Close-in phase noise in electrical oscillators

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ABSTRACT

Understanding the properties of close-in phase noise is crucial for analyzing the effects of low-frequency, colored noise on the frequency stability of electrical oscillators. This paper shows these properties are distinctly different from those of far-out phase noise, which are commonly studied in the literature. Unlike far-out phase noise, the spectrum of close-in phase noise caused by several uncorrelated noise sources is **not** the same as the sum of the phase noise spectra caused by individual sources. Furthermore, in the absence of colored noise, this spectrum is **not** necessarily Lorentzian as generally believed. We show that the phase noise spectrum of a periodic signal with zero cycle-to-cycle jitter is always Lorentzian and demonstrate the appearance of $1/f^4$ phase noise due to a Lorentzian noise source. We also study two methods for suppressing the effects of low-frequency, colored noise on phase noise: signal symmetrization and noise-source switching. We show that the suppression of $1/f^3$ phase noise in single-ended ring oscillators is due to switching and not because of symmetrization. Symmetrization is effective only for the noise sources which are constantly "on", such as the tail current source in differential ring oscillators. These findings provide effective guidelines for designing low-phase-noise oscillators.

Keywords: electrical oscillator, jitter, phase noise, ring oscillator, relaxation oscillator, frequency stability

1. INTRODUCTION

Despite its practical importance in communications, the formal definition of phase noise remains a matter of controversy. At least two distinct definitions are introduced by various authors. One of these definitions involves the power spectral density (PSD) of phase[1], the other one is based on the PSD of the signal itself[2]. The choice of definition is irrelevant at large offset frequencies (hereafter referred to as far-out phase noise) because the PSD of phase can be approximated by the PSD of the signal at far-out frequencies[3]. However, the numerical value of phase noise at small offset frequencies (the close-in phase noise) strongly depends on the definition. Furthermore, as we will see shortly, depending on which definition we use, some well-known properties of the far-out phase noise, such as the superposition of phase noise, can be violated at close-in frequencies.

"Close-in" is defined at small offset frequencies, where the phase noise spectrum does not have a $1/f^2$ shape. The analysis of phase noise at these frequencies is usually more complicated than that of the far-out phase noise mainly because close-in phase noise is, by definition, affected by low-frequency colored noise, such as generation/recombination noise and $1/f$ noise.

The analysis of close-in phase noise is often regrettably avoided in the literature on the ground that phase-locked-loops, which are used in most communication systems, suppress the phase noise at small offset frequencies. However, with the emergence of submicron MOSFETs with $1/f$-noise corner frequencies on the order of 100MHz, close-in phase noise can have a noticeable effect on the overall performance of future communication systems. Furthermore, a deep understanding of phase noise demands its characterization at all offset frequencies.

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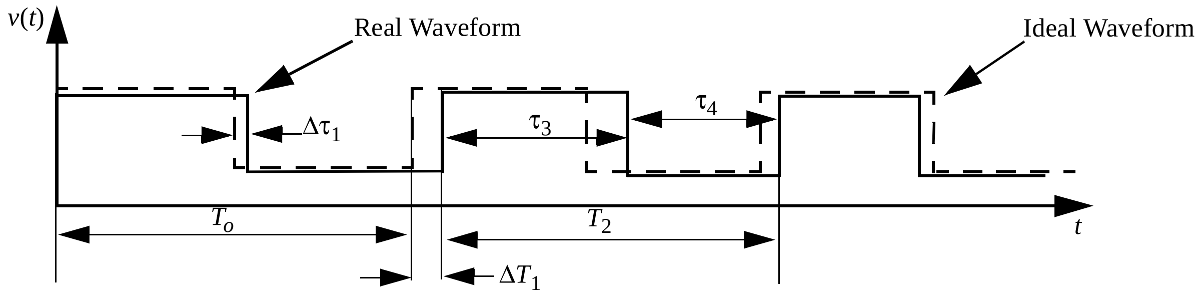

Fig. 1. Real and ideal waveforms for a rectangular oscillatory signal with parameter definitions.

In this paper, we focus on the characteristics of close-in phase noise of electrical oscillators and show that these characteristics are distinctly different from those of far-out phase noise. In order to perform a rigorous analysis of these characteristics, we study, in Section 2, the formal definitions of phase noise. We then use a simple, practical relaxation oscillator to study the properties of close-in phase noise. The output of this oscillator is represented by an oscillatory square-wave signal. Since such a signal can also represent the output of a limiting amplifier fed by an arbitrary oscillatory signal, its phase noise properties hold generally for any periodic signal. In Section 3, we present the analytical formulation of the phase noise of this oscillator first in the absence of colored noise and then in its presence. Using these formulations, Section 4 discusses various properties of close-in phase noise as well as various ways of suppressing the effect of low-frequency, colored noise on phase noise. This discussion provides useful insight about the frequency stability of electrical oscillators and practical guidelines for designing low-phase-noise oscillators.

