A New Perspective: Electromagnetic Theory is the Fundamental Foundation for all Mechanical Principles of Newton's Laws

Nirod Das [1]

[1]New York University

October 31, 2023

Abstract

The basic principles of Newton's laws, and related concepts of momentum and energy and their conservation, are derived from the electromagnetic theory. The electric and magnetic fields produced by an electric charge in uniform motion, as derived from Maxwell's equations, are used to find the forces the charge would exert on another charge, as measured in two inertial frames. These force transformation relations in the two frames are extended to apply to any general physical problem involving force. The force transformation relations are then used, together with the space-time relations of special relativity, to derive Newton's laws of motion applicable for velocity much smaller than the speed of light c(v << c), as well as to derive general expressions for mass, momentum and energy, applicable for any velocity v[?]c. Further, the momentum or energy as expressed in one inertial frame, are linearly related to the momentum and the energy expressed in another inertial frame. This result, when applied to a closed system with no external interaction, proves the momentum and the energy to be conserved, based on the required force-transformation relations. Fundamental and philosophical implications of the results and derivations are discussed. The basic principles of invariant electric and magnetic charge, upon which all electromagnetic concepts of Maxwell's equations are founded, are recognized to be complete, general and the fundamental origin of Newton's laws, making mechanical or material principles theoretically secondary.

A New Perspective: Electromagnetic Theory is the Fundamental Foundation for all Mechanical Principles of Newton's Laws

Nirod K. Das, *Member, IEEE*

***Abstract*—The basic principles of Newton's laws, and related concepts of momentum and energy and their conservation, are derived from the electromagnetic theory. The electric and magnetic fields produced by an electric charge in uniform motion, as derived from Maxwell's equations, are used to find the forces the charge would exert on another charge, as measured in two inertial frames. These force transformation relations in the two frames are extended to apply to any general physical problem involving force. The force transformation relations are then used, together with the space-time relations of special relativity, to derive Newton's laws of motion applicable for velocity v much smaller than the speed of light c ($v << c$), as well as to derive general expressions for mass, momentum and energy, applicable for any velocity $v \leq c$. Further, the momentum or energy as expressed in one inertial frame, are linearly related to the momentum and the energy expressed in another inertial frame. This result, when applied to a closed system with no external interaction, proves the momentum and the energy to be conserved, based on the required force-transformation relations. Fundamental and philosophical implications of the results and derivations are discussed. The basic principles of invariant electric and magnetic charge, upon which all electromagnetic concepts of Maxwell's equations are founded, are recognized to be complete, general and the fundamental origin of Newton's laws, making mechanical or material principles theoretically secondary.**

***Index Terms*—Maxwell's Equations, Special Theory of Relativity, Newton's Laws, Electromagnetic Perspectives.**

I. Introduction

It has been recently established in [1]-[3] that Maxwell's equations [4] can be derived from basic principle of invariance of the electric and magnetic charges, as fundamentally defined by Gauss' laws for the electric and magnetic fields, respectively, using only the space-time relations of special relativity [5], [6]. The principle of invariance of charge, unambiguously defined based on a self-consistent concept of force on the charge, using Gauss' laws applicable across reference frames, allows a fundamental derivation of Maxwell's equations from the basic charge principle. Newton's laws that govern the principles of the force and its resulting motion, or the principles of momentum and energy defined using the force, as conserved quantities, are not required in the fundamental derivation of Maxwell's equations.

Once Maxwell's equations are independently established, they can be solved for the electric and magnetic fields in any given problem involving charges, as seen in two reference frames, using which the associated forces are derived. The relationships between the solved forces in the two frames would establish the required relativistic force-transformation formulas in the two frames, without need for Newton's laws. Instead, Newton's laws can now be derived from the established force-transformation formulas.

Conventionally, the force-transformation formulas in special relativity are deduced starting from the basic Newton's laws and principles of momentum and energy conservation, by employing the space-time relations of special relativity [6]. This process derives the velocity-dependent functions for the mass, momentum and energy as intermediate steps, leading to the force transformation relations. Now that the force transformation relations are available directly from Maxwell's equations, one can then essentially retrace backwards the conventional derivations of the relativistic mechanics. Accordingly, one could derive the functional forms for the mass, momentum and energy, leading to fundamental "derivation" of Newton's laws and of the associated principles of momentum and energy conservation.

In this paper, we will follow such a derivation starting with a simple electrical problem having simple solutions from Maxwell's equations. Theoretical and philosophical significance of the different results and derivations are addressed. The fundamental nature of the electromagnetic principles, in contrast with the basic material principles of Newton's laws, are discussed.

This article would be a valuable companion to another article on fundamental electromagnetic theory presented in this magazine [1]-[3], which introduces Maxwell's equations as derived from simple concepts of charge and space-time relations. Understanding the electromagnetic theory with a new fundamental perspective, in relation to mechanical concepts of Newton's laws as studied in the present paper, would provide even deeper insight into the basic electromagnetic concepts, inspiring appreciation for their broader physical significance.

II. Force Transform Relations Derived From the Forces Between Two Charges

Consider two charges ($Q_1 = Q_2 = Q$) that are stationary with respect to each other. Their fields and mutual forces are measured in two reference frames, one (primed frame) where

N. Das is with the Department of Electrical and Computer Engineering, Tandon School of Engineering, New York University, Five Metrotech Center, Brooklyn, NY 11201.

Manuscript Submitted to IEEE Antennas and Propagation Magazine, June 28, 2023.

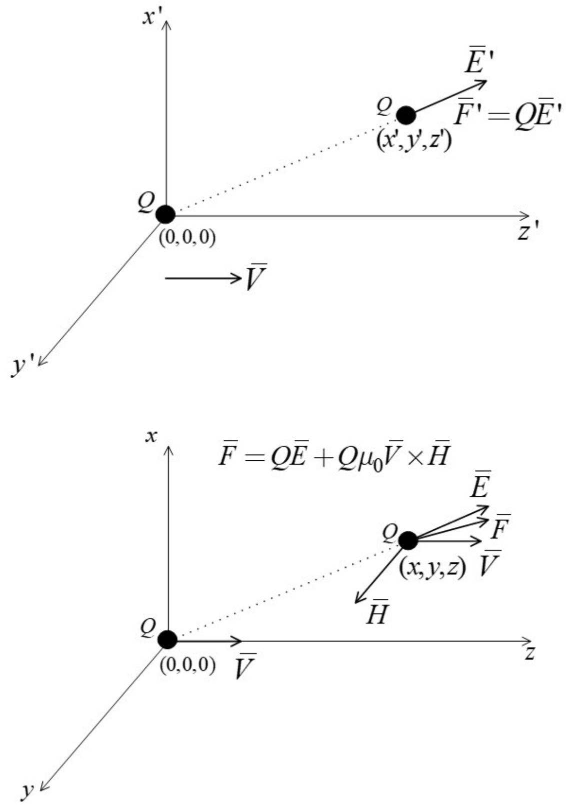

Fig. 1. Two charges, that are stationary with respect to each other, but located at different positions in space. The fields produced by one charge (source charge), acting upon the second charge (test charge), as seen in two different reference frames. The primed frame is moving with a velocity V along the z axis with respect to the unprimed frame.

