Factors Affecting the Stylus/Groove Relationship in Phonograph Playback Systems *

C. R. BASTIAANS

Westinghouse Research Laboratories, Pittsburgh, Pennsylvania

It is shown that a phonograph pickup stylus riding in the groove of a record partly penetrates the groove walls because of elastic and plastic deformation of the record material. At high bearing loads complete plastic flow sets in and the needle leaves a permanent indentation track, while at lower loads the elastic deformation is predominant. This leads to amplitude distortion in the reproduced signal which may be of two types: one which is a function of the recorded wavelength (*G* function or translation loss), the other a function of the dynamic moving mass of the stylus/armature (*H* function or stylus/groove resonance). A third phenomenon (*S* function or scanning loss) is caused by the finite size of the stylus/groove-wall contact surface.

Experiments with specially built pickups show the evolved theory to be valid even for very high frequencies. Special test records with recorded frequencies up to 100,000 Hz were used for these experiments.

INTRODUCTION In this paper, the mechanical behavior of a pickup stylus† tracking a phonograph record groove is analyzed, using the Hertzian theories on the elastic deformation of two curved bodies in mutual contact under the influence of a force. The validity of the theories is then checked by means of measured response curves of various pickups on specially made calibrated test records.

This analysis is based on the assumptions that: *a*) the pickup is an ideal mechanical system without any spurious resonances; *b*) stylus/groove contact is always maintained; *c*) the playback stylus is a cone with a spherical tip, made of a very hard material such as sapphire or diamond; *d*) the groove profile is always symmetric with respect to the normal to the record surface; *e*) the included bottom angle of the groove equals 90°; *f*) the groove modulation is sinusoidal, in the lateral plane; *g*) the radius of curvature of the modulated groove in the lateral plane is never less than 1.5 times the stylus tip radius; *h*) nonlinear distortion components and higher harmonics are neglected; *i*) the mechanical behavior of the record material obeys Hooke's Law; *j*) the record material is homogeneous; *k*) the turntable rotates with a constant angular velocity; *l*) extraneous forces other than the vertical bearing force exerted by the stylus in the groove are neglected (forces such as those due to stylus/groove friction, tone arm bearing friction, pinch effect, side-thrusts due to offset tone arms, non-level turntables, warped or eccentric records).

STYLUS/GROOVE FIT

Consider Fig. 1 which depicts a cross-section in a plane through the center of the stylus tip (pictured as a sphere), perpendicular to the record surface and the direction of the groove. This groove does not carry any information, i.e., it is a silent groove. Under the influence of the vertical needle force F_v, the groove walls will "give" and the needle tip will penetrate the walls until a static balance of forces is reached.

Hertz[1] has derived formulas to calculate the elastic deformation of two curved bodies in mutual contact under the influence of a force. In the case of a stylus in a groove (see Fig. 1), the equation expressing the normal force in terms of the vertical stylus force F_v reads*:

$$\delta_0 = \frac{\psi}{(2k^2)^{1/3}} \left[\left(\frac{F_v}{2\sin\beta} \right)^2 \cdot \frac{1}{R} \left(1 + \frac{R}{2\rho} \right) \right]^{1/3}. \quad (1)$$

Since $\beta = 90°/2$, Eq. (1) simplifies to:

$$\delta_0 = \frac{\psi}{(2k^2)^{1/3}} \left[\frac{F_v^2}{2R} \left(1 + \frac{R}{2\rho} \right) \right]^{1/3}. \quad (2)$$

From published tables[1] we find that ψ is practically constant and equal to 2 for the geometrical configurations prevailing in the stylus/groove fit (sphere/cylinder or

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† To conform with current usage, the term "stylus" rather than "needle" is used throughout this paper, despite the author's feeling that a semantic differentiation would be advisable between a (cutting) stylus and a (playback) needle—a differentiation that does exist in the other major languages.

sphere/plane), provided the radius of groove curvature is not less than 1.5 times the stylus tip radius R. Since

