Figure 1 is a simple model for a magnetic cartridge (either moving magnet or moving coil) connected to an RIAA-corrected amplifier and a weighting filter. The voltage-controlled voltage source with gain *H*(*s*) represents both the RIAA-correction and the noise weighting. The noise sources represent the noise of the amplifier. The thermal noise of the cartridge and record surface noise are not included.

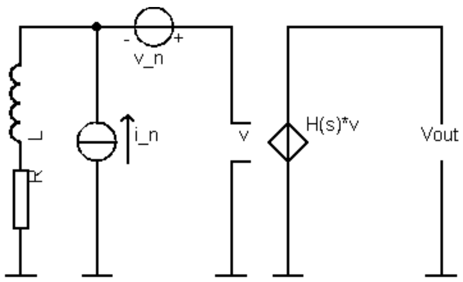

Figure 1 Simple model for a cartridge connected to an RIAA-corrected amplifier and noise weighting filter

The LR series network models the cartridge impedance. This is a reasonable model for the magnitude of the cartridge impedance but not necessarily for its phase, as typical moving-magnet cartridges have an effective series resistance that increases substantially with frequency. The calculations in this section, however, only depend on the magnitude of the impedance, which matches fairly well with an ideal LR series network.

The effect of cartridge loading is neglected. Especially for moving-magnet cartridges, the load has a significant impact on the frequency response above 10 kHz. As these high audio frequencies are much attenuated by the RIAA weighting, we shall assume that neglecting the cartridge loading is acceptable for noise calculations. (Simulations show that this is indeed the case.)

In any practical amplifier, the equivalent input noise voltage and current are correlated to some extent because any noise source after the first amplifying device (including its own collector, drain or anode noise) gives correlated contributions to the equivalent input noise voltage and current. Still, the **dominant** sources of equivalent input noise current and voltage in a gramophone amplifier are usually independent physical effects, so the correlation will generally be small enough to be neglected.

Under these assumptions, the power spectral density of the total noise at the output of the amplifier can be expressed as:

$$S(v_{out}) = S(v_n)\left|H(j2\pi f)\right|^2 + S(i_n)\left(4\pi^2 f^2 L^2 + R^2\right)\left|H(j2\pi f)\right|^2 \qquad \textbf{(1)}$$

$S(v_{out})$ is the power spectral density of the noise voltage after RIAA correction and weighting, $S(v_n)$ is the power spectral density of the equivalent input noise voltage and $S(i_n)$ is the power spectral density of the equivalent input noise current. $\left|H(j2\pi f)\right|$ is simply the combined gain of the RIAA-corrected amplifier and the noise weighting filter at frequency f.

The power spectral density of a noise voltage is by definition the squared voltage per unit bandwidth, SI unit V^2/Hz. The power spectral density of a current is the squared current per unit bandwidth, SI unit A^2/Hz. I very much dislike the way signal theorists use the word "power" to refer to quantities expressed in V^2 or A^2 instead of W, but there is nothing I can do about it.

The mean-square value (square of the RMS value) of the total noise over the band of interest after amplification and weighting is:

$$\int_{f_{\min}}^{f_{\max}} S(v_{out})\,df = \int_{f_{\min}}^{f_{\max}} S(v_n)\left|H(j2\pi f)\right|^2 df + \int_{f_{\min}}^{f_{\max}} S(i_n)\left(4\pi^2 f^2 L^2 + R^2\right)\left|H(j2\pi f)\right|^2 df \qquad \textbf{(2)}$$

$f_{\min}$ and $f_{\max}$ are the lower and upper limits of the band of interest.

When $S(v_n)$ and $S(i_n)$ are white over the band of interest, equation **(2)** can be rewritten as:

$$\int_{f_{\min}}^{f_{\max}} S(v_{out})\,df = S(v_n)\int_{f_{\min}}^{f_{\max}} \left|H(j2\pi f)\right|^2 df + S(i_n)\int_{f_{\min}}^{f_{\max}} \left(4\pi^2 f^2 L^2 + R^2\right)\left|H(j2\pi f)\right|^2 df \qquad \textbf{(3)}$$

The noise optimisation process comes down to taking design measures to tweak $S(v_n)$ and $S(i_n)$ such that the total noise becomes minimal. Substituting $f=f_1$ into equation **(1)** and comparing the right-hand sides of equations **(1)** and **(3)**, it is clear that both equations result in the same noise optimum when the ratio of the weighting factors of $S(v_n)$ and $S(i_n)$ is the same. That is,

$$\frac{\left(4\pi^2 f_1^2 L^2 + R^2\right)\left|H(j2\pi f_1)\right|^2}{\left|H(j2\pi f_1)\right|^2} = \frac{\int_{f_{\min}}^{f_{\max}} \left(4\pi^2 f^2 L^2 + R^2\right)\left|H(j2\pi f)\right|^2 df}{\int_{f_{\min}}^{f_{\max}} \left|H(j2\pi f)\right|^2 df} \qquad \textbf{(4)}$$

This can be written as:

$$R^2 + 4\pi^2 f_1^2 L^2 = R^2 + 4\pi^2 L^2 \frac{\int_{f_{\min}}^{f_{\max}} f^2\left|H(j2\pi f)\right|^2 df}{\int_{f_{\min}}^{f_{\max}} \left|H(j2\pi f)\right|^2 df} \qquad \textbf{(5)}$$

Both sides are equal when

$$f_1 = \pm\sqrt{\frac{\int_{f_{min}}^{f_{max}} f^2 \left|H\left(j2\pi f\right)\right|^2 df}{\int_{f_{min}}^{f_{max}} \left|H\left(j2\pi f\right)\right|^2 df}} \qquad \textbf{(6)}$$

The integrals on the right-hand side of equation **(6)** can readily be numerically evaluated, for example with a circuit simulator such as LTspice, using a circuit model for the various filters and a .measure-statement to do the integration. When you do this for IEC-modified RIAA correction and A-weighting and take 1 Hz and 1 MHz as f_{min} and f_{max}, the result is 3852 Hz.