the charges are at rest with respect to an observer, and the other (unprimed frame) where the charges are moving with respect to an observer at a constant velocity V along the z direction, as shown in Fig.1. This is equivalent to having the individual observer attached to the primed or the unprimed frames, who see the other observer moving with a uniform velocity V in the $-z$ and $+z$ directions, respectively. The origins of both the frames are aligned with the location of one of the charges at time $t = t' = 0$, whereas the other charge is located at (x', y', z') in the primed frame, or at (x, y, z) in the unprimed frame, timed at the instant $t = 0$ in the unprimed frame.

The two frames, moving with uniform velocity with each other, are assumed to be naturally "unbiased" in a uniform free-space medium. The basic invariant nature of propagation of light in a uniform free-space, and consequently the validity of the associated Maxwell's equations, are only ensured across all such equivalent, unbiased frames [1]-[3]. The invariant nature of light to follow a straight-line path in the uniform free-space medium, having the same magnitude of its velocity, across all unbiased frames, would require the reference frames to move with a uniform velocity with respect to each other, as a necessary condition. This condition may be verified using the space-time relations of special relativity, which were established based on the expected special nature of light [1]-[3]. This requirement of the uniform relative velocity is consistent with the two reference frames we selected in the electromagnetic analysis of Fig.1.

We will find the electric and magnetic fields produced by the charge at the origin, moving with velocity V along z direction in the unprimed frame. The fields seen in the primed frame is a specific case of that in the unprimed frame, when the velocity is substituted as $V = 0$ and the coordinates are changed from the unprimed to the respective primed variables. Using these fields due to the charge at the origin, we can find the total force applied on the second charge at the general location of Fig.1, for the two cases with observers in the primed and unprimed frames. Relating the components of the two forces $\bar{F}'$ and $\bar{F}$ would establish the required force transformation between the frames. These force transform relations, although derived for the specific simple situations of the two charges, would be applicable to any physical problem involving force, and considered fundamental relations with universal scope.

A. Derivation of the Force Fields

The electric $\bar{E}(x, y, z)$ and magnetic $\bar{H}(x, y, z)$ fields in the unprimed frame, produced by the charge located at the origin and moving in uniform velocity V along the z axis, are derived from Maxwell's equations. One simple approach is to express the moving charge as a superposition or integration of z-directed Fourier surface-current distributions $\tilde{J}_{sz}$ on a plane parallel to the charge velocity (xz-plane, y=0).

$$\begin{gathered}\bar{J}_s = \hat{z}J_{sz} = \hat{z}QV\delta(x)\delta(z-Vt)\\ = \frac{\hat{z}}{4\pi^2}\iint_{k_x,k_z} \tilde{J}_{sz}(k_x,k_z,t)e^{jk_xx}e^{jk_zz}dk_x dk_z;\ y=0,\\ \tilde{J}_{sz} = QVe^{-jk_zVt} = QVe^{-j\omega t},\ k = \frac{\omega}{c} = k_z\frac{V}{c}.\end{gathered} \tag{1}$$

The individual Fourier currents would produce plane waves propagating (evanescent) in the $\pm y$ directions [7], [8], satisfying proper boundary conditions across the currents at $y = 0$. The plane-wave fields are one of the simple solutions of Maxwell's equations (in the uniform free-space medium, observed in an unbiased reference frame).

$$\begin{gathered}H_z = 0\ (\mathrm{TM}_z\ \text{Field}),\ \bar{\nabla}\cdot\bar{H} = 0,\\ \tilde{H}_x(y=0_-) - \tilde{H}_x(y=0_+) = \tilde{J}_{sz},\\ \tilde{\tilde{H}} = (\mp\hat{x} - \frac{k_x}{k_y}\hat{y})\frac{\tilde{J}_{sz}}{2}e^{\mp jk_yy};\ y \gtrless 0,\\ k_y = \sqrt{k^2 - k_x^2 - k_z^2} = -j\sqrt{k_x^2 + k_z^2(1-\frac{V^2}{c^2})}\\ = -j\sqrt{k_x^2 + k_z'^2},\ k_z' = k_z\alpha,\ \alpha = \sqrt{1-\frac{V^2}{c^2}}.\\ \bar{E} = \frac{\bar{\nabla}\times\bar{H}}{-j\omega\varepsilon_0},\ \tilde{\tilde{E}} = \frac{-Q}{2\varepsilon_0 k_y}(k_x\hat{x} \mp k_y\hat{y} + k_z(1-\frac{V^2}{c^2})\hat{z})\\ \times e^{-j\omega t}e^{\mp jk_yy};\ y \gtrless 0.\end{gathered} \tag{2}$$

The total fields can then be obtained by Fourier integration of the plane-wave fields, that maybe verified with available expressions from physics and engineering texts [9], [10]. The fields at any general time t are expressed by substituting $z \to (z - vt)$ in the respective expressions at $t = 0$. Further, by close inspection of the Fourier integrals, the fields at $t = 0$ for a non-zero uniform velocity V are expressed in terms of those for a static charge with $V = 0$ (known, Coulomb's fields), through suitable substitution of variables. First, the electric fields are expressed as follows:

$$\bar{E} = \frac{1}{4\pi^2} \iint_{k_x, k_z} \tilde{\tilde{E}}\, e^{jk_x x} e^{jk_z z} dk_x dk_z =$$
$$\frac{1}{4\pi^2} \iint_{k_x, k'_z} \tilde{\tilde{E}}\, e^{jk_x x} e^{jk'_z z'} dk_x (dk'_z/\alpha) = \bar{E}_0(x, y, (z - vt)),$$
$$\bar{E}(t = 0) = \bar{E}_0(x, y, z) = \frac{1}{\alpha}\bar{E}(v \to 0, z \to z' = \frac{z}{\alpha},$$
$$k_z \to k'_z = k_z\alpha, \hat{z} \to \hat{z}' = \hat{z}\alpha; z'\hat{z}' = z\hat{z})$$
$$= \frac{Q}{4\pi\varepsilon_0 r'^3 \alpha}(x\hat{x} + y\hat{y} + z\hat{z}),\ r' = \sqrt{x^2 + y^2 + z'^2},$$
$$\bar{E}'(x = x', y = y', z = z'\alpha) = \frac{Q}{4\pi\varepsilon_0 r'^3}(x'\hat{x} + y'\hat{y} + z'\hat{z}),$$
$$x = x',\ y = y',\ z = z'\alpha. \tag{3}$$