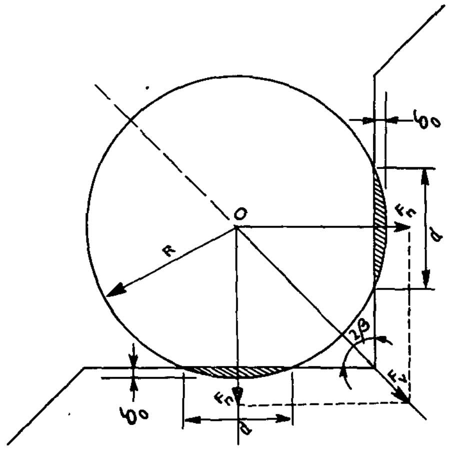

Fig. 1. Vertical cross-section of a spherical stylus tip with radius R penetrating the walls of a silent groove to a depth of δ_0 under the influence of stylus force F_v.

the stylus tip is very much harder than the record material, Hertz's coefficient k is given by

$$k = 8E/[3(1-\nu^2)] = (8/3)E_0. \quad (3)$$

Therefore, Eq. (2) becomes:

$$\delta_0 = \left(\frac{9}{16E_0^2}\right)^{1/3}\left[\frac{F_v^2}{2R}\left(1+\frac{R}{2\rho}\right)\right]^{1/3} \quad (4)$$

$$= K\left[\frac{F_v^2}{2R}\left(1+\frac{R}{2\rho}\right)\right]^{1/3},$$

defining K as: $K = \psi/(2k^2)^{1/3} = (9/16E_0^2)^{1/3}$.

In a silent groove, the radius of groove curvature $\rho = \infty$, so that Eq. (4) becomes:

$$\delta_0 = K(F_v^2/2R)^{1/3}. \quad (5)$$

The span of the contact surface or indentation width d is, according to Hertz, equal to:

$$d = 2(\delta_0 R)^{1/2}. \quad (6)$$

As an example, the actual value of indentation has been calculated in a concrete case. Given a pickup with the following characteristics: $F_v = 2 \times 10^{-2}$ N (approx. 2 g); $R = 17.8 \times 10^{-6}$ m (approx. 0.7×10^{-3} in.); $E = 3.3 \times 10^9$ N/m^2 for Vinylite (a copolymer of vinyl chloride acetate); $\nu = 0.35$ for most plastics; $E_0 = 3.76 \times 10^9$ N/m^2 (5.45×10^5 psi); $K = 3.42 \times 10^{-7}$ m$^{4/3} \cdot$ N$^{-2/3}$. Then by Eq. (5) the *depth of penetration* $\delta_0 = 0.765 \times 10^{-6}$ m ($.03 \times 10^{-3}$ in.), and by Eq. (6) the *width of contact surface* $d = 7.37 \times 10^{-6}$ m ($.29 \times 10^{-3}$ in.).

It should be pointed out again that the very often used expression "stylus pressure" actually refers to the stylus *force*. Pressure is defined as *force per unit area*, and a little calculation will show that the actual stylus *pressure* exerted by this 2 gram pickup amounts to $.331 \times 10^9$ N/m^2 (4.8×10^4 psi). This pressure is much higher than the yield point of record plastics (around 10^8 N/m^2 = 14,500 psi). It may therefore be expected that *plastic flow* will set in, either wholly or partly.

Figure 2 shows two straight lines giving the theoretical indentation width d caused by 2 different stylus tip radii ($17.8\ \mu = 0.7$ mil and $11\ \mu = 0.43$ mil) exerting normal forces in the range from .001 to .1 N (approximately .1 to 10 g) as calculated by Eqs. (4), (5) and (6).

Practical measurements of the widths of indentation tracks left on a blank Vinylite record by styli with the above-mentioned tip radii under a range of bearing loads of 1 to 10 g show that, within the range of interest, high bearing loads leave track widths that measure as predicted by the Hertzian formulas for elastic deformation (refer to Fig. 2). This leads to the conclusion that under these circumstances *complete plastic flow* set in after the initial elastic deformation, and a permanent indentation of a width equal to that of the elastic deformation was left.