2. THE FORMAL DEFINITION OF PHASE NOISE

An electronic oscillator is responsible for generating a periodic signal with a stable oscillation frequency. In an ideal oscillator, this frequency would remain constant over time. In a real oscillator, however, the frequency of oscillation is modulated by electronic noise, present in all real systems. Because of this electronic noise, the oscillation frequency randomly fluctuates with time. These frequency fluctuations degrade the performance of the system in which the oscillator is used.

To evaluate the performance of a communication system in the presence of noise, we need to characterize the frequency fluctuations of its oscillator. This characterization can be performed using time-domain or frequency-domain analysis and different measures can be defined correspondingly. From a practical point of view, the best measure is the one that best facilitates the performance assessment of the communication system. Thus, depending on which kind of system is under consideration, different measures of frequency instability will be favorable. One instability measure, often referred to in the literature, is phase noise. Its formal definition is, however, still a matter of controversy. In this section, we first present the existing definitions of phase noise as a measure of frequency instability. We then discuss the effect of frequency instability on the performance of various types of communication systems. In light of this discussion, we choose an appropriate definition of phase noise for our study.

2.1 Existing definitions of phase noise

Figure 1 shows an ideal periodic square-wave signal along with a real signal, which has nonzero frequency instability. The nominal oscillation period for this signal is denoted by T_o. In the presence of noise, the real value of the duration of period at the ith period is a random variable denoted by T_i. For stationary oscillators, the expected value of this random variable is independent of i and, by definition, is the nominal period of oscillation. Demir et al. explain the properties of stationary oscillators[2]. The duration of the ith half-period of oscillation is denoted by τ_i. Thus

$$T_i = \tau_{2i-1} + \tau_{2i} . \tag{1}$$

We define jitter in the ith period, ΔT_i, as the difference between the actual and the nominal duration of this period,

$\Delta T_i = T_i - T_o$. The period jitter, $\overline{\Delta T^2}$, is the variance of ΔT_i. For a stationary oscillator, this is independent of *i*. The cycle-to-cycle jitter, $\overline{\Delta T_{i,j}^2}$, is defined as the expected value of $\Delta T_i \Delta T_j$. This is normally only a function of *i-j* and not of *i* or *j* alone. Similarly, we define half-period jitter and half-cycle-to-cycle jitter as the variance of $\Delta\tau_i$ and the expected value of $\Delta\tau_i\Delta\tau_j$, respectively. In most practical situations, having $\overline{\Delta T_{i,j}^2}$ for all (*i-j*)'s provides enough information for the characterization of frequency instability in the time domain.

The characterization of the frequency instability in the frequency domain is more complicated and is based on the definition of phase noise. At least two distinct definitions are used by various authors: one based on the PSD of the phase[1] and the other based on the PSD of the signal itself[2].

According to the first definition, phase noise is the PSD of the phase. The main advantage of this definition is that it keeps the phase noise independent of the amplitude noise. However, this choice of definition also generates some mathematical and practical difficulties. For example, phase is not a stationary variable and its PSD is mathematically undefined[*]. Although it is possible to define a generalized PSD for phase, this would complicate the already involved mathematics for two reasons. First, the total power of the generalized PSD would be infinite, making it impossible to normalize. Second, the generalized PSD would grow without bound around zero frequency. Such an ill-behaved function is hard to work with, especially when close-in phase noise is of interest.

According to the second definition, the phase noise is the PSD of the signal itself, normalized to the total signal power. Using this definition, the phase noise can be calculated analytically and is a well-behaved function around zero offset frequency[2,4]. However, the PSD of the signal is then a function of both jitter and amplitude noise.

It has been shown that the behavior of phase noise at large offset frequencies is independent of the choice of definition[3]. At small offset frequencies, however, these two definitions provide significantly different values for phase noise. To decide which definition is more appropriate for a specific application, we first need to study the effect of frequency instability on the performance of communication systems.

2.2 Phase noise in communication

RF communication systems normally require an accurate time reference because of their multi-user nature. In these systems, several users share the same communication channel, necessitating modulation/demodulation of the messages. Reliable modulation and demodulation is highly dependent upon the accuracy of the frequency of the oscillators used in these systems. On the other hand, in high-speed digital communication systems, the necessity of having an accurate time reference stems from the desire to reach higher data rates. In both cases, the frequency instability of the carrier or clock degrades the performance of the system. However, because of the different nature of these systems, different sets of tools are required to assess performance.

Figure 2 shows a typical RF communication system. The desired signal and an interfering one are located at ω_{RF} and $\omega_{RF}+\Delta\omega$ respectively. Note that the presence of a high-power interfering signal is the result of using the same transmission media for several users. This interfering signal is multiplied by the local oscillator signal in the mixer. Thus the noise of the local oscillator is modulated by this interfering signal and appears at the output of the mixer. At the output of the mixer, the noise power at IF is proportional to the magnitude of the PSD of the local oscillator signal in the vicinity of $\omega_{LO}+\Delta\omega$. Since the IF signal power is proportional to the total power of the local oscillator at the output of the mixer, the degradation of the signal-to-noise ratio due to phase noise is proportional to the phase noise of the local oscillator if we adopt the second definition of phase noise. Therefore, the performance assessment of RF communication systems is greatly simplified if we adopt this definition.