Then, the magnetic fields maybe derived from the electric fields, by noting the relationships between their Fourier-domain expressions above.

$$\tilde{H}_x = -\tilde{E}_y\varepsilon_0 V,\ \tilde{H}_y = \tilde{E}_x\varepsilon_0 V,\ \tilde{H}_z = 0,$$
$$H_x = -E_y\varepsilon_0 V,\ H_y = E_x\varepsilon_0 V,\ H_z = 0,$$
$$\bar{H} = \bar{H}_0(x, y, (z - Vt)),$$
$$\bar{H}(t = 0) = \bar{H}_0(x, y, z) = \frac{QV}{4\pi r'^3 \alpha}(-y\hat{x} + x\hat{y}). \tag{4}$$

The special case for a stationary charge (in the primed frame) consists of only the electric field $\bar{E}'$ given by Coulomb's law, with a zero magnetic field $\bar{H}' = 0$. This may be verified from the general field expressions in the unprimed frame with any uniform velocity V, by simply substituting $V = 0$ and changing the unprimed to the primed coordinate parameters. Now, the forces $\bar{F}'$ and $\bar{F}$ in Fig.1, observed respectively in the primed and unprimed frames, are expressed using the above fields, timed at $t = 0$ in the unprimed frame.

$$\bar{F}(x, y, z) = Q\bar{E} + Q\mu_0(V\hat{z} \times \bar{H})$$
$$= \frac{Q^2}{4\pi\varepsilon_0 r'^3\alpha}(\alpha^2 x\hat{x} + \alpha^2 y\hat{y} + z\hat{z}),$$
$$\bar{F}'(x', y', z') = Q\bar{E}' + Q\mu_0(V\hat{z} \times \bar{H}')$$
$$= \frac{Q^2}{4\pi\varepsilon_0 r'^3}(x'\hat{x} + y'\hat{y} + z'\hat{z}), \tag{5}$$

$$F_x = \alpha F'_x,\ F_y = \alpha F'_y,\ F_z = F'_z. \tag{6}$$

B. Generality of the Force Transform Relations, Extended to Any Physical System Involving Force

The above result (6), although is derived for a simple problem, may be properly interpreted and extended for a general configuration. The charge at the origin is the "source" charge, which produces all the force fields we derived that act upon the second charge, called the "test" charge. The same result (6) would apply for any arbitrary location of the source charge, as well as for any arbitrary values of the source and test charges that may not be equal to each other. By principle or superposition, the same final result (6) would be obtained as well for an arbitrary spatial distribution of the source charges, producing an arbitrary distribution of the force field $\bar{F}(x, y, z)$. Further, the result (6) requires the velocity of the test charge to be directed along the z axis, with a magnitude equal to zero and V as seen in the primed and unprimed frames, respectively, only at the time of observation $t = 0$. The same result (6) would be valid for any arbitrary path and velocity function of the test charge, with any other velocity $\bar{v}(t)$ at times $t \neq 0$ before or after the observation. This is because, the force (5) acting upon the test charge is dependent only on the location and velocity of the test charge at the time of observation $t = 0$, independent of all time-derivatives of the velocity at $t = 0$ or of the velocity function $\bar{v}(t)$ of the test charge at other times $t \neq 0$.

In summary, the force-transform relationship (6) would work for a general force field as well as a general path or velocity function of the test body. Considering such generality, the above relations between the forces in the two frames may be declared to be valid for a physical problem involving any possible force field and any possible motion of the test body.

A force applied on a given body is meant to be an agent to produce change in motion of the body, as time passes. Accordingly, the amount of the applied force $\bar{F}$ may be defined as the time-derivative of certain physical quantity, called the momentum $\bar{p}$, associated with the body in general motion. The momentum and force are vector quantities, representing the directed, vector nature of the motion and its change. Accordingly, the component of the force vector in any given direction is equal to the time derivative of the component of the momentum vector in the particular direction. For mathematical generality, the time variation of the momentum may be expressed in the form of a general momentum function dependent on the position $\bar{r}$, velocity $\bar{v}$, acceleration $\bar{a}$ and all higher-order time-derivatives of the velocity.

$$\bar{F}(t) \triangleq \frac{d\bar{p}(t)}{dt},\ \bar{F}'(t) \triangleq \frac{d\bar{p}'(t)}{dt'},$$
$$\bar{p}(t) = \bar{p}(\bar{r}, \bar{v}, \bar{a}, \bar{a}_1, \bar{a}_2, \ldots),\ \bar{p}'(t') = \bar{p}(\bar{r}', \bar{v}', \bar{a}', \bar{a}'_1, \bar{a}'_2, \ldots),$$
$$\bar{r} = x\hat{x} + y\hat{y} + z\hat{z},\ \bar{r}' = x'\hat{x} + y'\hat{y} + z'\hat{z},$$
$$\bar{v} = \frac{d\bar{r}}{dt} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z},\ \bar{v}' = \frac{d\bar{r}'}{dt'} = v'_x\hat{x} + v'_y\hat{y} + v'_z\hat{z},$$
$$\bar{a}_n = \frac{d^n\bar{a}}{dt^n} = \frac{d^{n+1}\bar{v}}{dt^{n+1}},\ \bar{a}_0 = \bar{a} = \frac{d\bar{v}}{dt},$$
$$\bar{a}'_n = \frac{d^n\bar{a}'}{dt'^n} = \frac{d^{n+1}\bar{v}'}{dt'^{(n+1)}},\ \bar{a}'_0 = \bar{a}' = \frac{d\bar{v}'}{dt'},$$
$$\bar{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z},\ \bar{F}' = F'_x\hat{x} + F'_y\hat{y} + F'_z\hat{z},$$
$$\bar{p} = p_x\hat{x} + p_y\hat{y} + p_z\hat{z},\ \bar{p}' = p'_x\hat{x} + p'_y\hat{y} + p'_z\hat{z}. \tag{7}$$

III. Dependence of the Momentum of a Body on its Motion, Derived from the Force-Transformation Relations

The specific dependence of the momentum on the parameters of motion of a body can be deduced from the fundamental force-transformation relations of (6), based on the space-time transformation relations of special relativity.