In the lower range of bearing loads (see Fig. 2), however, the width of the (permanent) indentation track left by the stylus is much less than predicted by the theory. Viscoelastic materials like Vinylite do not exhibit a sharp division between the elastic and plastic region. Any stress will produce a response with both elastic deformation and viscous flow, the magnitudes of which depend not only on the properties of the material but also on the length of time during which the stress is applied.

A convenient way to find the track width due to both elastic and plastic deformation is to deposit a soap film on the record surface. Such a film is microscopically thin, and should not interfere with the phenomena to be investigated since it is very much softer than the Vinylite.

The needle will now leave a track with a width equal to the total deformation, purely elastic in the low range, plastic in the high range and both elastic and plastic in between. The findings are shown in the measured curves of Fig. 2. A surprising fact shows up: below a certain load the track width is even smaller than the theoretical value! Above this load, the same widths as without the soap film are found, confirming that these

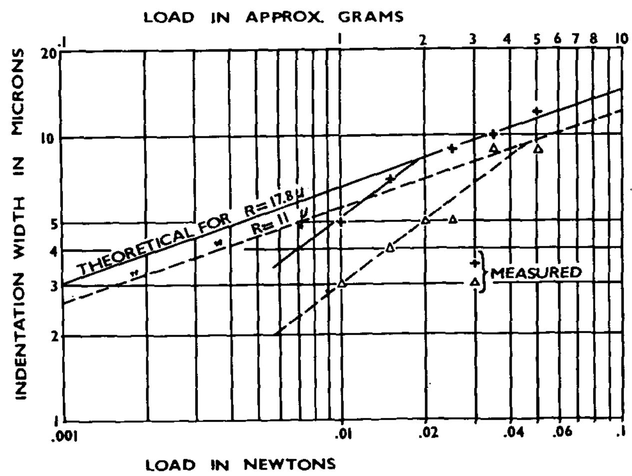

Fig. 2. Track widths left by two different stylus tips on Vinylite, calculated and measured for a range of stylus forces.

* Refer to Appendix I for definitions of all symbols.

tracks are indeed caused by complete plastic flow with very little or no elastic springback. Below the critical load the material seems to behave as a much stiffer material; note the steeper slope of the lines obtained when the measured values are connected.

This "knee" in the elastic behavior of Vinylite has been found by others, via other approaches. Hunt[2] has suggested the existence of a *size effect* similar to the effect observed in tiny metal whiskers that are known to be extremely strong only because of their complete lack of flaws. Barlow[3] has objected to this size effect and explains the thinner permanent track widths at lower loads by elastic component recovery. His theory, however, is contradicted by our findings with the soap film. An interesting explanation was given by Walton[4] who suggested a phenomenon very aptly termed "surfboard action." When the groove speed exceeds some critical value, the applied load apparently is supported only partially by the elastic reaction of the material, the balance being supported by the dynamic lifting action on the stylus.

This investigation has been far from exhaustive and a vast field of exploration in the rheological behavior of the record material remains. In measuring the indentation track widths on the record surface it is found that there is a *variation* in the value of the stiffness with position around the record and with radial distance from the center of the record. Work hardening and/or skin effect are possible causes of the anomalies sometimes found in practical measurements of indentation tracks on records.

A last word on plastic flow: It is a known fact that the highest shear stress in the material below an indenter occurs at a point about half a contact-circle radius *below* the center of contact (Davies[5]). It has been suggested[6] that in the stylus/groove contact, sub-surface yielding begins near a load of .150 g and plastic yielding at the surface starts at loads in the range from 1 to 1.6 g (using a stylus tip radius of 17.8 μ).

