

Tech Design...

Examination of crossover induced transient distortion.

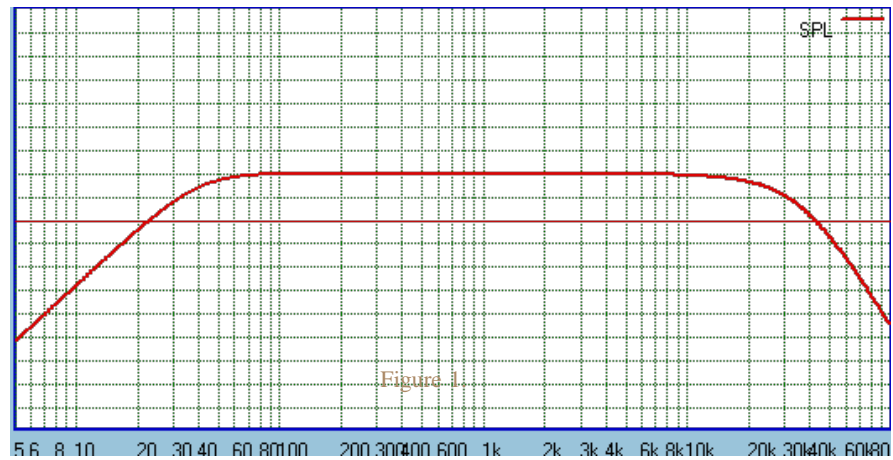
The following discussion address how we can look at a multi-way speaker's (the "system") transient response by decomposing the response into a minimum phase component and an all pass component. The minimum phase component of the system represents the best performance which could be obtained without the used of various types of digital processing. The all pass component represents the additional phase distortion introduced by the crossover. Since the all pass response will be, by definition, flat from DC to infinity, if it is not to introduce any additional phase/transient distortion it must be either minimum (zero) phase or linear phase. Linear phase reduces to a pure time delay which introduces no transient distortion. Here we shall limit the discussion to what can be engineered using analog filters. We shall also address the performance only on the design axis and at the design listing distance. Off axis effects, power response, polar response, etc are not considered since they not only depend on the crossover but on driver directionality, spacing, system format (dipole, monopole, line source), etc.

Before we start a few general comments about crossovers should be presented.

- 1) All crossovers which sum flat yield an all pass responses.
- 2) Any crossover which sums flat when connected with one or more sections inverted relative to the remaining sections can not be minimum or linear phase and therefore, can not be transient perfect.
- 3) Any crossover that sums flat in phase (LR type crossovers and the notched variants by Thiele [1]) can not be minimum phase or transient perfect. (Linear phase crossovers with LR amplitude response do sum flat in phase, and are transient perfect, but not minimum phase.)
- 4) Any speaker that uses physical offset of the driver acoustic centers can not be minimum or linear phase, thus not truly transient perfect.

I make these statements up front because I have seen over the web speaker systems presented as minimum phase and/or transient accurate which use in phase crossovers and, for example, have the tweeter connected with inverted polarity. While these speakers may be otherwise well designed, the claims of minimum phase and/or transient accuracy are exaggerations.

We now begin by observing that every multi-way speaker system is a band pass device with representative low and high frequency cut offs. For the sake of this discussion we shall consider a two-way system with 2nd order 40 Hz low frequency cut off and a 25 k Hz high frequency cut off that initially follows a nd order roll off and then decays into as steeper 4th order roll off. The anechoic response of such a system would appear as shown below in Figure 1.



The transient response of such a system is governed by both the amplitude and phase of the system. Here we shall be concerned mainly with the effect of the system phase on the transient response.

For any system it is possible to decompose the response into a minimum phase component and an all pass component. Thus, we can write the system transfer function as

$$S(s) = MP(s) \times AL(s)$$

where s is the complex frequency, $j\omega$, MP is the minimum phase component and AP is the all pass component. The minimum phase component of the system response is just that of a band pass filter having the same amplitude response. The Figure 2, below, shows the phase of the minimum phase component in red and the associated group delay in blue. The group delay is

