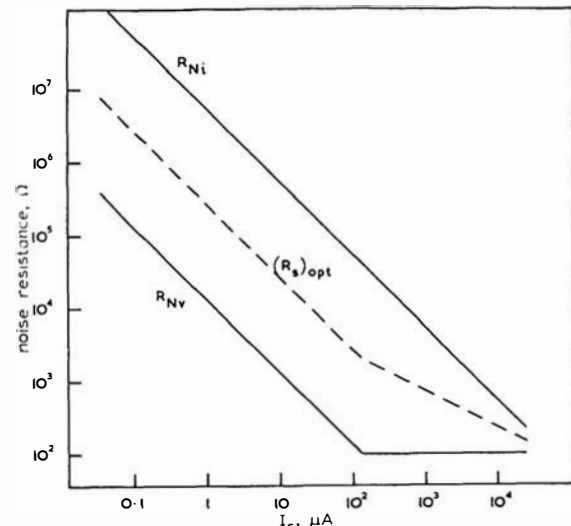


Fig. 1 Noise resistances for an ideal transistor, with  $\beta_0 = 100$  and  $r_b = 100\Omega$

because the limiting series noise resistance is normally between  $100$  and  $400\Omega$  for typical small-signal transistors. If this were entirely due to the ohmic base resistance, it should be possible, in principle, to obtain a good noise figure from an arbitrarily low source resistance, by the choice of a sufficiently large transistor; the fact that this approach is not generally a useful one (because of considerations of current gain, frequency response and cost) is implicit in the familiar principle (Reference 4, for example) that, in order to obtain a satisfactory noise figure with a low source impedance, one must use a transformer.

The use of an input transformer in a low-noise audio-frequency circuit is usually highly undesirable, and in many cases impossible. We shall now describe a technique which has been used in this laboratory to improve the noise figure of audio-frequency amplifying systems where the source impedance is  $1\text{ k}\Omega$  or less, based on the fact that, when  $n$  identical amplifiers are connected in parallel, the series noise resistance and parallel noise resistance are both reduced by a factor  $n$  compared with the corresponding value for one of the amplifiers.

To prove this result, we consider the signal/noise ratio given by the parallel combination for very low and very high source impedances. When  $R_S \rightarrow 0$ , the mean-square signal current delivered into the load is increased by a factor  $n^2$  compared with a single amplifier, but the mean-square noise current is only increased by a factor  $n$ , because the noise generators of the individual amplifiers are not correlated; thus the signal/noise ratio is improved by a factor  $n$  compared with that given by a single amplifier. When  $R_S \rightarrow \infty$ , the source current is shared between the amplifiers, and the signal-output current is only the same as that available from a single amplifier; so the total signal/noise ratio is deteriorated by a factor  $n$ . Comparison of these results with eqn. 3 shows that both  $R_{Nv}$  and  $R_{Ni}$  have been reduced by a factor  $n$ , owing to the parallel connection of the amplifiers. An extension of this argument shows that, if the  $n$  amplifiers have different values of  $R_{Nv}$ , the appropriate formula for the series noise resistance of the combination is  $n^{-2}\Sigma R_{Nv}$ .

Fig. 2 shows the basic configuration (feedback components being omitted) of a 4-transistor input stage employing this principle. The series and parallel noise resistances of the circuit are still related to the individual collector current  $I_C$  by curves of the form of Fig. 1; but the resistance scales, and therefore the lower limit of the low-noise régime, are shifted by a factor of 4. With this circuit, we obtain an improvement of 6dB in noise figure from a very low-impedance source, compared with that given by a single transistor without a

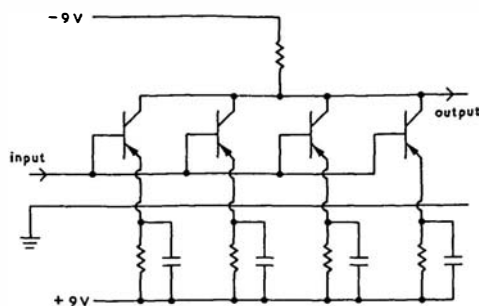


Fig. 2 Low-noise input circuit

Transistors: 2N3702  
Emitter resistors: 8.2 kΩ

transformer; and a noise figure of 3dB is obtainable from a 30Ω source. This technique should be particularly applicable to field-effect transistors, whose noise performance is basically limited by the voltage-generator, rather than the current-generator, noise, and we are carrying out experimental work along these lines.