The fact that a phonograph record is not completely ruined by a first playing at even 5 g bearing weight is most probably due to the fact that fresh material is constantly presented to the stylus tip when the pickup plays the moving groove. Microscopic examination of a record groove that has been played once with a stylus force of only 2 g will, however, show that a slight permanent indentation track is already left on both groove walls. It has also been shown that repeated playings (with no intervening periods of time) of a small band of the record bring about an accelerated wear of the record groove. Apparently the sub-surface shear stresses are not immediately dissipated and the accumulating abuse leads to an early breakdown of the record material. Clearly the time factor plays an important part in the rheological behavior of the plastic.

PLAYBACK LOSS

It is interesting to examine the mechanical phenomena that occur in a modulated groove because of the effects discussed above, limiting the discussion with the justifiable assumption that we have to deal with *elastic deformation* only. As early as 1941[7] and 1942[8] analyses have been made of the playback loss due to elastic deformation of the modulated groove. A very elaborate investigation was conducted by Miller in 1950[9]; a few years later, Kantrowitz worked through Miller's analysis again, with special regard to vertical recording.[12] A different ap-

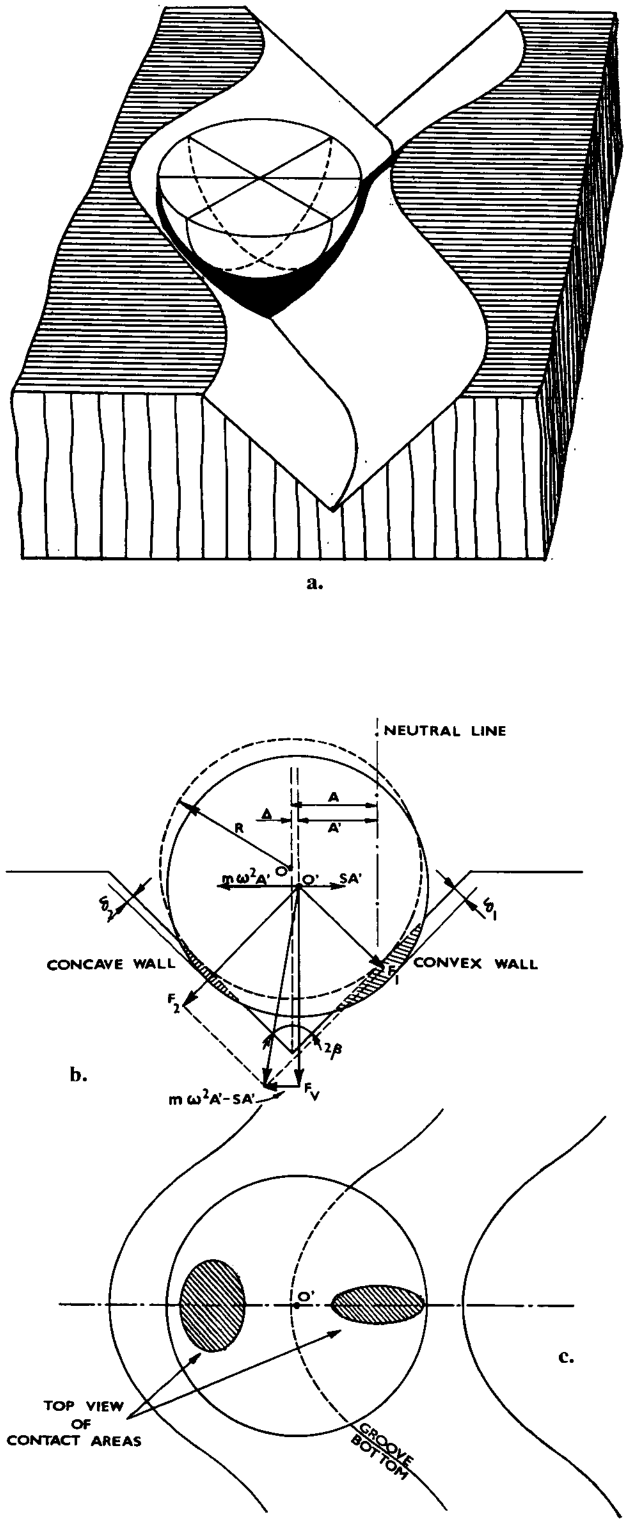

Fig. 3. **a.** Perspective view of a stylus tip (shown as a semi-sphere) sitting in the crest of a sinusoidally modulated groove. **b.** Vertical cross-section of a spherical stylus tip with the radius R, penetrating the walls of a modulated groove. The penetration into the convex wall is deeper than in the concave wall. **c.** Top view of the situation depicted in Figure 3b.