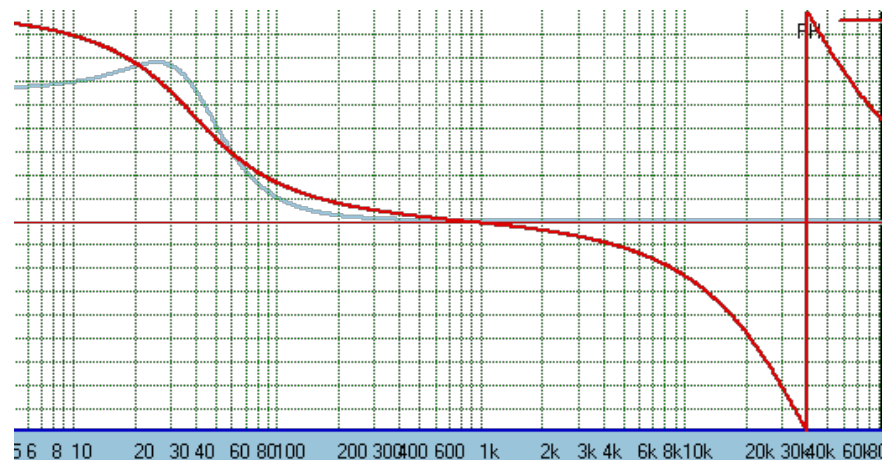


Figure 2.

zero at frequencies well above the high frequency limit of the system and then increases to a more or less constant value mid band associated with the low pass nature of the high frequency limit of the system. (A low pass filter has constant group delay over much of its pass band.) As the frequency continues to decrease we observe an increase in the group delay becoming obvious somewhere around 400 Hz. The Figure 3 shows the square wave response of this minimum phase system at frequencies between 100 Hz and 10,000 Hz. Note this over this frequency range the amplitude response is flat. The 100 Hz response shows a slanted top and bottom of the wave, This is a result of the varying group delay at low frequency. At 200 Hz the response is still slanted, but to a lesser degree. Similarly at 400 Hz. As the frequency rises we see flatter and flatter tops and bottoms of the square wave because the group delay is becoming more constant. However, by 1600 Hz we start to see a slight rounding of the leading edge of the rise and fall of the wave. The primary reason for this is because the higher frequency components of the square wave are being attenuated in amplitude by the low pass nature of the system at high frequency

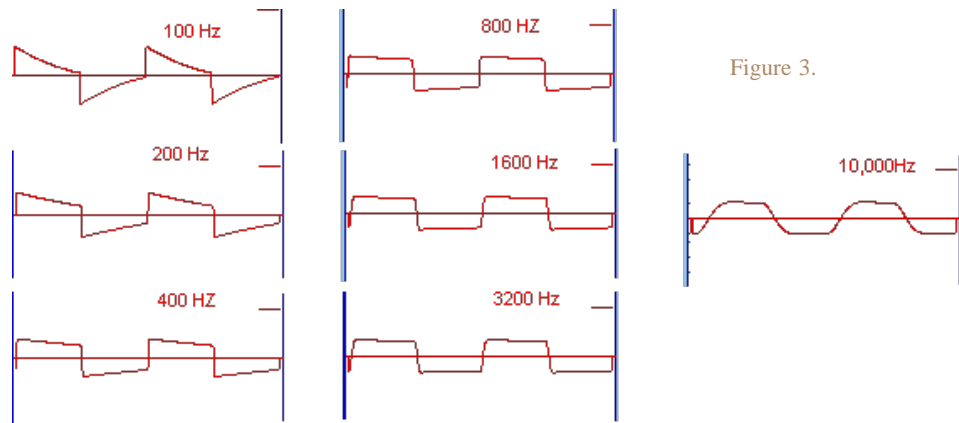


Figure 3.

The square wave responses shown above represent the best possible result for a conventional speaker system which possesses the band pass response as indicated.

We must next consider what degradation in the response we might see when we include the contributions from the all pass component of the system. Since an all pass response is flat from DC to infinity by definition, the all pass component can only introduce phase distortion. First we must ask, where does the all pass component come from? There are two sources for a multi-way speaker. The first is the propagation delay from the sources to the listening position. If the drivers are correctly aligned, with their acoustic centers the same distance from the listening position, then this aspect of the all pass response is nothing more than a constant time delay. Since a constant time delay does not introduce any transient distortion we can place this aside for the moment. The second aspect of the all pass response arises from the crossover. Any 2-way crossover which has a low pass and high pass section that sums flat in amplitude at the listening position is, again by definition, an all pass response. Thus, all odd order Butterworth crossovers are all pass crossovers as are the even order Linkwitz Riley crossovers. In addition to these common crossovers there are also any number of transient perfect crossovers such as the constant voltage (subtractive) crossovers discussed by Small [2], the transient perfect 2nd order crossovers discussed by Kreskovsky [3,4], and the family linear phase crossovers defined by subtraction and delay as originally discussed by Lipshitz and Vanderkooy[5].