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*Note that phase can be made stationary if it is kept between 0 and 2π. We do not consider this interpretation of phase here.

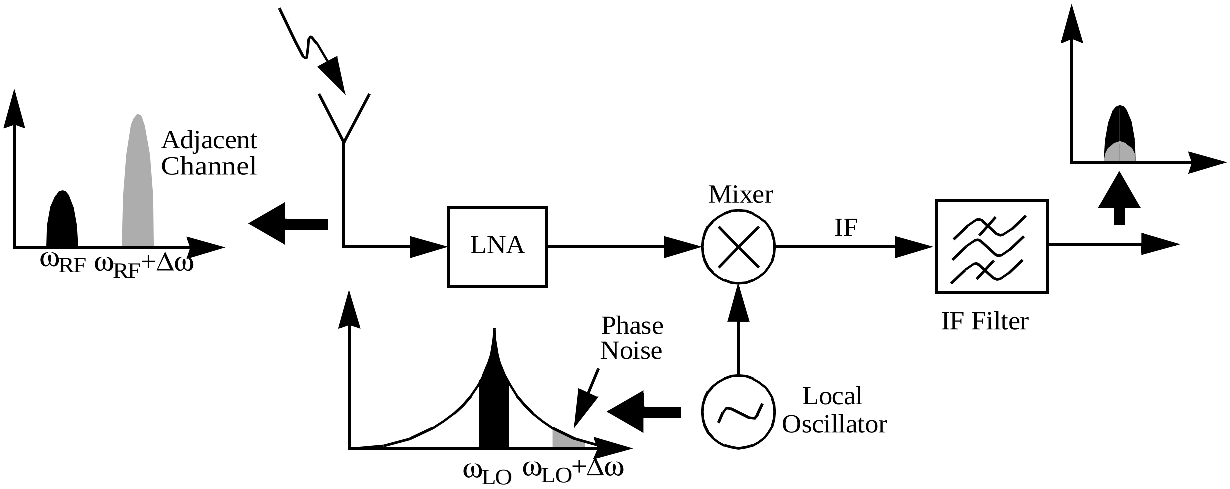

Fig. 2. The front-end of a typical RF receiver.

In high-speed digital communication systems, the frequency instability of the clock increases the bit error rate. The bit error rate in these systems is solely a function of jitter and is independent of amplitude noise. Since the amplitude noise affects the PSD of the signal but not that of phase, it might seem that the first definition of phase noise is more convenient for these systems. However, the analysis of bit error rate in communication systems is most easily performed in the time domain. Consequently, the choice of the definition of phase noise is of little importance for these systems.

The second definition of phase noise also facilitates experimental work. The measurement of the PSD of the signal using a spectrum analyzer is a routine measurement procedure. On the other hand, the process of measuring the PSD of phase is usually much more involved. Furthermore, as we will see in the next section, the analytical calculation of the PSD of the signal is relatively straightforward.

The comparison of the two definitions of phase noise reveals that defining the phase noise as the PSD of the signal normalized to the total power facilitates its measurement and analytical calculations and is usually more helpful for assessing the performance of RF communication systems. The main drawback of adopting this definition is that the amplitude noise affects the PSD of the signal. The effect of this amplitude noise can usually be suppressed using a limiting amplifier and should be distinguished from effects of jitter, which are impossible to suppress. To circumvent this problem, we need to perform phase noise analysis **after** taking into account the effect of the limiting amplifier. We believe that the benefits of defining the phase noise as the normalized PSD of the signal outvalues this complexity and use this definition.

3. ANALYTICAL FORMULATION OF PHASE NOISE OF RELAXATION OSCILLATORS

In this section, we present the analytical formulation of the phase noise of the signal shown in Fig. 1. This signal can represent the output of a relaxation oscillator as well as the output of an arbitrary oscillator after passing it through a limiting amplifier. Thus, many of the phase noise properties of this signal are general and applicable to all kinds of oscillators. We first introduce a relaxation oscillator whose output can be represented by the signal given in Fig. 1. We then present the analytical formulation of phase noise due to white noise and low-frequency colored noise. Unless otherwise stated, our formulation assumes that the signal of Fig. 1 is generated by the simple relaxation oscillator shown in Fig. 3. This assumption does not affect the generality of the final results. The formulations presented in this section will be used in the next section to discuss the properties of close-in phase noise.

3.1 Formulation of jitter

The relaxation oscillator of Fig. 3 is composed of a Schmitt comparator in an *RC* feedback loop. The details of the operation of this oscillator are explained elsewhere[5]. In this paper we assume that the output of this oscillator has a duty cycle of fifty percent.

