# Damping: Loudspeakers In Series

**Dick Pierce**
**Professional Audio Development**
**Hanover, MA**
*"Debunking audio myths for over a fiftieth of a millenium"*

## 1 INTRODUCTION

An entire mythology surrounds the notion of damping in loudspeakers. We have separately treated the oft-cited and nearly useless "damping factor" specification in a separate article. Here, we discuss much of the myth surrounding the behavior of loudspeakers connected in series and the seemingly intuitive effect on damping tat results. Of late, this topic is relevant or several reasons.

First, we have seen a gain in use of multiple-woofer systems, exemplified by the popular, so-called "D'Appolito" configuration (often the woofers are connected in parallel, but there are applications where a series connection could be appropriate as well). Second, we have observed an unfortunate trend in consumer electronics, especially in home theater receivers. More and more, these receivers have skimped on power supply and/or output stage design and are quite unable to drive the lower impedances oft found in today's speakers. The question often arises whether speakers can be hooked in series, thus raising the impedance.

The answer often given to both of these scenarios is "Oh, no, you can't do that. The damping of each speaker will be severely reduced because of the series impedance of the other speaker!

As you might have guessed, we're going to show why this is wrong. We'll do this by taking an analytical approach, and test our analysis by actually measuring actual systems.

## 2 WHAT IS DAMPING?

The term "damping" is has a very specific and unambiguous definition: technically, it is a measure of how quickly energy is removed from a resonant system. This definition stands despite attempt to co-opt the term for otherwise imprecise and often incorrect uses.[1] It is a measure of how quickly a resonant or oscillatory system is brought under control by removing energy that would otherwise keep the resonance going.

Energy is stored in reactive elements. These include masses and compliances (or springs) in the mechanical world, and inductances and capacitances in the electrical world. Mechanically, energy of a mass is the kinetic energy due to the motion of a mass. The kinetic energy of a moving mass equal to the mass times the velocity squared. In a spring or compliance, the energy is stored as potential energy in the compression or extension. The potential energy is equal to the spring constant times the compression or extension squared).

Electrically, kinetic energy is in the magnetic field around an inductor created by the current flowing through the inductor. The energy is equal to the inductance times the current squared. In a capacitor, it's the potential energy in the electric field caused by the impressed voltage on the plates of a capacitor, equal to the capacitance times the voltage squared.

Energy is removed through loss mechanism, such as frictional losses in the mechanical domain or ohmic losses in the electrical. These losses convert energy to heat, and once that happens, the energy is no longer available.

In loudspeakers, there is a direct measure of the ratio of energy stored to energy lost, and that is the

---

[1] One often encounters hi-fi accessories, for example, that utilize "mass damping" to control resonances. That adding a mass will change a resonant system is hardly in dispute, that it 'damps" a resonance is altogether a different and quite incorrect claim.

so-called "$Q$" factor. And we find that in most loudspeakers, there are two predominant loss mechanisms, each with there own $Q$ measurement. The $Q$ due to mechanical losses is designated as $Q_M$, while that for electrical losses is designated as $Q_E$ [2]. We can calculate these $Q$ factors knowing the energy storage and losses mechanisms involved

The mechanical $Q_M$ results from the energy storage in the moving mass of the cone and the frictional losses in the suspension. It is calculated thus:

$$Q_M = 2\pi F \frac{M_M}{R_M}$$

where $F$ is the resonant frequency of the system in Hertz, $M_M$ is the mechanical mass of the system and $R_M$ is the frictional loss in the system. Increase the mass, and more energy is stored in the system. Increase the friction, and more energy is dissipated from the system

The electrical $Q_E$ of the system results, again, from the energy stored in the moving mass, but now dissipated by the electrical resistance in the system. It is calculated as:

$$Q_E = 2\pi F \frac{M_M}{B^2 l^2} R_E$$

Here, $B$ represents the magnetic flux density in the voice coil gap, $l$ is the length of wire in the magnetic field, and $R_E$ represents the DC resistance of the voice coil.