Before we begin to look at the different possibilities for the all pass component of the multi-way speaker's response we must emphasize that when the term crossover is used it is taken to mean that the acoustic output of the drivers has been shaped to exactly match the transfer function defined by the target response. That is, if an LR4 crossover is referred to, it means that at the listening position the acoustic output of the driver would exactly match the LR4 HP or LP target, except for the roll offs associated with the system response at the frequency extremes. We must also accept that these idealized crossovers sum perfectly flat only if the phase variation due to the high and low frequency cut off of the system response does not interfere with the targeted crossover phase. This is predominantly a problem with lower order crossovers which will result in small amplitude errors in the response. These small amplitude variations should be included as part of the minimum phase component of the decomposition, but for convenience, here we shall assume the minimum phase component is always given by that presented above where the amplitude in the pass band is perfectly flat. As such, the all pass components to be presented may also include a small amplitude variation to account for equalizing the system response to the flat minimum phase component.

To obtain the characteristic of the all pass response we divide the system response $S(s)$ by the response of the minimum phase component;

$$AP(s) = S(s) / MP(s)$$

This all pass response then defines the additional phase distortion introduced by the crossover, relative to the listening position. The ideal all pass response would be a linear phase response representative of a pure time delay of finite magnitude. Note that zero phase vs frequency is just a linear phase response associated with a zero time delay. Now let's look at the effects of different crossover. In all cases we shall look at a crossover at 1.5 K Hz. To demonstrate the

effect of the crossover on transient response we shall examine reproduction of an 800 Hz square wave since the frequency components of such a wave will span the crossover frequency.

1st order Butterworth:

The all pass component for the B1 crossover is shown in Figure 4 with the amplitude (red) and phase (green) at the left and the 800 Hz square wave reproduction at the left. The all pass response shows a small error in amplitude and phase at high frequency which is due to the effects discussed above. Other than that, it is apparent the square wave reproduction is near perfect as would be expected for a B1 crossover since it introduces no amplitude or phase distortion, as is well known. That is, the response is transient perfect. In this regard, a speaker system using a correctly implemented B1 crossover with band pass response as shown in Figure 1 would exhibit the square wave response shown for the minimum phase band pass system response shown in Figure 3 without further degradation by the crossover.

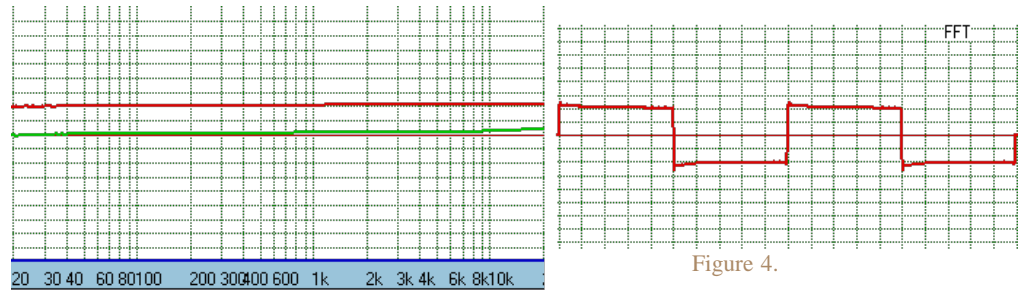


Figure 4.

3rd order Butterworth, in phase:

The all pass component for the B3 crossover is shown in Figure 5. It is apparent that the all pass response introduces significant phase distortion and the square wave response is highly degraded compare to the 800 Hz square wave shown in Figure 3 for the minimum phase system component. The square wave response shows the initial pulse (from the tweeter) going in the correct direction but the woofer pulse is slow to respond and tweeter output has decayed to near zero before the woofer output rises to complete the response. The phase response of the all pass component is clearly neither minimum phase nor linear phase. The minimum phase for a flat, all pass response is zero phase vs frequency. If the all pass were linear phase then the square wave reproduction would be perfect, but shifted slightly to the right due to the constant time delay linear phase represents.

The slow woofer response has led to speculation that "time aligning" the system by moving the tweeter back to offset the woofer delay could improve the response. This intentional miss-alignment can also result in introducing response irregularities. The amplitude irregularities can be corrected using appropriate equalization, but it remains to be seen if the resulting all pass response is minimum or linear phase. This will be discussed further later on.