Consider first a simple case with a force $\bar{F} = F_z\hat{z}$ in the z direction, resulting in motion with changing position $\bar{r}(t) = z(t)\hat{z}$ and velocity $\bar{v}(t) = v_z(t)\hat{z}$ only along the z direction. Let the body at a given point be seen by a stationary observer (unprimed frame), and another observer (primed frame) moving with the same velocity $V = v_z$ as that of the body at the time of observation. We will derive results for this simple case of linear motion, based on the general force transformation relations (6). The results for the simple motion can be extended as well for a general motion.

A. Zero Force on a Stationary Body, and the Principle of an Inertial Frame

For the simple case with a motion along the z direction, consider the simplest situation of a given body in the primed frame, placed at a given location (at the origin $z' = 0$). Due to the "unbiased" natures of the reference frame and the surrounding free-space, which we assumed in the above analysis to begin with, the particular body is expected to naturally remain stationary at the specified location, with no intrinsic bias for its movement in any one way or another. And, this natural stationary state observed in the unbiased reference must be maintained without any assistance of force. Accordingly, any valid definition of force must be associated with this basic condition. In other words, the force must be defined such that it is zero for a stationary body observed in an unbiased frame, as assumed in (7).

For the naturally stationary body at the origin $z' = 0$ of the unprimed frame, with no spatial motion as time progresses, there would be no time variation of the momentum $\bar{p}'$, and therefore the associated force $\bar{F}'$ be zero, as per the definition of (7). However, it may be realized that such a definition of force is to be applicable only in an unbiased frame. A stationary body in a biased frame would instead require a force in order to maintain its stationary position, in which case the force definition (7) would be clearly invalid.

Accordingly, an observer attached to the reference frame, would also naturally remain fixed at the origin of the frame as a stationary body, without any influence of force. In other words, the reference frame may be considered to be naturally "free-floating" in space. In this sense, the reference frame may be called an "inertial frame," in reference to the mechanical concept of inertia of the observer, with its natural tendency to maintain its fixed position in absence of any force. The "unbiased" nature of the frame, originally defined electromagnetically, where light is observed to propagate in straight lines with an invariant speed, is now explained to be equivalent to its free-floating, inertial nature, defined in mechanical terms. This is a fundamental understanding.

B. Position Independence of the Momentum

The above stationary body in the primed frame would be seen in the unprimed frame with a uniform velocity V in the z direction, having no acceleration or other time-derivatives of its velocity. As per the force transformation relation (6), the force F_z seen in the unprimed frame is required to be zero, given that the force F'_z in the primed frame is known to be zero. In other words, the body must not need any force in order to sustain a uniform linear motion, as observed in the unprimed frame.

$$\begin{gathered} F_z = \frac{dp_z}{dt} = \frac{dp_z}{dz}\frac{dz}{dt} = \frac{dp_z}{dz}V,\ \frac{dz}{dt} = v_z = V, \\ F'_z = \frac{dp'_z}{dt'} = \frac{dp'_z}{dz'}\frac{dz'}{dt'} = 0,\ \frac{dz'}{dt'} = v'_z = 0, \\ F_z = F'_z = 0,\ \frac{dp_z}{dz} = 0. \end{gathered} \tag{8}$$

Recall that the unprimed frame, like the primed frame, was also originally selected to be electromagnetically unbiased. Therefore, like the primed frame discussed earlier, the unprimed frame may also be considered an inertial frame in mechanical terms. Accordingly, the above conclusion regarding the body in uniform linear motion, specifically deduced in the unprimed frame, may be generally stated for validity in any inertial frame. That is, a body would maintain a uniform linear motion in an inertial frame, without any assistance of force.

Mathematically, the above conclusion (8), derived using (6) and (7), is equivalent to having the momentum p_z to be independent of the position z. This leaves the momentum p_z to be a function of its remaining variables - the velocity v_z, acceleration a_z, and other higher-order time-derivatives of the v_z (see (7)).

C. Independence of Momentum With All Time-Derivatives of Velocity, and Newton's First Law

The position independence of momentum established that a stationary body or a body with uniform velocity does not require a force. We would like to know other possible motions, if any, that also may not require force.

Consider a linear motion along the z axis, with a non-zero acceleration a_z in the unprimed frame, having the velocity $v_z = V$, and all time derivatives of the velocity v_z except the first derivative (or, acceleration $a_z = a_{0z}$), to be zero. The associated force component F'_z and F_z along the z direction, as seen in the two frames, defined in (7) as time-derivatives of the momentum p'_z or p_z in the respective frames, must satisfy the transform relations (6). This would require the momentum p'_z to be independent of all time derivatives of v'_z.

The space-time relations of the special relativity may be used to deduce the consequent relations for the velocity, as well as for its time-derivatives, in the primed frame with those in the unprimed frame. It may be shown that all time-derivatives of the velocity v'_z in the primed frame would be non-zero functions of $v_z = V \neq 0$, even though only the first time-derivative (acceleration $a_z = a_{0z}$) of the velocity v_z is non-zero in the unprimed frame. This is due to the non-linear nature of the relativistic relation (11) between the velocities v'_z and v_z in the two frames. Further, the time-derivatives of the velocity v'_z of increasingly higher order can be shown to be proportional to increasing exponents of the acceleration $a_z = a_{0z}$. The above conditions, applied with the force transformation relations (6) in the two frames, would lead to the requirement of the momentum p'_z to be independent of all time-derivatives of the velocity v'_z.