The author is much indebted to D. W. Harding of this laboratory, who carried out the experimental work.

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## TOPOLOGICAL ANALYSIS OF NETWORKS CONTAINING NULLATORS AND NORATORS

A topological analysis of networks containing nullators and norators is presented. Only one topological rule of determinant expansion is necessary; summing of small numbers with big ones can be avoided when using impedances or admittances as branches.

A letter by Davies<sup>1</sup> dealt with the topological analysis of networks containing nullators and norators; the same subject was discussed in a lecture read at the Prague Summer School on Circuit Theory in 1965.<sup>2</sup> However, a different derivation of fundamental formulas enables a certain generalisation.

The following theorems are used as a basis:

(i) Generalised Ohm's law expressed in the form

$$y v_b = z i_b \quad (1)$$

where  $v_b$  and  $i_b$  are branch voltages and currents, respectively, and  $y$  and  $z$  are regular diagonal matrixes selected so that the product:

$$Y = z^{-1} y \quad (2)$$

is the diagonal matrix of all nonzero admittances.

(ii) Kirchhoff's law expressed in the form

$$A i_b = 0 \quad (3)$$

$$B v_b = 0 \quad (4)$$

where  $A$  is an arbitrary cutset matrix (e.g. incidence matrix of the nodes and branches),  $B$  is an arbitrary loopset matrix, and  $v_b$  and  $i_b$  are voltage and current vectors, respectively, in all branches under consideration. These may be branches with either nonzero admittances or nullators, norators and independent sources. The former will later be called regular branches and the latter (i.e. nullators, norators and sources) will later be called singular branches. The nullators and norators must appear in any circuit in pairs.

Under the stated assumption, it is possible to write the system equations in the form

$$DX = E \quad (5)$$

$$D = \begin{bmatrix} r' & s'_1 & s'_2 & t'_1 & t'_2 & r'' & s''_1 & s''_2 & t''_1 & t''_2 \\ y & 0 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline & B & & & & & 0 & & & \\ \hline & 0 & & & & & A & & & \end{bmatrix} \begin{matrix} r \\ s_1 \\ s_2 \\ t_1 \\ t_2 \\ b \\ a \end{matrix} \quad (6)$$

Here  $I$  and  $0$  are the corresponding identity and zero matrixes,  $r$  and  $r'$ ,  $r''$  are the rows and columns corresponding to the regular branches,  $s_1$  and  $s'_1$ ,  $s'_1$  are those corresponding to nullators,  $s_2$  and  $s'_2$ ,  $s'_2$  are those corresponding to the norators,  $t_1$  and  $t'_1$ ,  $t'_1$  are those corresponding to the independent sources of voltage,  $t_2$  and  $t'_2$ ,  $t'_2$  are those corresponding to the independent sources of current,  $a$  are the rows of matrix  $A$ ,  $b$  are the rows of matrix  $B$ ,  $X' = [v'_b, i'_b]$ ,

$$E' = \begin{bmatrix} r & s_1 & s_2 & t_1 & t_2 & b & a \\ 0 & 0 & 0 & v_0 & i_0 & 0 & 0 \end{bmatrix}$$

$v_0$  and  $i_0$  are the vectors of the voltages and currents of independent sources.

By modification of the matrix  $D$  (eqn. 6) and application of the Laplace rule, it is possible to derive all the network functions used from the system equation (eqn. 5). The

	network	determinant	branch 1	fundamental network	branch 2
a	N	$\Delta$			
b	$N_{1,1}$	$\Delta(1,1)$			
c	$N_{2,2}$	$\Delta(2,2)$			
d	$N_{1,2}$	$\Delta(1,2)$			
e	$N_{2,1}$	$\Delta(2,1)$			
f	$N_{2,12}$	$\Delta(12,12)$			

Fig. 1 Fundamental networks

Input (branch 1)	Output (branch 2)
a Omitted	Omitted
b Short circuit	Omitted
c Omitted	Short circuit
d Nullator	Norator
e Norator	Nullator
f Short circuit	Short circuit