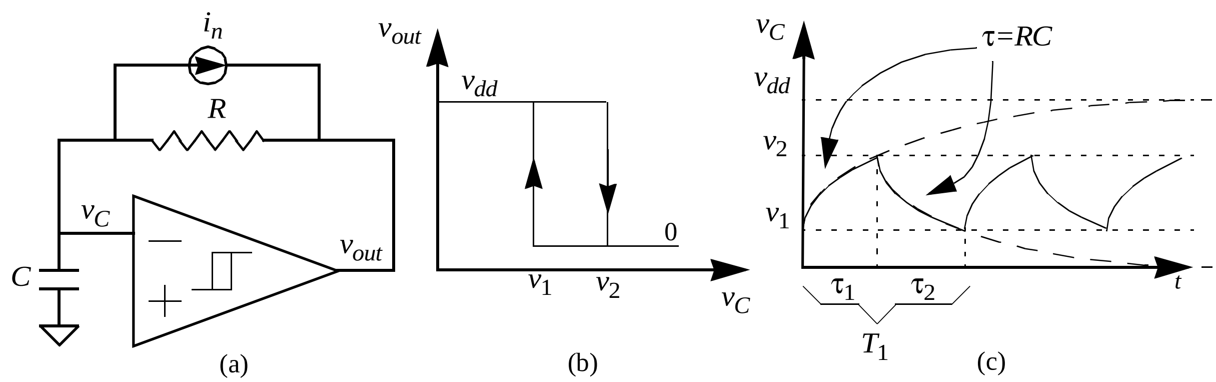

Fig. 3. (a) A typical RC relaxation oscillator. (b) The Schmitt comparator transfer function. (c) The waveform for the capacitor voltage.

For the analysis of jitter and phase noise of this oscillator, we assume that the only noise source of the system is i_n, which is in parallel with the resistor (the comparator is noise-free). The jitter is the result of the uncertainty of the capacitor voltage at the end of each half period, which is in turn the result of the resistor noise. The value of the capacitor voltage at the start of each half period is a deterministic variable because the comparison levels v_1 and v_2 are assumed to be noise-free. We define the series of random variables Δv_i to characterize the uncertainty of the capacitor voltage at the end of the ith half-period. These random variables as functions of the noise source and circuit parameters are given by[5]

$$\Delta v_i = \frac{e^{-\frac{T_o}{2RC}}}{C}\int_0^{\frac{T_o}{2}} e^{\frac{x}{RC}} i_n\left(x + \frac{(i-1)T_o}{2}\right) dx \,,$$

where we have approximated the duration of the ith half-period by its nominal value. Using this equation, we can calculate the fluctuation properties of Δv_i's:

$$\overline{\Delta v_i} = 0 \,,$$

$$\overline{\Delta v_i \cdot \Delta v_j} = \frac{e^{-\frac{T_o}{RC}}}{C^2}\int_0^{\frac{T_o}{2}}\int_0^{\frac{T_o}{2}} e^{\frac{x+y}{RC}} \overline{i_n\left(x + \frac{(i-1)T_o}{2}\right) i_n\left(y + \frac{(j-1)T_o}{2}\right)} dx dy \,.$$

The random variable $\Delta\tau_i$ characterizing the fluctuations of the duration of the ith half period is merely Δv_i divided by the slope of the capacitor voltage at the transition time[5]. Consequently, we can write the half-cycle-to-cycle jitter as

$$\overline{\Delta\tau_i \cdot \Delta\tau_j} = \frac{e^{-\frac{T_o}{RC}}}{S_i S_j C^2}\int_0^{\frac{T_o}{2}}\int_0^{\frac{T_o}{2}} e^{\frac{x+y}{RC}} \overline{i_n\left(x + \frac{(i-1)T_o}{2}\right) i_n\left(y + \frac{(j-1)T_o}{2}\right)} dx dy \,, \tag{2}$$

where S_i is the slope of the capacitor voltage at the end of the ith half-period (a signed number). Evaluation of this integral is possible only after knowing the fluctuation properties of i_n.

3.2 Formulation of phase noise generated by white noise

In the case of white noise, $\overline{i_n(t) i_n(t')} = i_{nw}\delta(t-t')/2$, where i_{nw} is the amplitude of the single-sided PSD of the white noise source. Equation (2) dictates that in this case the half-cycle-to-cycle jitter is zero for any $i \neq j$. That is, the variations of the duration of all half-periods are mutually independent. By setting $i = j$ in (2), the half-period jitter is found to be

$$\overline{\Delta\tau_i^2} = \frac{Ri_{nw}}{4S_i^2 C}\left(1 - e^{-\frac{T_o}{RC}}\right),$$

where, for simplicity, we have assumed that the slope of the waveform is the same for all falling and rising edges. Using (1) we can calculate the period jitter as

$$\overline{\Delta T_i^2} = \frac{Ri_{nw}}{2S_i^2 C}\left(1 - e^{-\frac{T_o}{RC}}\right). \tag{3}$$

The variance of the duration of k consecutive periods, called cumulative jitter, is k times this number and grows linearly with k (or, equivalently, with the total duration under consideration). This result is essential for the formulation of phase noise presented in this sub-section. Although our proof of the linear dependency of cumulative jitter on k is limited to the circuit of Fig. 1, it is a valid approximation if the following conditions are satisfied. First, all of the noise sources in the system should be white, and second, all poles of the system should be significantly larger than the offset frequency at which we calculate phase noise.