$$F'_z = \frac{dp'_z}{dt'} = \frac{dp'_z}{dz'} v'_z + \frac{dp'_z}{dv'_z} a'_z + \sum_{n=0}^{\infty} \frac{dp'_z}{da'_{nz}} a'_{(n+1)z} =$$
$$\frac{dp'_z}{dv'_z} \rho_0 a_z + \sum_{n=0}^{\infty} \frac{dp'_z}{da'_{nz}} \rho_{n+1} a_z^{n+2},$$
$$a'_{nz} = \frac{d^{n+1} v'_z}{dt'^{(n+1)}} = \rho_n a_z^{n+1};\ a'_{0z} = a'_z = \frac{dv'_z}{dt'},$$
$$\rho_n (v_z = V \neq 0) \neq 0,\ n \geq 0,$$
$$F_z = \frac{dp_z}{dt} = \frac{dp_z}{dv_z} a_z,\ a_z = a_{0z} = \frac{dv_z}{dt},$$
$$a_{nz} = \frac{d^{n+1} v_z}{dt^{n+1}} = 0,\ n \geq 1,$$
$$F'_z = F_z,\ \frac{dp'_z}{da'_{nz}} = 0,\ n \geq 0. \quad (9)$$

In the above derivation, the F_z is expressed proportional to the a_z, with no dependence on higher exponents of the a_z. The F_z expression may be viewed as a power-series of the a_z, with only one term involving the first-exponent of the a_z. On the other hand, the F'_z is expressed as a power-series of the a_z, involving all exponents of the a_z. The expressions of F'_z and F_z must be equated, as required by the force transformation relations (6). This would require the individual terms in the power-series expressions of the F'_z and F_z, with different exponents of the a_z, to be equated. Given that the coefficients ρ_n can be shown to be non-zero for all $n \geq 0$, as discussed in the following section, the above process leads to requiring the momentum p'_z to be independent of all time-derivatives of the velocity v'_z.

The above process maybe similarly repeated for a linear motion along the z axis with a non-zero acceleration a_{nz}, $n = N > 0$, in the unprimed frame, having the velocity $v_z = V$, and all other time derivatives of the velocity v_z except the $(N+1)-$th derivative (or acceleration a_{Nz}), to be zero. In this case, it may be shown by extending (11,12), that the n-th time-derivative of the velocity v'_z in the primed frame would be zero for all orders $n < (N+1)$, but would be non-zero functions of $v_z = V \neq 0$ for all orders $n \geq (N+1)$, even though only the $(N+1)-$th time-derivative (acceleration a_{Nz}) of the velocity v_z is non-zero in the unprimed frame. Further, the $n-$th time-derivatives of the velocity v'_z of increasingly higher order $n \geq (N+1)$ can be shown to be proportional to increasing exponents of the acceleration a_{Nz}. The above conditions, together with the primary result (9) for $N = 0$, applied with the force transformation relations (6) in the two frames, would lead to requiring the momentum p_z to be independent of all time-derivatives of the velocity v_z.

Like the position independence of the momentum deduced earlier, the independence of the momentum with all time-derivatives of the velocity is also a significant deduction from the electromagnetic theory. This leaves the momentum to be dependent only upon the velocity. Using the definition of force in (7), this means that a non-zero force would be required only when the velocity of a massive body is changed. In other words, a stationary body would remain stationary, and a body in uniform motion would maintain the uniform motion, without any force, whereas an accelerating body would certainly require a non-zero force. However, any change in the acceleration would not require any additional force. This is Newton's first law of motion, although the first law does not specify that higher-order time derivatives of velocity beyond the first derivative (acceleration) do not require additional force. This aspect is implied only through Newton's second law, to be derived in the following.

D. Functional Dependence of the Momentum with Velocity

As explained above, the momentum function p_z is left with the velocity v_z as its only valid variable. The functional expression of p_z with the variable v_z can be deduced from the above result (9), by using the expression of ρ_0 derived from the space-time relations of special relativity.

$$F_z = F'_z,\ \frac{dp_z}{dv_z} = \rho_0 \frac{dp'_z}{dv'_z} = \frac{1}{\alpha^3} \frac{dp'_z}{dv'_z} = \frac{1}{(1 - \frac{v_z^2}{c^2})^{3/2}} \frac{dp'_z}{dv'_z},$$
$$p_z(v_z) = \frac{m_0 v_z}{(1 - \frac{v_z^2}{c^2})^{1/2}},\ m_0 = \left.\frac{dp'_z}{dv'_z}\right|_{v'_z=0} = \left.\frac{dp_z}{dv_z}\right|_{v_z=0}, \quad (10)$$

$$x' = x,\ y' = y,\ z' = \frac{z - Vt}{\alpha},\ t' = \frac{t - zV/c^2}{\alpha},$$
$$v'_x = \frac{v_x \alpha}{(1 - v_z V/c^2)},\ v'_y = \frac{v_y \alpha}{(1 - v_z V/c^2)},\ v'_z = \frac{v_z - V}{(1 - v_z V/c^2)},$$
$$dt' = \alpha(dt),\ dv'_x = \frac{dv_x}{\alpha},\ dv'_y = \frac{dv_y}{\alpha},\ dv'_z =$$
$$\frac{dv_z}{\alpha^2};\ v_z = V,\ v_x = v_y = v'_x = v'_y = v'_z = 0,$$
$$\frac{dv'_z}{dt'} = \rho_0 \frac{dv_z}{dt} = \alpha^{-3} \frac{dv_z}{dt};\ \rho_0 = \alpha^{-3}. \quad (11)$$

Relations between the higher order time-derivatives of the velocity v'_z in the primed frame and the acceleration in the unprimed frame can be similarly obtained by further differentiating the above relations between the velocities in the two frames. This would provide the expressions for all other ρ_n, $n > 0$, which are non-zero functions of $v_z = V$ as we needed in the above derivation (9).

$$a'_{nz} = \frac{d^{n+1} v'_z}{dt'^{(n+1)}} = \rho_n \left(\frac{dv_z}{dt}\right)^{n+1} = \rho_n a_z^{n+1},$$
$$\rho_n (v_z = V, v'_z = 0) = (2n+1)!! \left(\frac{V}{c^2}\right)^n \alpha^{-(n+3)},$$
$$(2n+1)!! = (2n+1)(2n-1)(2n-3) \cdots (1). \quad (12)$$

E. Expressions in the Small-Velocity Limit, and Newton's Second Law

Note that we have "derived" the expression (10) for the momentum component p_z, as a function of the velocity component v_z, starting from Maxwell's equations. In the limit of a small velocity, the expression of momentum in (10), and the associated force defined in (7), would take the form of Newton's second law. In the small-velocity limit, the momentum is shown to be proportional to velocity, with the constant of proportionality m_0 recognized as the rest mass. The force, defined in (7) as the time-derivative of the momentum, is therefore equal to the rest mass times the acceleration (time-derivative of the velocity), in the small-velocity limit. This is the most basic mechanical formula constituting Newton's second law [11] of motion for a body of constant rest mass m_0. Accordingly, we have "derived" Newton's second law of motion, from the electromagnetic theory based on Maxwell's equations. This is a significant development.