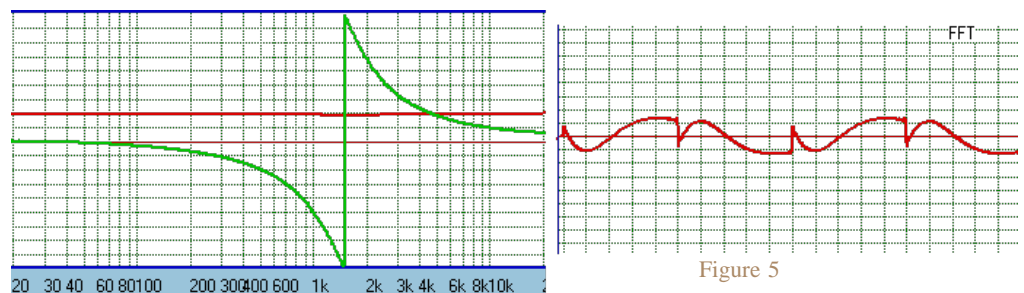


Figure 5

2nd Order Linkwitz/Riley

The all pass component for the LR2 crossover is shown in Figure 6. As with the B3 crossover, it is apparent that the all pass response introduces phase distortion and the square wave response is highly degraded compare to the 800 Hz square wave shown in Figure 3. The phase response of the all pass component is still neither minimum phase nor linear phase. However, we must be careful when interpreting the result for the LR2 crossover. We note that the initial

spike in the square wave response is in the inverted direction. Thus, when the polarity of the input changes suddenly the output yields a spike in the wrong direction with the appearance of an exponential decay to what should be the level of the top or bottom of the square wave. If we examine this at a low frequency the result appears as shown in Figure 7 where the frequency is 400 Hz. Again, this is only the response of the all pass component of the system response. I present this figure because I have seen such responses presented claiming that it shows good square wave reproduction and transient accuracy. They say a picture is worth 1000 words, and in such a case, believe the picture, not the words. Any system that shows the initial part of a step or square wave response going in the wrong direction can not be transient accurate regardless of what other virtues the system may possess.

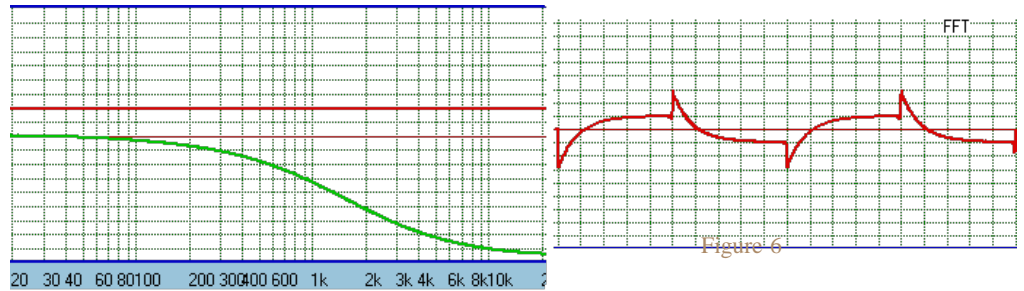


Figure 7. 400 Hz square wave reproduction.

4th Order Linkwitz/Riley

The all pass component for the LR crossover is shown in Figure 8. Close examination of the plots will reveal that there is great similarity between the all pass response of the LR4 crossover and that of the B3 crossover. In fact, this is the case with the exception that the phase rotation near the crossover point is somewhat more gradual.

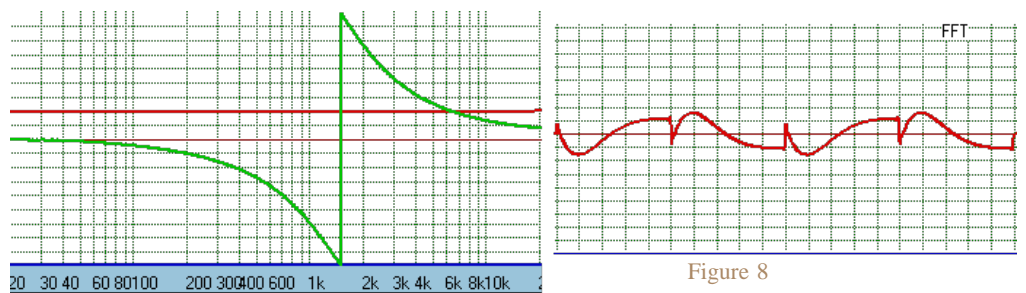


Figure 8

2nd Order Transient Perfect Crossover:

The 2nd order transient perfect crossover is described by Kreskovsky [3,4] and is based on the concepts presented by Vanderkooy and Lipstiz [6] where the corner frequencies of the HP and LP sections are overlapped so that the summed response, while no longer flat, is minimum phase. Then, minimum phase equalization is applied to obtain flat response. The result is an all pass response with flat amplitude and zero phase shift; transient perfect, in which the HP and LP sections have 2nd order asymptotic slopes. The all pass response is shown in Figure 9. A speaker system build with this type of crossover will produce the transient response of the band pass system of Figure 3 with no additional crossover induced transient distortion.