The proof of this supposition is based on elementary circuit theory. Consider an oscillator with P state variables and, therefore, P poles, the smallest of which is denoted by P_s. Also assume that there are several white noise sources in this system. We select the time interval T_T significantly larger than $1/P_s$ and much smaller than $1/\Delta f$ when $\Delta f = f - f_o$ is the offset frequency. Note that this is possible only if P_s is much larger than Δf as required above. Since T_T is much larger than all of the poles in the system, the values of the state variables at time $2T_T$ are approximately independent of their values at time T_T. Thus, state variables are only a function of the behavior of noise sources in the time interval between T_T and $2T_T$. Since these sources are assumed to be white, their behavior between T_T and $2T_T$ is independent of their behavior between 0 and T_T. Thus the jitters, which are uniquely given by the value of the state variables at the end-points of the cycles, are mutually independent for different T_T-long intervals of time. For a total time of kT_T the total cumulative jitter grows linearly with k and is k times the cumulative jitter in each of these intervals. The distribution of jitter inside each of these time intervals is insignificant for the phase noise at Δf because T_T is much smaller than $1/\Delta f$. We can then assume that the distribution of jitter inside each of these time intervals is uniform. With this assumption, jitter will grow linearly with time inside each T_T-long interval of time as well. Consequently our formulation of phase noise due to white noise, which is based on the linear growth of jitter with time, is valid for all circuits that satisfy the two aforementioned conditions.

Knowing that the variance of the cumulative jitter grows linearly with time, we can analytically calculate the PSD of the signal given in Fig. 1. This analysis is performed elsewhere[6] for the Guassian distributed jitter, and it is shown that the spectrum of phase noise around the first harmonic can be approximated by

$$PN(\Delta f) = \frac{f_o^3 \overline{(\Delta T)^2}}{\left(\pi f_o^{\,3} \overline{(\Delta T)^2}\right)^2 + (f - f_o)^2}. \tag{4}$$

Equation (4) dictates that the phase noise around the first harmonic can be approximated by a Lorentzian function. Stratonovich[4] shows that the phase noise of a noisy sinusoidal signal can also be approximated by such a function. In fact, this result is quite general and applies to any periodic signal (regardless of its shape) as long as the square root of the period jitter is much smaller than the period and the cumulative jitter grows linearly with time. The first of these conditions is satisfied in any circuit that one could practically call an oscillator. The second condition was discussed earlier.

Equation (4) shows that the far-out phase noise drops as $1/(\Delta f)^2$ when $\Delta f = f - f_0$ is the offset frequency. This far-out phase noise behavior is well-known from measurement results[7] and other theoretical work[3] and is independent of the choice of definition for phase noise. The phase noise, however, becomes flat in the vicinity of the carrier. This latter result is dependent upon the choice of definition for phase noise.

3.3 Formulation of phase noise generated by colored noise

In the presence of colored noise, the formulation of phase noise becomes complicated because the autocorrelation of the i_n is no longer a delta function and cycle-to-cycle jitter can be non-zero for $i \neq j$. In this subsection, we assume that the autocorrelation function of colored noise is Lorentzian. The effect of 1/*f* noise can be captured by modeling it as the sum of several Lorentzian sources.

The power spectral density and autocorrelation function of a single Lorentzian-shape current noise source are given by $S_i(\omega) = i_{nl}/(1+\omega^2\theta^2)$ and $\overline{i_n(t)i_n(t')} = i_{nl}/(4\theta)\exp(-|t-t'|/\theta)$, respectively, where i_{nl} is the amplitude of the single-sided PSD at ω=0, and θ determines how fast the autocorrelation function drops with time. Using this autocorrelation function, (2) reduces to:

$$\overline{\Delta\tau_i^2} = \frac{i_{nl}R^2}{4S_i^2(R^2C^2-\theta^2)}\left(-\theta-\theta e^{-\frac{T_o}{RC}} + RC - RCe^{-\frac{T_o}{RC}} + 2\theta e^{-\frac{T_o}{2RC}-\frac{T_o}{2\theta}}\right)$$

and

$$\overline{\Delta\tau_i \cdot \Delta\tau_j} = \frac{i_{nl}R^2}{4S_iS_j(R^2C^2-\theta^2)}\left(-\theta-\theta e^{-\frac{T_o}{RC}} + \theta e^{-\frac{T_o}{2RC}-\frac{T_o}{2\theta}} + \theta e^{-\frac{T_o}{2RC}+\frac{T_o}{2\theta}}\right)e^{\frac{-|i-j|T_o}{2\theta}}$$

for any $i \neq j$.

The above equations can be combined with (1) to calculate the period jitter and cycle-to-cycle jitter. This calculation shows that period jitter can be minimized by equalizing the slope of the signal at all transitions (rising and falling edges). This result is consistent with previous findings[7] and can be explained intuitively. The change in the duration of each half-period due to noise can be compensated by the change in the duration of the adjacent half period because the fluctuation properties of the noise source vary slowly with time, and the S_i's have different signs at the end of two consecutive half-periods. In the fully symmetric case, $S_{2i-1} = -S_{2i}$, and the effect of low-frequency colored noise is greatly suppressed.