We can also speak, of course, of the total Q, or $Q_T$ of the system due to the combined mechanical and electrical damping, and it's calculated by the familiar formula:

$$Q_T = \frac{Q_M \times Q_E}{Q_M + Q_E}$$

The lower the Q, the more damped the system is. The higher the Q, the less damped.

## 3 THE INTUITIVE PREMISE

Here's the claim: putting speakers in series is a bad idea because the series resistance of one speaker destroys the damping of the other. Why, even the equation for electrical $Q_E$ above says so: having two voice coils in series doubles the voice coil resistance (assuming the voice coils are the same, for simplicity). So it must logically follow that adding two speakers in series must severely destroy the damping, and the equation above shows that it should double.

It makes intuitive sense. It even seems to appeal to technical authority. But will it stand up to analytical and empirical scrutiny? Is this, perchance, another widely held belief that might not be so?

## 4 ANALYSIS

We'll consider the case where we are connecting two of the same thing in series, be it two identical woofers in an enclosure or two identical speakers in series. We're using identical woofers or systems to make the analysis simpler[3]

Let's first look at the effects of two speakers in connected together in the mechanical domain. It might seem obvious, but since no electrical effects are considered in the mechanical domain, it makes no difference on the mechanical damping or $Q_M$ whether two speakers or two woofers are connected in series or parallel. Indeed, it doesn't even make any difference if they aren't connected at all electrically.

In the mechanical case, we have doubled the moving mass $M_M$ to $2\,M_M$ (we have twice as many cones, after all), but we have also double the amount of frictional loss from $R_M$ to $2\,R_M$ as well (twice as many surrounds and spiders, too). Plugging

[2] There are other loss mechanisms, most notable the acoustical losses. However, for direct radiator loudspeakers, these loss mechanisms are quit insignificant, most often representing less than 1% of the total losses. Not coincidentally, this number is not too dissimilar form the acoustical efficiency of such speakers as well, because in order to produce sound, real work has to be done, and it is the work done into these acoustical "losses" that actually is the produced sound. Eliminate the acoustical loss, say by taking away the radiation load by putting the speaker in a vacuum, and you've eliminated the sound. Not an entirely useful exercise for something like a loud "speaker."

[3] While the case of non-identical drivers or systems is more complicated, the general principles apply, though there are confounding factors such as frequency-dependent attenuation resulting from different frequency-dependent impedances.

these changes into the equation for mechanical damping, we find:

$$Q_M = 2\pi\ F\ \frac{2\ M_M}{2\ R_M}$$

We can simplify: $\frac{2}{2}$ in the equation is equal to 1, and we thus end up with:

$$Q_M = 2\pi\ F\ \frac{M_M}{R_M}$$

This equation, describing the effect on $Q_M$ of connecting two speakers in series, is precisely the same equation for the case of a single speaker by itself.

Now, let's look at the electrical damping or $Q_E$. Here, we have, indeed, doubled the resistance $R_E$ to $2\ R_E$ (the voice coils are hooked in series), but we have also doubled the moving mass from $M_M$ to $2\ M_M$ as well *and* we've also double the length of the voice coil wire from $l$ to $2\ l$ sitting in the magnetic field as well. Now, let's plug all those factors of two into the equation for electrical Q:

$$Q_E = 2\pi\ F\ \frac{2 M_M}{B^2\ (2l)^2}\ 2 R_E$$

The next step expands and combines terms:

$$Q_E = 2\pi\ F\ \frac{2 M_M}{B^2\ 4l^2}\ 2 R_E$$

In another step, let's accumulate all these new factors (2 from the doubling of mass, 2 from the doubling of the voice coil resistance, and 4 from the *square* of the doubling of the length of the wire) together for the numerator and the denominator:

$$Q_E = 2\pi\ F \frac{4}{4}\ \frac{M_M}{(Bl)^2}\ R_E$$

And, since the fraction $\frac{4}{4}$ is equal to 1, we can reduce this equation to:

$$Q_E = 2\pi\ F\ \frac{M_M}{B^2\ l^2}\ R_E$$

Again, this result, showing the electrical $Q_E$ of two speakers connected in series, is identical to the case of just a single speaker.