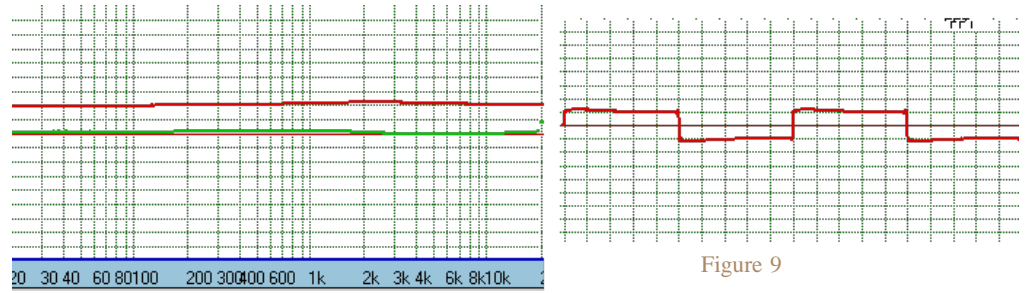


Figure 9

Transient Perfect crossover Derived by Subtraction:

Transient perfect crossovers derived by subtraction define an infinite family of all pass responses which are either of the minimum (zero) phase or linear phase. The fundamental idea is that if the summed response is to be all pass and either zero or linear phase then it is given as

$$AP(s) = \exp(-s \times T_d)$$

where T_d is a time delay which. The phase vs frequency is given as

$$\Phi(\omega) = -\omega \times T_d.$$

It is obvious that such a relationship yields a phase shift which varies linearly with frequency. In the case where $T_d = 0$, $AP(s) = 1$, flat amplitude and zero phase. In this case we can assure $AP(s)$ will remain 1.0 if we define high pass and low pass responses as

$$HP(s) = 1 - LP(s)$$

for a given, arbitrary LP response. When the HP response is initially specified we have

$$LP(s) = 1 - HP(s)$$

Thus by construction

$$HP(s) + LP(s) = 1 = AP(s)$$

Such crossovers are representative of the Constant Voltage crossover discussed by Small [2].

When T_d is non-zero useful results can be obtained by specifying the low pass response and subtracting it from an all pass response with linear phase, thus defining the high pass section:

$$HP(s) = AP(s) - LP(s) = \exp(-s \times T_d) - LP(s)$$

The caveat is that in these types of crossovers T_d must be equal to (or very close to) the DC group delay of the specified low pass section. The usefulness of this approach is limited to the choice of the low pass section. The basic concept was first introduced by Lipshitz and Vanderkooy [5]. It is important to realize that this form of "Subtractive-Delayed" crossover is in **no way** equivalent to physically offsetting the tweeter to compensate for the group delay of the woofer response. As discussed above, such offsetting will never yield an all pass response with zero or linear phase and flat amplitude. There are cases where it may approximate such a response but there is always some error.

CV crossover using an LR4 LP response:

Figure 10 shows the all pass response of a CV crossover using an LR4 low pass response. To within the small errors arising from processing, as discussed previously, it is observed that the all pass response is flat with zero phase. The 800 Hz square wave reproduction shows no transient degradation. A speaker system build with this type of crossover will produce the transient response of the band pass system of Figure 3 with no additional crossover induced transient distortion.

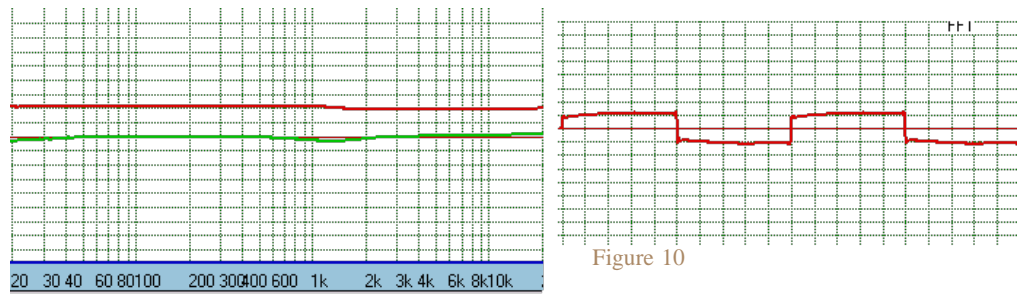


Figure 10

SD crossover using an LR4 LP response:

Figure 11 shows the all pass response and 800 Hz square wave reproduction for a SD type crossover implemented using an LR4 LP response. That the phase varies linear with frequency (a pure time delay) is shown more clearly in Figure 12 using a linear frequency scale. A speaker system build with this type of crossover will produce the transient response of the band pass system of Figure 3 with no additional crossover induced transient distortion.