In the fully unsymmetrical case $S_{2i-1} = \infty$ and S_{2i} is finite. Since we are usually interested in the effect of low frequency colored noise on close-in phase noise, we have $\theta \gg T_o$ and $\theta \gg RC$. The calculations of cycle-to-cycle jitter using these relationships for the fully unsymmetrical case leads to

$$\overline{T_i \cdot T_j} \approx A_\theta e^{\frac{-|i-j|T_o}{\theta}}, \tag{5}$$

where A_θ is given by

$$A_\theta = \frac{i_{nl}R^2}{4S_{2i}^2(R^2C^2-\theta^2)}\left(-\theta-\theta e^{-\frac{T_o}{RC}} + \theta e^{-\frac{T_o}{2RC}-\frac{T_o}{2\theta}} + \theta e^{-\frac{T_o}{2RC}+\frac{T_o}{2\theta}}\right). \tag{6}$$

Equation (5) is exact only for $i \neq j$.

Equation (5) shows that the cycle-to-cycle jitter drops in a manner similar to a Lorentzian autocorrelation function. This result is essential for the phase noise formulation presented in this sub-section. Although our proof of this phenomenon is limited to the circuit of Fig. 1, it is a valid approximation if the following conditions are satisfied. First, the only noise source of the circuit should be a Lorentzian noise, and second, the smallest pole of the system should be much larger than the offset frequency at which we calculate phase noise. The proof is similar to the argument presented in the case of white noise, where the cumulative jitter grows linearly with time. Thus, the formulation presented in this section is valid for all circuits in which the smallest pole of the system is much larger than the offset frequency at which we calculate the phase noise.

The phase noise of a signal with cycle-to-cycle jitter given in (5) is calculated elsewhere[6] as

$$PN(j\omega) = \frac{\displaystyle\sum_{k'=0}^{\infty} \frac{\left(-\frac{\omega^2 E_\theta}{2}\right)^{k'}}{k'!} \frac{f_o^3\left(D_\theta + \frac{2k'T_o}{\omega^2\theta}\right)}{\left(\pi f_o^{\,3}\left(D_\theta + \frac{2k'T_o}{\omega^2\theta}\right)\right)^2 + (f-f_o)^2}}{\displaystyle\sum_{k'=0}^{\infty} \frac{\left(-\frac{\omega^2 E_\theta}{2}\right)^{k'}}{k'!}} \tag{7}$$

which is in fact the sum of several Lorentzian functions. C_θ, D_θ and E_θ are given by

$$C_\theta = -2A_\theta \frac{e^{-\frac{T_o}{\theta}}}{\left(1 - e^{-\frac{T_o}{\theta}}\right)^2} \quad ,$$

$$D_\theta = A_\theta\left(1 + \frac{2e^{-\frac{T_o}{\theta}}}{\left(1 - e^{-\frac{T_o}{\theta}}\right)}\right) \quad ,$$

$$E_\theta = 2A_\theta \frac{e^{-\frac{T_o}{\theta}}}{\left(1 - e^{-\frac{T_o}{\theta}}\right)^2} .$$

In the presence of multiple independent noise sources in a system, the total cycle-to-cycle jitter will be the sum of the cycle-to-cycle jitters generated by individual sources. It is easy to show that in this case the cumulative jitter is also the sum of the cumulative jitters generated by individual sources. After calculating the accumulated jitter, we can calculate the phase noise using the method presented by Navid et al[6].

4. THE CHARACTERISTICS OF THE CLOSE-IN PHASE NOISE

In this section we use our phase noise formulation to discuss the properties of close-in phase noise. We first examine the general shape of the phase noise spectrum in the presence of various kinds of noise sources and discuss the validity of some of the generally-accepted beliefs about this spectrum. We then explain the differences between the properties of the close-in phase noise and that of the far-out phase noise. Our analysis shows that some of the approximations, which are valid for far-out phase noise, are not acceptable for close-in phase noise. Finally, we discuss various ways of suppressing

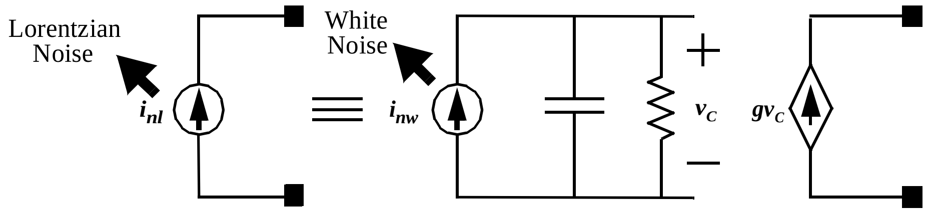

Fig. 4. White-noise equivalent network for a Lorentzian noise source.