Thus the analysis clearly shows that the damping is not severely compromised by connecting two systems or drivers in parallel, because the measures of damping, $Q_M$ and $Q_E$, remains the same for both the mechanical and electrical domains, and thus the total $Q_T$ also remains the same. *Q.E.D.*

## 5 ATTENUATION OF 'BACK-EMF'

One claim that is made is that the so-called 'back-EMF' caused by the driver's motion and the subsequent motion of the voice coil through the magnetic field now sees twice the series resistance and thus can only be half as effective[4]. While, again, this seems to be intuitively correct, it's wrong on several counts. The previous analysis based on Q is, in and of itself, sufficient to dispel this error. However, we'll look at an analysis based on the back-EMF picture specifically.

The notion is that with twice the series resistance due to the second speaker, the amount of current resulting from the back-EMF is half as much, and thus contributes to only half as much electrical damping. Proponents of this view cite Ohms law as support. If the available 'damping current' in the normal case is:

---

[4] The concept of back-EMF, while rooted in physical reality, has proven to be a source of great confusion and myth all its own in the realm of loudspeakers. Special properties have been attributed to this phenomenon, some of them almost magical and supernatural. In fact, as seen by the amplifier, there is no difference, under any circumstances, between the behavior of a loudspeaker with a voice coil, a magnet, a mass and a compliance or a capacitor, an inductor and a resistor in parallel. To get further down to the physical behavior, there is no difference in the current that is generated, for example, dues to a voice coil moving through a magnetic field *vs.* a fixed inductor immersed in a time-varying magnetic field. In fact, the equations describing the position and velocity of the coil are identical to the equations describing the voltage and current in a resonant tank circuit, save for the difference in units (mechanical *vs.* electrical). The voice coil simply acts as a 'transformer" handily doing the conversion of mechanical to electrical units for us. Nothing more.
This suggests that some of the bizarre behaviors of otherwise impossible current flows in and out of speakers, for example, may not exist as some proponents have claimed. This may be one of the reasons why these same proponents have been either unwilling or unable to show a single verifiable instance of such behaviors.

$$I_{DAMPING} = \frac{E_{EMF}}{R_E}$$

then, with two speakers in series, and thus *twice* the series resistance, the current *must* be half as much:

$$I_{DAMPING} = \frac{E_{EMF}}{2\ R_E}$$

and, thus, the electrical damping that results *must* be half as much.

Again, as simple and obvious as such an assertion and its supporting equations must seem, it ignores one important and very glaring fact: the speakers are driven together, and thus both drivers are producing the same back EMF. Appropriately amended, the equation now reveals that:

$$I_{DAMPING} = \frac{2\ E_{EMF}}{2\ R_E}$$

Simplifying this equation leads us back to the original:

$$I_{DAMPING} = \frac{E_{EMF}}{R_E}$$

Thus, viewed from the point of the damping afforded by the generation of back-EMF, series connection does not have the effect of reducing damping as claimed.

## 6 EMPIRICAL SUPPORT

The intuitive premise makes one clear prediction: the damping is seriously compromised by placing two speakers in series. This must be manifested by a substantial increase in the Q factors of the speaker. Specifically, the premise predicts that the electrical Q factor should be much greater. How much greater is not clear, because the premise is woefully short of analytical precision. But let's say that we should see at least a doubling of the electrical Q. And since the electrical damping predominates in most speakers, the total Q should be similarly affected.

On the other hand, our analysis above predicts that the Q factors should remain essentially unchanged. Such an unambiguous difference makes this discussion an ideal candidate for falsification by experiment[5].

For the first experiment, I selected two woofers, a pair of Seas PR17RC 6½" woofer-midrange drivers. I measured the relevant parameters, the DC resistance, resonance and the mechanical, electrical and total damping of each separately, and then with the two connected in-phase in series to see the effect on damping of such a series connection. The actual results are shown in Table 1:

The data would seem to strongly support the analytical method's predictions, and refute those of the intuitive model. That's fine for single speakers, and this result has been validated in numerous home-built systems using multiple drivers in series. It might be a different question, though, for complete speaker systems, often the situation found in some installations.