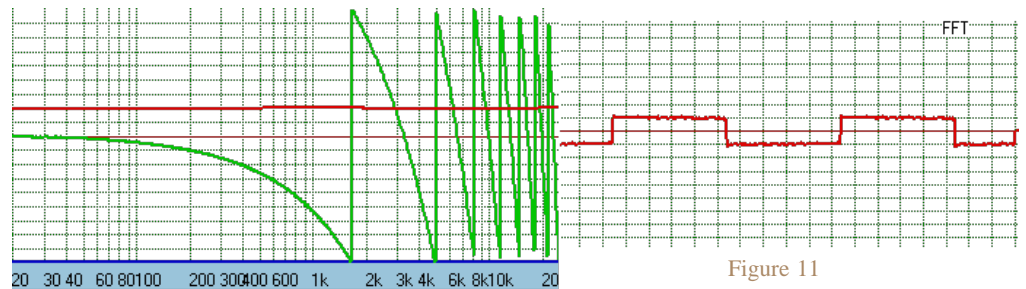


Figure 11

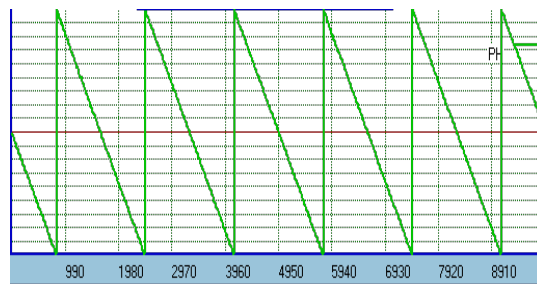


Figure 12. Phase with linear frequency scale.

Approximately Transient Perfect Crossovers using Time Alignment

It was mentioned above in the 3rd order butterworth discussion that it has been contemplated that, due to the slow response of the woofer in a 2-way system, some semblance of transient perfect response may be obtainable by physically offsetting the tweeter to account for this slow woofer response. First we ask what the origin of the slow woofer response is? In fact, it is due to the nature of any low pass filter. All minimum phase low pass filters introduce a frequency dependent group delay. As we move somewhat below the cut off frequency of the filter this group delay become constant and is equivalent to a linear phase or constant time delay. This observation is what lead to the higher order, SD type filters originally proposed by Lipshitz and Vanderkooy [5]. However, this same observation also lead to consideration of physically offsetting the tweeter to account for the woofer low pass response delay. The idea was that by offsetting the tweeter a crossover could be constructed which, while not necessarily having flat response, would hopefully result is a response which was composed of a linear phase component and a minimum phase component with the minimum phase component representing the deviation from flat response. If this were possible then minimum phase equalization of the response deviation from flat would result in a linear phase crossover. Unfortunately, Vanderkooy and Lipshitz [6] showed that for conventional all pass crossovers higher than 1st order (odd order Buterworth and even oder Linkwitz/Riley) this is not possible. That is, they showed that after removal if the linear phase component of the response, the remaining phase component is not minimum phase. Thus minimum phase equalization would not result in a linear phase all pass response.

However, Vanderkooy and Lipshtiz did not consider filter other than conventional all pass crossovers. In the early 80's Spica, for example, introduce a series of speakers which employed tweeter offset to time align the system and these speakers were said to have very good transient response. These speakers used a 1st order electrical high pass filter with impedance compensation for the tweeter and a 2nd order electrical low pass for the woofer with an additional notch filter for. When combined with the acoustic response of the drivers the results was a intended to be a 4th order Bessel low pass and a quasi 1st order high pass acoustic response. Quasi 1st order because ultimately the 1st order electrical filter combined with the tweeter acoustic output would yield a 3rd order roll off at frequencies below the crossover point. The tweeter off set was then adjusted to yield optimal time response. To see how well the potential performance of such a system would be we shall examine the acoustic crossover response alone. In Figure 13, however, it is assumed that the tweeter roll off remains 1st order and the excess phase associated with the tweeter offset has been removed. The phase of the speaker is shown in green. The lower red line is the minimum phase for the system amplitude. From this and from the square wave response to the right, while an improvement over standard crossovers, the response is not truly linear phase.