proach was given by Kerstens[10] in 1956. The interested reader is referred to the relevant publications for more detailed information. In the following analysis a very much simplified approach is given, neglecting higher order components and tracing distortion.

When the tip of a stylus sitting on the crest of a sinusoidally modulated groove (see Fig. 3a) is moved in the lateral plane by the modulated groove, two lateral forces come into play; these are (see Fig. 3b):

1. A stiffness force SA', where S is the suspension stiffness (or the inverse of the compliance) of the needle-armature and A' is the actual excursion, relative to the neutral line (silent groove), made by the tip.

2. An acceleration force $m\omega^2A'$ required to move the dynamic mass m of the stylus-armature (assumed to be concentrated in the center of the stylus tip).

These forces oppose each other, so that the vertical stylus force F_v is altered by a lateral component ($m\omega^2A' - SA'$). Decomposed components F_1 and F_2 exert forces, respectively, on the convex and the concave groove wall. Notwithstanding the fact that $F_2 > F_1$, the indentation in the concave wall will be less than that in the convex wall; this is because much more supporting area is presented by the concave wall to the sphere, so that a balance between the indenting force and the opposing force exerted by the groove wall is obtained with less penetration. Figure 3a makes it apparent that the concave wall "wraps around" the sphere, whereas the convex wall presents a "ridge" to it.

The net result is that the center of the sphere will be deflected from the ideal position (obtainable only with an infinitely stiff record) by an amount

$$\Delta = A - A' = (\delta_1 - \delta_2)\cos\beta. \tag{7}$$

The wall forces F_1 and F_2 can be expressed as follows:

$$F_{1,2} = \frac{F_v}{2\sin\beta} \pm \frac{(S - m\omega^2)A'}{2\cos\beta}. \tag{8}$$

Using the definition of K given in Eq. (4), the elastic deformation formula proposed by Hertz[1] becomes, in this case,

$$\delta = K\left[\frac{F_n^2}{R}\left(1 + \frac{R}{2\rho}\right)\right]^{1/3}. \tag{9}$$

The radius of curvature as calculated in Appendix II is:

$$\rho = V^2/(\omega^2A); \tag{10}$$

this, however, is the curvature in the lateral plane. In a plane perpendicular to a groove wall the radius of curvature equals:

$$\rho_{1,2} = \pm\,[V^2/(\omega^2A\cos\beta)] \tag{11}$$

In practice, the included groove angle is very close to 90°; assume it to be exactly 90° in order to simplify the analysis to some extent. Substituting in Eq. (9),

$$(F_{1,2})^2 = \frac{F_v^2}{2}\left[1 \pm \frac{(S - m\omega^2)A'}{F_v}\right]^2 \text{ and } 2\rho_{1,2} = \pm\frac{2\sqrt{(2)}V^2}{\omega^2A}$$

one obtains

$$\delta_{1,2} = K\left(\frac{F_v^2}{2R}\right)^{1/3}\left(1 \pm \frac{(S - m\omega^2)A'}{F_v}\right)^{2/3}\left(1 \pm \frac{R\omega^2A}{2\sqrt{(2)}\,V^2}\right)^{1/3}. \tag{12}$$