So, I went to my storage room and grabbed a pair of rather ordinary bookshelf speakers, some ancient ones made by the old H. H. Scott company. I could well have used any two speakers, but there were handy and fully functional. I measured the resonant frequency, the DC resistance of the voice coil, and the relevant Q factors for each speaker alone, and the two in series. Along with these numbers, I also present the predictions made by the two competing theories, the "intuitive" premise, and the "analytical" theory described above. The results are shown in Table 2.

It would seem that the empirical data strongly supports our analytical model, and strongly refutes the intuitive premise. *Q.E.D.*

## 7 FREQUENCY RESPONSE ERRORS

One problem with two speakers in series is that the frequency dependent impedance variations of one will upset the frequency response of the other,

[5] "Falsification" is a vital part of the scientific method. A theory must be falsifiable, that is, it must make a prediction that, by experiment or observation, can be clearly shown to be right or wrong. In the case we have here, either one theory, the other theory, or neither theory will be supported by the experimental data. No data can support both. If a theory makes a prediction that can't be tested, it's no good as a theory. You might have a theory: "I can levitate myself while no one is looking." It's impossible for anyone else to construct a test, because they can never look at you doing what you claim, thus the "theory" has no value scientifically.

and vice versa. As it turns out, this is not the case as well.

Consider how the attenuation arises. Take the case of two resistances in series, $R_1$ and $R_2$. Given an impressed voltage of $V_S$, we can calculate the voltage across $R_2$. The current through the entire circuit, $I$ will, by Ohm's law, be:

$$I = \frac{V_S}{R_1 + R_2}$$

Given that current, again, by Ohm's law, the voltage across $R_2$ will be:

$$V_{R2} = I\ R_2$$

And, combining these two equations and simplifying, we find that:

$$V_{R2} = V_S \frac{R_2}{R_1 + R_2}$$

Now, in the case where $R_1 = R_2$, this reduces to simply:

$$V_{R2} = \frac{1}{2} V_S$$

Now, this can be generalized for impedances. If the impedances are the same, we can say that:

$$V_{Z2} = V_S \frac{Z_2}{Z_1 + Z_2}$$

$Z_1$ and $Z_2$ represent the complex, frequency dependent impedances of our loudspeakers. If $Z_1 = Z_2$, which would be the case if our two speakers are the same (and this includes the frequency-dependent impedance variations as well), then our equation reduces to the fact that the voltage across each speaker would be:

$$V_Z = \frac{1}{2} V_S$$

Notice the complete absence of any frequency-dependent terms in this final equation: with two identical speakers in series, the voltage across each is simply ½ that of the voltage the amplifier is producing across the total, and is independent of frequency. There are, thus, no frequency-dependent variations in frequency as a result of putting two identical speakers in series. Q.E.D.

| | Measurement | | | Prediction | |
|---|---|---|---|---|---|
| **Parameter** | **A** | **B** | **A+B** | **Analytical** | **Intuitive** |
| Resonance $F_S$ | 71.20 | 69.03 | 70.1 | | |
| DC resistance $R_E$ | 5.72 | 5.70 | 11.42 | | |
| Damping | | | | | |
| Mechanical $Q_M$ | 1.29 | 1.33 | 1.32 | | |
| Electrical $Q_E$ | 0.88 | 0.92 | 0.91 | ~0.90 | >1.80 |
| Total $Q_T$ | 0.52 | 0.54 | 0.55 | ~0.53 | >1.06 |

**Table 1: Driver Measurements**

| | Measurement | | | Prediction | |
|---|---|---|---|---|---|
| **Parameter** | **A** | **B** | **A+B** | **Analytical** | **Intuitive** |
| Resonance $F_S$ | 110.5 | 113.7 | 112.6 | | |
| DC resistance $R_E$ | 6.86 | 6.87 | 13.90 | | |
| Damping | | | | | |
| Mechanical $Q_M$ | 2.77 | 3.06 | 3.15 | | |
| Electrical $Q_E$ | 1.02 | 1.09 | 1.14 | ~1.06 | >2.28 |
| Total $Q_T$ | 0.75 | 0.80 | 0.83 | ~0.80 | >1.6 |

**Table 2: Speaker System Measurements**