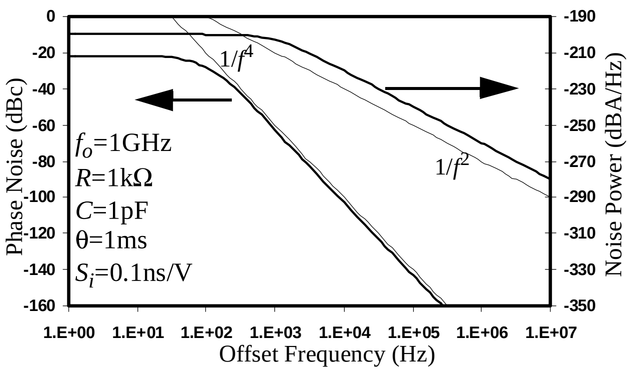

Fig. 5. Phase noise spectrum generated by a Lorentzian noise sources

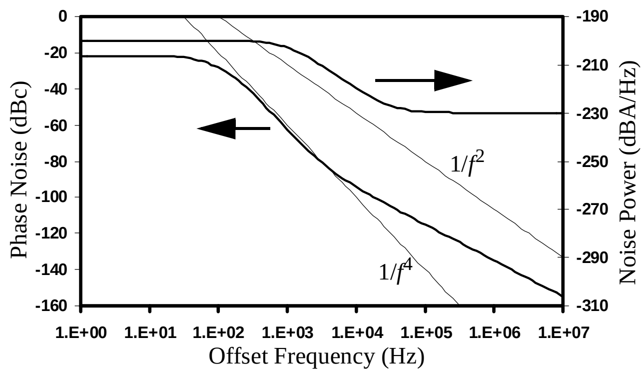

Fig. 6. Phase noise generated by a combination of Lorentzian and white noise sources

the effect of low-frequency, colored noise on phase noise.

Contrary to popular belief, the spectrum of the phase noise is not necessarily Lorentzian in the absence of colored noise. According to our formulation, the phase noise has a Lorentzian shape in the vicinity of the oscillation frequency only if we assume that the cumulative jitter grows linearly with time. As discussed earlier, the validity of this assumption requires that not only all of the noise sources in the system be white but also all poles of the system be significantly larger than the offset frequency at which we calculate the phase noise. This second condition is not necessarily satisfied for circuits containing only white noise sources. In fact, from a circuit-theory point of view, a colored noise source can often be reconstructed using a network of white noise sources and noise-free electronic components. Fig 4 shows the reconstruction of a Lorentzian noise source as an example. The reconstruction of $1/f$ noise sources is straightforward if we notice that a $1/f$ spectrum is the sum of several independent Lorentzian sources. Using such reconstruction networks, we can start from an arbitrary oscillatory system and replace the colored noise sources with their white-noise equivalent networks to arrive at a system with only white noise sources. We would expect the phase noise spectrum of this system to have a Lorentzian shape if the absence of colored noise sources were a sufficient condition for having a Lorentzian-shape phase noise spectrum. That is, we would expect the phase noise to be Lorentzian in all systems. However, this result is experimentally shown to be invalid[7].

The fallacy of this result can also be shown using our phase noise formulation. This formulation in the presence of colored noise sources shows that the phase noise of a system with one Lorentzian noise source is the sum of several Lorentzian functions. Figure 5 shows such a phase noise spectrum along with the PSD of the noise source generating this phase noise. The numerical parameters used in this simulation are given in the inset of this figure. Note that the spectrum of phase noise has a $1/f^4$ shape at far-out frequencies.

Figure 6 shows that in the co-presence of independent Lorentzian and white-noise sources, the spectrum of phase noise eventually returns to a nearly-$1/f^2$ shape at far-out offset frequencies. This result is expected because at high frequencies the white noise eventually dominates the Lorentzian noise. This analysis is performed using the superposition properties of cumulative jitter, as explained earlier. The numerical parameters used in this simulation are the same as the ones given in the inset of Fig. 5.

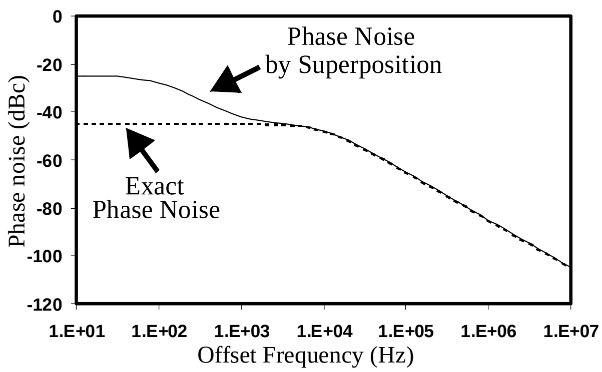

Fig. 7. Phase noise calculation using superposition.

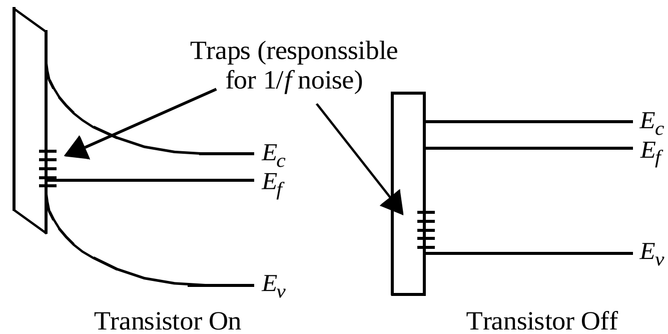

Fig. 8. Relative position of traps and the Fermi level in On and Off states.