$$
\begin{gathered}
p_z(v_z) = \frac{m_0 v_z}{(1-\frac{v_z^2}{c^2})^{1/2}},\ p_z(v_z \to 0,\ v_z << c) = m_0 v_z, \\
F_z = \frac{dp_z}{dt} = \frac{d(m_0 v_z)}{dt} = m_0 \frac{dv_z}{dt} = m_0 a_z;\ v_z << c, \\
p_z(v_z) = m(v_z) v_z = \frac{m_0 v_z}{(1-\frac{v_z^2}{c^2})^{1/2}}, \\
m(v_z) = \frac{m_0}{(1-\frac{v_z^2}{c^2})^{1/2}}.
\end{gathered}
\tag{13}
$$

F. Relativistic Mass

In consistency with the momentum expression in the small-velocity limit, which we deduced above to be the product of the velocity v_z and the rest mass m_0, the general expression of the momentum $p_z(v_z)$ in (10) may also be expressed as a product of the velocity v_z and a general mass term $m(v_z)$. This new mass term, as shown in (13), is a function $m(v_z)$ of the velocity v_z of motion, unlike a fixed mass m_0 assumed in Newton's law. Further, this velocity-dependent mass, referred to as the relativistic mass, would increase indefinitely as the velocity v_z increases approaching the speed of light c. We have succeeded to derive the required velocity function of the relativistic mass, directly from Maxwell's equations.

The same dependence of the relativistic mass, as a function of the velocity v_z for the linear motion along z, is extended in the following to apply as well for a general motion along any arbitrary path, where v_z may be substituted with the magnitude v of the general velocity vector $\bar{v}$.

G. Generalization to Motion in the Three Dimensions

The above derivations assumed a simple linear motion along the z axis. The direction of motion along the z axis for the simple motion is an arbitrary choice. Similar results would work as well for a motion along any general direction. Accordingly, the result in (10) may be used to relate the magnitude of a general momentum function $\bar{p}(\bar{v})$ to the magnitude of the velocity vector $\bar{v}$. The small-velocity limit for the general case would be an extension of the equivalent limit (13) for the simple case. Further, consistent with the small-velocity limit, the general momentum vector is also directed along the velocity $\bar{v}$.

The momentum vector in the general direction can then be decomposed into its individual components p_x, p_y and p_z in the x, y and z directions, respectively.

$$
\begin{gathered}
\bar{p}(\bar{v}) = m(v)\bar{v},\ p(v) = m(v)v = \frac{m_0 v}{(1-\frac{v^2}{c^2})^{1/2}}, \\
m(v) = \frac{m_0}{(1-\frac{v^2}{c^2})^{1/2}}, \\
\bar{p}(\bar{v}) = m\bar{v} = m(v_x\hat{x} + v_y\hat{y} + v_z\hat{z}) = \frac{m_0(v_x\hat{x}+v_y\hat{y}+v_z\hat{z})}{(1-\frac{v^2}{c^2})^{1/2}}, \\
p_x = \frac{m_0 v_x}{(1-v^2/c^2)^{1/2}},\ p_y = \frac{m_0 v_y}{(1-v^2/c^2)^{1/2}}, \\
p_z = \frac{m_0 v_z}{(1-v^2/c^2)^{1/2}},\ v^2 = v_x^2 + v_y^2 + v_z^2,
\end{gathered}
\tag{14}
$$

$$
\begin{gathered}
p_z(v_z) = m_0 v_z,\ p_x(v_x) = m_0 v_x,\ p_y(v_y) = \\
m_0 v_y;\ v_x, v_y, v_z << c, \\
\bar{p}(\bar{v}) = p_x\hat{x} + p_y\hat{y} + p_z\hat{z} = \\
m_0(v_x\hat{x} + v_y\hat{y} + v_z\hat{z}) = m_0\bar{v};\ v << c, \\
m_0 = \left.\frac{dp_x}{dv_x}\right|_{v=0} = \left.\frac{dp_y}{dv_y}\right|_{v=0} = \left.\frac{dp_z}{dv_z}\right|_{v=0} = \left.\frac{dp}{dv}\right|_{v=0}.
\end{gathered}
\tag{15}
$$

We derived the above general expressions (14) for the momentum components, starting with a simple motion along the z direction. This derivation explicitly satisfied the required transform relationship (6) only between the force components F_z and F'_z, for the simple case, which led to relating the momentum p_z to the velocity v_z in (10), also for the simple case. The results were then generalized to (14) for velocity along an arbitrary direction by reorienting the velocity axis, from which the expressions for the individual momentum components for the general case were decomposed. The required transform relationships (6) between all three force components, for the general case, were expected to be implicitly satisfied by the final expressions of the momentum components of (14), through the followed process of generalization and coordinate reorientation. This is a theoretically simple, valid approach.

However, the expressions for the individual momentum components in (14) for the general case may also be explicitly verified to satisfy the required transform relationships (6) for all three force components. This is possible by first differentiating the momentum components in (14), expressed in the two frames, with respect to the individual velocity components. The results are then used in steps similar to (9) to relate respective force and momentum components using the space-time relationships (11), leading to verification of the force transform relations (6).

H. Energy Expression Derived From the Force and Momentum

Now, let us derive the expression for the energy of a moving body, adopting the conventional definition of energy used in the Newtonian mechanics. The derivation would make use of the momentum expression we established above. At this point, we do not question any reasoning behind the choice of the definition of energy. The definition is likewise introduced in the Newtonian mechanics, without any justification for its special form. It is simply expected without "proof" that the conventional energy definition would provide a useful conserved quantity, which is one of the foundational principles in the Newtonian mechanics. The validity of definition of the energy used, and the proof of its conserved nature, will be addressed in the section IV-B.

$$dW = \bar{F}\cdot\overline{ds} = \frac{d\bar{p}}{dt}\cdot\overline{ds} = d\bar{p}\cdot\bar{v} = v_x dp_x + v_y dp_y + v_z dp_z \tag{16}$$
$$= \frac{m_0(v_x dv_x + v_y dv_y + v_z dv_z)}{(1-v^2/c^2)^{3/2}} = \frac{(m_0/2)d(v^2)}{(1-v^2/c^2)^{3/2}}$$
$$= d\left(\frac{m_0}{(1-v^2/c^2)^{1/2}}\right)c^2 = (dm)c^2,$$
$$W = \int \frac{(m_0/2)d(v^2)}{(1-v^2/c^2)^{3/2}} = \frac{m_0c^2}{(1-v^2/c^2)^{1/2}} = mc^2. \tag{17}$$

The incremental energy dW is shown to be equal to the incremental mass dm times c^2, which now establishes the basic mass-energy relationship $W = mc^2$, derived directly from Maxwell's equations. Accordingly, all forms of energy and mass may be treated in equivalent terms using (17). This would allow mechanical treatment of general systems which may include conventional massive bodies as well as electromagnetic radiation. Any exchange of energy and momentum between the conventional bodies and the radiation may be implemented using concepts of electromagnetic field-mass/energy and field-momentum [12].