At this point replace S by:

$$S = m\omega_0^2, \tag{13}$$

where ω_0 is the free resonance of the pickup system. Equation (7) can then be rewritten as follows:

$$\Delta = \frac{K}{\sqrt{(2)}}\left(\frac{F_v^2}{2R}\right)^{1/3} \left\{\left[1 + \frac{m(\omega_0^2 - \omega^2)A'}{F_v}\right]^{2/3}\left(1 + \frac{R\omega^2A}{2\sqrt{(2)}\,V^2}\right)^{1/3} - \left[1 - \frac{m(\omega_0^2 - \omega^2)A'}{F_v}\right]^{2/3}\left(1 - \frac{R\omega^2A}{2\sqrt{(2)}\,V^2}\right)^{1/3}\right\}. \tag{14}$$

This expression can be simplified, since only the high frequency region is of interest, so that $\omega_0 < \omega$ (f_0 is usually around 1500 Hz for most pickups); thus, $\omega_0^2 << \omega^2$. Further applying the binomial theorem (introducing an error of less than 5% in the worst case):

$$\frac{R\omega^2A}{2\sqrt{(2)}\,V^2} < 1, \text{ therefore}$$

$$\left(1 \pm \frac{R\omega^2A}{2\sqrt{(2)}\,V^2}\right)^{1/3} \approx 1 \pm \frac{R\omega^2A}{6\sqrt{(2)}\,V^2}$$

and

$$\frac{m(\omega_0^2 - \omega^2)A'}{F_v} << 1, \text{ therefore}$$

$$\left(1 \mp \frac{m\omega^2A'}{F_v}\right)^{2/3} \approx 1 \mp \frac{2m\omega^2A'}{3F_v}.$$

The lateral displacement of the sphere's center can now be written as:

$$\Delta \approx \frac{K}{\sqrt{(2)}}\left(\frac{F_v^2}{2R}\right)^{1/3}\left[\frac{R\omega^2A}{3\sqrt{(2)}\,V^2} - \frac{4m\omega^2A'}{3F_v}\right]. \tag{15}$$

The actual amplitude of excursion is thus, after some rearranging, found to be:

$$A' = A - \Delta = A \cdot \frac{[1 - K(F_v^2/2R)^{1/3}(R\omega^2/6V^2)]}{[1 - K(F_v^2/2R)^{1/3}(2\sqrt{(2)}m\omega^2/3F_v)]}. \tag{16}$$

The derivation of this equation involves no mechanical damping, either in the pickup system proper or manifested as viscous losses in the record material. Consequently, we may find a situation where the actual amplitude A' is infinitely high, i.e., poles of A' (denominator of the fractional portion of Eq. (16) equals zero) and a situation where the actual amplitude A' is zero, i.e., zeros of A' (numerator equals zero). The numerator is generally called the translation loss function; the denominator is termed the stylus/groove resonance function.

STYLUS/GROOVE RESONANCE

Equating the denominator in Eq. (16) to zero, and substituting K from Eq. (4), one obtains:

$$\omega_r^2 = \frac{3(2F_vR)^{1/3}}{2\sqrt{(2)}\,mK} = \frac{(96E_0^2F_vR)^{1/3}}{2\sqrt{(2)}\,m} \qquad (17a)$$

or

$$f_r = [0.6362/\pi\sqrt{(m)}](E_0^2F_vR)^{1/6} \qquad (17b)$$

where f_r = stylus/groove resonant frequency.

The stylus/groove resonance may also be expressed as follows:

$$f_r = 1/[2\pi\sqrt{(mC)}] = \sigma^{1/2}/2\pi m^{1/2}$$

where C = record compliance and σ = record stiffness, from which we obtain:

$$C = 0.617(E_0^2F_vR)^{-1/3} \qquad (18a)$$

and

$$\sigma = 1.619(E_0^2F_vR)^{1/3}. \qquad (18b)$$

From Eq. (17b) it is seen that the stylus/groove resonance frequency varies with the 1/6th power of stylus force F_v, all other parameters being held constant:

$$f_r \propto F_v^{1/6}.$$

Table I is computed using this expression; it also shows the magnitude of the record compliance C based on a tip radius R = 17.8 μ (0.7 × 10^{-3} in.) and a Vinylite record with E_0 = 3.76 × 10^9 N/m² (5.45 × 10^5 psi):