One of the generally-accepted approximations about the spectrum of far-out phase noise is superposition[7]. Unfortunately, this approximation is invalid for close-in phase noise. To see this, consider the oscillator in Fig. 3 with only one noise source $\overline{i_n(t)i_n(t')} = i_{nw}\delta(t - t')/2$. The phase noise of this oscillator is given by (4) in the vicinity of the fundamental frequency. We can rewrite the noise source as $\overline{i_n(t)i_n(t')} = i_{nw}\delta(t - t')/200 + 99i_{nw}\delta(t - t')/200$. According to superposition, the spectrum of phase noise would be the sum of the phase noise spectra generated by individual sources (with appropriate normalization). Figure 7 shows the phase noise calculated using superposition and the directly-calculated phase noise. This graph clearly shows that the superposition approximation is valid only for far-out phase noise and breaks down at small offset frequencies.

Our analysis of phase noise can explain why the effect of low-frequency, colored noise on oscillators' phase noise can be suppressed by noise source switching. To suppress the effect of non-white noise, we need to force the cumulative jitter to grow linearly with time. The cumulative jitter grows linearly with time if, and only if, the jitter in each period is independent of the jitters of the previous cycles. If the system does not have a memory of the jitters induced in the previous cycles, its phase noise will be Lorentzian and the effect of the colored noise will be suppressed. The memory of the system can be reduced by periodically switching the noisy devices on and off. For example, the basic device physics for MOS devices shows that switching these devices moves the relative location of the Fermi level to the trap sites responsible for 1/f noise (Fig. 8). Thus, the trap sites that are located in the vicinity of the Fermi level during the 'on' state move to locations significantly higher or lower than the Fermi level due to switching and their occupancy becomes relatively deterministic during the off time. Once the device is switched back on, its noise properties are only functions of the initial conditions generated during the off time and are relatively independent of what had happened in the previous on-time. In effect, if we periodically switch the device on and off, it loses its memory of what had happened in the previous 'on' times, which means that it will have less colored noise. The experimental data supports this suppression of 1/f noise in switched MOS circuits[8,9]. This phenomenon is responsible for the experimentally-observed suppression of $1/f^3$ phase noise in single-ended ring oscillators[7].

Another way of suppressing the effect of low-frequency, colored noise on phase noise is symmetrization. Since low-frequency colored noise sources have a rich content at low frequencies, their fluctuation properties change slowly with time. Consequently, if we symmetrize the signal in terms of duty cycle and rise/fall slope, we can compensate for the effect of jitter in one half-period by its effect in the other half-period. However, the symmetrization techniques can only be useful for the noise sources which are active during the whole period. For example, this technique is effective for suppression of the effect of the noise sources associated with the tail current source in differential ring oscillators. On the other hand, this technique is ineffective for noise sources which are present only in half of the period such as MOSFET device noise in single-ended ring oscillators. In this case, the symmetrization of the waveform has an insignificant effect on phase noise because the noise of the PMOS and NMOS devices are independent, and only one of them is active in each half-period. In single-ended ring oscillators, the symmetrization can only suppress the effect of the noise of the short circuit time during which both devices conduct. It is then clear that the main mechanism of suppression of phase noise in single-ended ring oscillators is the switching effect described earlier.

5. CONCLUSION

We have studied the characteristics of close-in phase noise in ring oscillators and showed that some of the approximations which are routinely used for far-out phase noise are not acceptable at close-in frequencies. Unlike the far-out phase noise, the behavior of close-in phase noise is dependent upon the choice of definition between the two widely accepted definitions of phase noise. We compared these two definitions of phase noise and chose the definition of phase noise as the normalized PSD of the signal for this study. We then presented analytical formulation of phase noise for a square-wave periodic signal and discussed some of its properties. These properties were shown to be general and not dependent upon the choice of oscillator.

We showed that phase noise has a Lorentzian spectrum if we assume that the cumulative jitter grows linearly with time. To satisfy this condition, in addition to the absence of any colored noise, the system must not have any poles at frequencies comparable to the offset frequency at which we calculate the phase noise. Therefore white noise sources can, in principle, generate non-Lorentzian phase noise spectra. In practice, however, the deviation of phase noise spectrum from a Lorentzian shape is usually an indication of the presence of a non-white noise source because well-designed oscillators rarely have a pole at frequencies comparable to the offset frequency at which we measure phase noise. We also discussed the superposition approximation and showed that this approximation is valid only for far-out phase noise and breaks down at small offset frequencies.

We showed that the suppression of the effect of low-frequency colored noise on the oscillator's phase noise is possible by switching the noise sources on and off periodically or by symmetrization of the waveform. In single-ended ring oscillators, the switching of transistors is the main suppression mechanism of the effect of $1/f$ noise on the phase noise. On the other hand, symmetrization is most effective for the noise sources which are always on, such as the tail current source in differential ring oscillators. These findings provide insight for efficient design of low-phase-noise electrical oscillators.

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