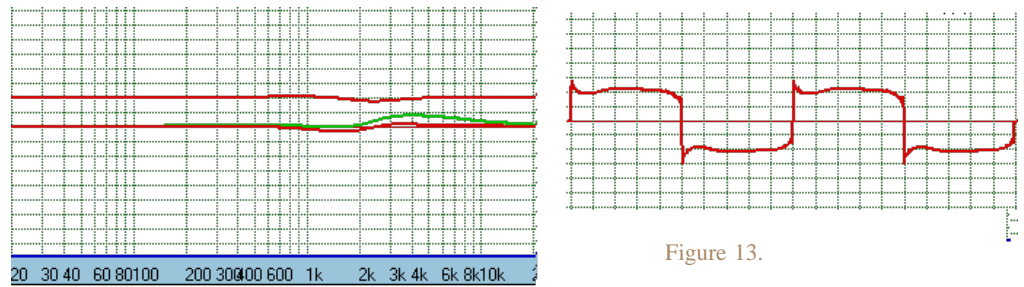


Figure 13.

When the roll off of the tweeter is included in the result the response appears as shown in Figure 14. The dip in the response could be compensated for, but as is indicated by the comparison of the actual and minimum phase response, and by the square wave distortion, the response is not minimum or linear phase.

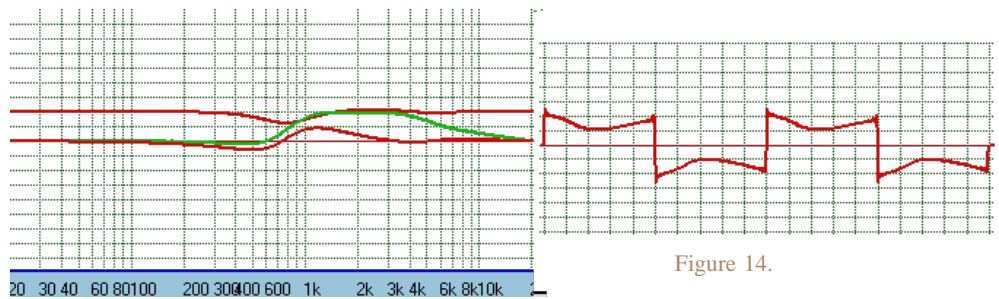


Figure 14.

Thus it appears that the conclusions of Vanderkooy and Lipshitz [6] regarding the used of physical offset, or "time alignment", to compensate for the GD of the low pass response applies to crossovers of mixed types as well. However, while not perfect, it is readily apparent that these types of crossovers do offer an improvement in transient response over the standard crossover; Butterworth of order greater than 1, and the Linkwitz/Riley crossovers. However, with any system there are the other consideration regarding off axis and polar response, and the complexities introduced by physically offsetting drivers.

We can analyze the result more deeply. The question is, can a linear (or zero) phase crossover be developed by introducing physical offset, resulting in delay, to only the high pass section? In accordance with the analysis of [6] it is not required that the crossover sum flat, but only that the summed response be composed of a minimum phase component and a linear phase component. The minimum phase component accounts for any response irregularities and application of minimum phase equalization would render the summed response flat and linear phase, thus transient perfect. We have seen this failed above, but what about in general?

We again start with the definition of the system response, ignoring the low and high frequency cutoffs.

$$S(s) = \exp(-sT_{dl}) \times MP(s)$$

where $\exp(-sT_{dl})$ is the linear phase component and $MP(s)$ is the minimum phase component. Since the response deviation from flat is assumed to be contained in the minimum phase component the system response can be equalized to an all pass response by applying minimum phase equalization,

$$AP(s) = EQ(s) \times S(s) = EQ(s) \times \exp(-sT_{dl}) \times MP(s)$$

where $EQ(s) = 1 / MP(s)$

We next consider the high pass function, $HP(s)$ representing the on axis response of the filtered tweeter. If the tweeter AC is offset from the woofer AC by some distance, d , then it is equivalent to a time delay of $T_d = d/c$ where c is the speed of sound. Thus the offset tweeter response may be expressed as

$$T(s) = \exp(-sT_d) \times HP(s)$$

when referenced to the plane containing the woofer AC, as shown in Figure 16.