IV. Energy and Momentum Conservation in a Closed System

A. Momentum-Energy Transformation Relations in the Two Frames

Let us express the momentums (14) in two inertial frames, moving with velocity V with respect to each other along the z direction. This is possible using the space-time relations (11) of special relativity. The momentums in the unprimed frame can now be linearly related with those in the primed frame in terms of the energy expression of (17) in the primed frame. Using symmetry of results between the two frames, similar relationship between the momentums in the two frames and the energy in the unprimed frame can be obtained by interchanging the primed and unprimed variables, and replacing V with $-V$.

$$\bar{p} = \frac{m_0\bar{v}}{(1-\frac{v^2}{c^2})^{1/2}},\ \bar{p}' = \frac{m_0\bar{v}'}{(1-\frac{v'^2}{c^2})^{1/2}},$$
$$p_x = p'_x,\ p_y = p'_y,\ p_z = \frac{p'_z + m'V}{\alpha} = \frac{p'_z + W'V/c^2}{\alpha},$$
$$p'_z = \frac{p_z - mV}{\alpha} = \frac{p_z - WV/c^2}{\alpha}, \tag{18}$$

$$v_x = \frac{v'_x\alpha}{1+v'_zV/c^2},\ v_y = \frac{v'_y\alpha}{1+v'_zV/c^2},\ v_z = \frac{v'_z+V}{1+v'_zV/c^2},$$
$$(1-\frac{v^2}{c^2}) = (1-\frac{v_x^2+v_y^2+v_z^2}{c^2}) =$$
$$\frac{\alpha^2}{(1+v'_zV/c^2)^2}(1-\frac{{v'_x}^2+{v'_y}^2+{v'_z}^2}{c^2}) =$$
$$\frac{\alpha^2}{(1+v'_zV/c^2)^2}(1-\frac{v'^2}{c^2}),\ \alpha = (1-V^2/c^2)^{1/2}. \tag{19}$$

The energy expression of (17) in the unprimed frame can also be similarly related to that in the primed frame in terms of the z directed momentum in the primed frame. Similar relationship in terms of the momentum component in the unprimed frame can also be obtained by interchanging primed and unprimed variables and replacing V by $-V$. This is by symmetry of results between the two frames.

$$W = mc^2 = \frac{m_0c^2}{(1-\frac{v^2}{c^2})^{1/2}},\ W' = m'c^2 = \frac{m_0c^2}{(1-\frac{v'^2}{c^2})^{1/2}},$$
$$W = \frac{m_0c^2(1+\frac{v'_zV}{c^2})}{\alpha(1-\frac{v'^2}{c^2})^{1/2}} = \frac{m'c^2(1+\frac{v'_zV}{c^2})}{\alpha} = \frac{(W'+p'_zV)}{\alpha},$$
$$W' = \frac{(W-p_zV)}{\alpha}. \tag{20}$$

B. Concept of Energy as a Conserved Parameter

The transformation relation (6) simply requires that a given body with no external force, as observed in any one inertial frame, would also be observed with no external force in any other inertial frame. However, the body could consist of an arbitrary number of internal parts, having general forces of interaction and relative motion between them, but with the sum of all the forces equal to zero. Therefore, the above simple condition (6) needs to be consistently expanded to require that any such set of zero-sum forces to be as well measured with the same zero-sum condition in all the inertial frames. This would be independent of the constitution of the individual parts and the nature of their interacting forces.

Now, consider a system with all its individual forces added to zero, as seen by an inertial frame (primed frame). As discussed above, the system would also be seen with the zero total force in the unprimed inertial frame, which is moving with a uniform velocity V along the $-z$ axis with respect to the primed frame. Although the total force is zero, the system is free to undergo any change of state of its individual parts, produced due to the forces of interactions between the parts. This would be characterized by change of velocity, momentum and energy of the individual parts.

Based on the basic definition of force (7), having the total force zero would mean that the total change of momentum $\Delta\bar{p}$ and $\Delta\bar{p}'$ of all constituent parts of the system, over any time interval, would also be zero. Given the required momentum expressions of (14) and their relationships (18) in the two frames, it would additionally require the energy of (17) to be conserved ($\Delta W' = \Delta W = 0$).

$$\sum \bar{F}' = 0,\ \Delta\bar{p}' = 0;\ \sum\bar{F} = 0,\ \Delta\bar{p} = 0,$$
$$\Delta p_x = \Delta p'_x = 0,\ \Delta p_y = \Delta p'_y = 0,\ \Delta p_z = \Delta p'_z = 0,$$
$$\Delta p_z = \frac{\Delta p'_z}{\alpha} + \frac{(\Delta W')V/c^2}{\alpha} = 0,\ \Delta W' = 0;$$
$$\Delta p'_z = \frac{\Delta p_z}{\alpha} - \frac{(\Delta W)V/c^2}{\alpha} = 0,\ \Delta W = 0. \tag{21}$$

This is a significant result, which proves that the energy, as conventionally defined in the Newtonian mechanics using the incremental form (16), is in fact conserved in a system with zero total force. The conservation of energy for a zero-force system no longer needs to be accepted as a foundational mechanical principle, without proof, simply based on theoretical and observational success of the principle. Conversely, if we are looking for a useful scalar parameter to be conserved in a system with zero total force, then the conventional definition of energy (16) (written in incremental form) is now theoretically proved to be one such conserved quantity. Other possible expressions of the energy one might think of may not succeed to maintain the desired energy conservation, consistent with the force transform relations (6) and special relativity.