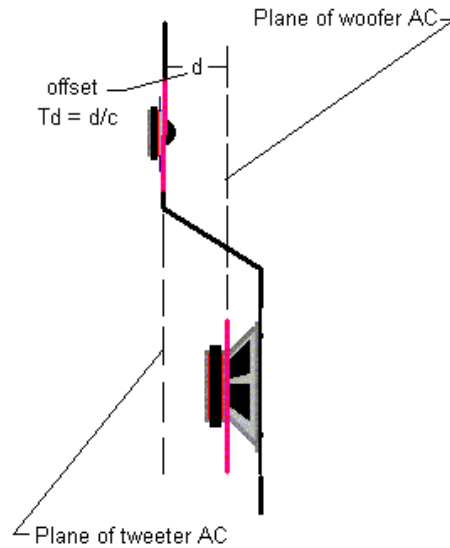


Figure 16.

We now need to find the woofer response, $W(s)$, which when summed to the offset tweeter will yield the system response, $S(s)$, or,

$$W(s) = S(s) - T(s)$$

Using the responses defined above,

$$W(s) = \exp(-sT_{dl}) \times MP(s) - \exp(-sT_d) \times HP(s)$$

We can factor out the linear phase component of the system response to yield,

$$W(s) = \exp(-sT_{dl}) \times [MP(s) - \exp(-s(T_d')) \times HP(s)]$$

where $T_d' = T_d - T_{dl}$

Now, assume that $W(s)$ has the form of some kind of low pass response. As such, the output of the system as the frequency goes to infinity will be solely that of the tweeter. Thus, as the frequency goes to infinity the delay would be that of the tweeter response, $HP(s)$, plus the delay due to the offset. If $HP(s)$ is a high pass, minimum phase response the delay associated with $HP(s)$ will be zero

as the frequency goes to infinity and the delay of the output will be that due to the tweeter offset. However, this leads to a contradiction because by construction the system output (when $T_{dl} = 0$.) will have zero delay since MP(s), being minimum phase, will have zero delay as f goes to infinity. [In the case where T_{dl} is not zero the system output would be delayed by T_{dl} while the tweeter response at high frequency would be delayed by $T_{dl} + T_{d'} = T_{d'}$.] So what is wrong? Either MP(s) can not be minimum phase (as was shown above in the in the 2nd order Bessel / 1st order Butterworth case), or the assumption that $W(s)$ has the form of a low pass response is incorrect. This proof by contradiction shows that it is not possible to construct a transient perfect crossover (linear or zero phase) by offsetting the tweeter as is commonly attempted in so called "time aligned" systems. That is, there are no possible high pass / low pass filter combinations which will combined to reduce the system response to a minimum phase component plus a linear phase component when the tweeter is offset behind the woofer. While the approach taken, for example, by Spica years ago may yield an acceptable approximation to a linear phase crossover, there will always be some error and deviation from linear phase in the crossover region and the crossover will not be transient perfect.

There is one last interesting case to consider; What if $T_{d'} = 0.0$. That is, what if the tweeter offset is equal to the system delay, T_{dl} ? In this case we have

$$W(s) = \exp(-sT_{dl}) \times [\text{MP}(s) - \text{HP}(s)]$$

The term in the red brackets is just the form the low pass function for Small's [2] CV crossover, $W_{cv}(s)$, with result that

$$W(s) = \exp(-sT_{dl}) \times W_{cv}(s).$$

The linear phase term, $\exp(-sT_{dl})$ can be interpreted as either an electronic delay, or a physical offset of the woofer. In either case this delay is identical to that of the tweeter since $T_{d'} = 0$ and should be interpreted as shifting either the apparent or actual acoustic center of the woofer to lie in the same plane as the tweeter AC.

In the final analysis what has been shown is that any crossover which is transient perfect must have the AC of the drivers aligned to lie in the same plane. This can be accomplished by physical offset of the correct amount or by electronic delay. However, the use of excess offset, positioning the tweeter AC behind that of the woofer, to attempt to compensate for the GD of the woofer response, will never yield a crossover which is truly transient perfect.

References

1. Thiele, N.: Loudspeaker Crossovers with Notched Responses, 108th AES Convention, Feb. 2000.
2. Small, R. H.: Constant-Voltage Crossover Network Design, JAES, V 19, No. 1, 1971.
3. Kreskovsky, J. P.: A Transient-Perfect Second-Order Passive Crossover, Audio Express, May 2001.
4. Kreskovsky, J. P.: Design an Active Transient-Perfect Second-Order Crossover, Audio Express, Dec. 2002.
5. Lipshitz, S. P. and Vanderkooy, J.: A Family of Linear-Phase Crossover Networks of High Slope Derived by Time Delay, JAES, B 31, No. 1/2, 1983.
6. Vanderkooy, J. and Lipshitz, S. P., Is Phase Linearization of Loudspeaker Crossover Networks Possible by Time Offset and Equalization?, JAES, V 32, Dec, 1984.