C. Momentum Conservation in a Closed System, and Newton's Third Law

Now consider a system physically contained inside a definite volume of space, identified with an entirely closed surface boundary, with no interaction with the external free space across the boundary surface. And, this is the case as seen by any inertial observer (primed and unprimed frames). The non-interaction condition across the closed boundary may be characterized in terms of no flow of energy, or its mass equivalent as per (17), across any part of the boundary. Accordingly, the total energy or equivalent mass would remain constant inside the system ($\Delta W = \Delta W' = 0$). This assumes that no energy or mass can spontaneously appear or disappear at any locaation inside the closed system, without a definite trace of flow of the energy occurring across the closed boundary surface.

Under the above condition, it may be shown from (20) that the total momentum p_z, p'_z inside the closed system in each frame would remain unchanged ($\Delta p'_z = 0 = \Delta p_z$). The choice of the z direction is arbitrary in the above discussion of the energy conservation in the closed system. Therefore, component of the momentum along any direction, or equivalently the total momentum vector, would remain unchanged ($\Delta \bar{p} = 0 = \Delta \bar{p}'$). This is the principle of momentum conservation in a closed system.

Further, because the total momentum would remain unchanged, the total of all forces in the closed system would be zero, as per the definition of force in (7). Equivalently, every force in the closed system would be balanced by a counter reaction force that is equal in magnitude but oppositely directed. This is Newton's third law of motion. We have now proved Newton's third law from the electromagnetic theory and special relativity.

$$\begin{gathered}\Delta W = \Delta W' = 0,\ \Delta W = \frac{\Delta W'}{\alpha} + \frac{V \Delta p'_z}{\alpha} = 0,\ \Delta p'_z = 0,\\ \Delta W' = \frac{\Delta W}{\alpha} - \frac{V \Delta p_z}{\alpha} = 0,\ \Delta p_z = 0,\ \Delta \bar{p}' = \Delta \bar{p} = 0,\\ \Delta \bar{p}' = 0,\ \sum \bar{F}' = 0;\ \Delta \bar{p} = 0,\ \sum \bar{F} = 0. \qquad (22)\end{gathered}$$

It may be noted, that the two results (21) and (22) are mutually complementary to each other. That is, the condition of zero total force, or equivalently the conservation of total momentum, would require the total energy to be conserved. And conversely, the conservation of the total energy would require the total momentum to be conserved, as well as the total force to be zero.

D. Conservation of Total Energy and Momentum in the Universe

Consider the entire universe, which in principle contains all physical space there is, and therefore does not have any other external space across which any energy or mass can be exchanged with. Accordingly, the entire universe is in principle a closed system. Therefore, as per the above deductions, the total momentum as well the energy in the entire universe must be conserved, with every possible force in the universe balanced by an opposing force of equal magnitude, at all times. This is the universal principle of conservation of energy and momentum.

V. Discussion: Basic Concepts of Electro-Magnetic Charge and Space-Time Supersede Newton's Laws

We have succeeded to derive all basic mechanical principles of Newton's laws from Maxwell's equations. Further, we known that Maxwell's equation can be established [1] directly from the basic principles of electric and magnetic charge and their invariance, using only the space-time concepts of the special relativity. Accordingly, the principles of the electric and magnetic charge and the space-time relativistic transformation relations, constitute a complete set of basic rules or laws to govern the electrical *as well as* mechanical characteristics of the nature.

In other words, we have established that the basic concepts of electro-magnetic charge and space-time are complete, which "supersede" all mechanical principles making them redundant. This interpretation may at first seem counter-intuitive. This is because we come to be educated about the mechanical principles first, which are more instinctively experienced as we come in contact with our physical world on a daily basis. Based on the mechanical principles, we are then gradually educated about more advanced principles of the electrical or magnetic forces, and their associated fields. This learning process leads to a common impression that the mechanical principles that are academically established first must be independent of, and therefore fundamentally supersede, the more advanced electromagnetic principles we learn later on. As we now understand, this impression is misleading.

The mechanical principles are introduced based only on our common-sense faith in Newton's laws without any objective "proof", by essentially relying on our everyday experiences and experimental observations. Although the electromagnetic principles, in the form of Maxwell's equations, are established later based upon these mechanical principles, with deeper insights we come to understand that the electromagnetic principles could be more fundamental. The governing basic principles of invariant electric and magnetic charges are recognized to be complete and minimal, and the underlying mechanical concepts will now have to be constrained in order to be consistent with the fundamental electromagnetic concepts. These constraints provide the desired "proof" or explanation for Newton's laws and the associated momentum and energy conservation, which no longer have to be accepted only on faith in their agreement with experimental observations and common-sense experiences. This is a significant, new scientific view.

REFERENCES

[1] N. Das. Introducing Maxwell's Equations as Derived from Simple Relativity Transformation Principles . *IEEE Antennas and Propagation Magazine*, 63(6):122–136, December 2021.
[2] N. Das. Deriving Maxwell's Equations from First Principles of Relativistic Charge Invariance and Space-Time Relations. *2021 IEEE International Symposium on Antennas and Propagation (APSURSI)*, December 2021.
[3] N. Das. *Introduction to Electromagnetic Fields, Radiation and Antenna Theory*. UEGM Publishing, ISBN:978-1734063004, 2021.
[4] James Clark Maxwell. *A Treatise on Electricity and Magnetism, Vol. I and II (Reprint from 1873)*. Dover Publications, 2007.
[5] Albert Einstein and Anna Beck (English Translator). *The Collected Papers of Albert Einstein, Volume 2: The Swiss Years: Writings 1900-1909 (see Documents 23 and 24)*. Princeton University Press, 1989.
[6] Ray Skinner. *Relativity for Scientists and Engineers*. Dover Publications, Inc., 1982.
[7] N. N. Rao. *Elements of Engineering Electromagnetics, Ch.4,8*. Prentice Hall, New Jersey, 2000.
[8] R. F. Harrington. *Time Harmonic Electromagnetic Fields, Ch.2,3*. John Wiley and Sons; IEEE Press, 2001.
[9] L. B. Felsen and N. Marcuvitz. *Radiation and Scattering of Waves (Ch.4)*. Prentice Hall, 1973.
[10] Richarrd P. Feynman, Robert B. Leighton, and Mathew Sands. *Lectures on Physics, Vol.II, Ch.25,26*. Addision Wesley, 1964.
[11] Sir Isaac Newton. *Principia: Mathematical Principles of Natural Philosophy. I. B. Cohen, A. Whitman and J. Budenz, English Translators from 1726 Original*. University of California Press, 1999.
[12] Richarrd P. Feynman, Robert B. Leighton, and Mathew Sands. *Lectures on Physics, Vol.II, Ch.27,28*. Addision Wesley, 1964.