TRANSLATION LOSS

Equating the numerator in Eq. (16) to zero, we obtain:

$$\omega_c^2 = \frac{6V^2}{K}\left(\frac{2}{F_v^2R^2}\right)^{1/3} = 12V^2\left(\frac{2E_0}{3F_vR}\right)^{2/3} \qquad (19a)$$

or

$$f_c = 1.513(V/\pi)(E_0/F_vR)^{1/3}. \qquad (19b)$$

where f_c = the cut-off frequency at which the stylus does not move and hence no output from the pickup is obtained. This cutoff frequency varies inversely with the 1/3 power of stylus force F_v:

$$f_c \propto F_v^{-1/3}.$$

Again, it is illustrative to compile the cutoff frequencies that will prevail when considering a 12-in. Vinylite record played with a given pickup at various bearing weights (Table II). The same tip radius as before is used (R = 17.8 × 10^{-6} m = 0.7 × 10^{-3} in.). In all cases the outer groove diameter (O.D.) equals 292 mm (11.5 in.) and the inner groove diameter (I.D.) equals 100 mm (3.94 in.) for 78 and 45 rpm, 120 mm (4.7 in.) for 33⅓ rpm, the standard diameters in the recording field.

TABLE I. Record compliance in Terms of the stylus force.

Stylus Force F_v	*Normalized* f_r*	*Record Compliance*
(Approx. grams)		(10^{-5} m/N)
10	1.3	2.16
8	1.255	2.33
6	1.2	2.56
4	1.12	2.96
2	1	3.68
1	0.89	4.6
0.5	0.795	5.92
0.1	0.607	10

RELATIVE PICKUP RESPONSE

Introducing ω_r from Eq. (17a) and ω_c from Eq. (19a) in Eq. (16), the relative pickup response may be written as

$$\frac{A'}{A} = \frac{1-(\omega/\omega_c)^2}{1-(\omega/\omega_r)^2} = \frac{1+k_1s^2}{1+k_2s^2} = G(s)H(s). \qquad (20)$$

F. G. Miller[9] evolved an expression for the function $H(\omega/\omega_r)$ which includes a damping factor

$$\epsilon = \zeta/m\omega_r$$

where ζ = damping coefficient of the combined system pickup and record (kg/sec):

$$H(\omega/\omega_r,\epsilon) = \{[1-(\omega/\omega_r)^2]^2+\epsilon^2(\omega/\omega_r)^2\}^{-1/2}. \qquad (21)$$

This equation is presented in graphical form in Fig. 4 with parameter ϵ = 0.1 . . . 2.0. The damping factor ϵ tends to reduce f_r by a slight amount. Beyond the resonant frequency the H function approaches a decrease of 12 dB/octave asymptotically. This function may be recognized as a simple low-pass function having a pair of complex conjugate poles. Damping factor ϵ equals the inverse of the Q factor of the tuned circuit.

The function $G(\omega/\omega_c)$ is graphically depicted in Fig. 5a which will be discussed later on. Note that for $\omega > \omega_c$ the term reverses its sign; in other words, the stylus tip starts moving in a direction *opposing* the lateral groove excursions. Mathematically, A' may become larger than the original groove amplitude A. Not only would this be hard to visualize physically, but things become obscured because a third function (see section on scanning loss) comes into play, so that beyond the cutoff frequency A' will not become larger than A.

SCANNING LOSS

In the foregoing, the size of the contact surfaces between stylus tip and groove walls has been neglected.

* Based on the resonant frequency obtained with a given moving mass m at a bearing weight of 2 g. In other words, if a certain pickup exhibits a resonance of 50 kHz at a stylus force of 2 g, decreasing this force to 0.1 g (provided it can track at this low force) will bring down the resonance to .607 × 50 = 30 kHz.

TABLE II. Cutoff frequencies with various bearing weights.

Turntable Speed	*Diameter*	*Groove Speed*	*Cutoff Frequencies in kHz at*							
			5	4	3	2	1	.5	.1	Approximate Grams
			.05	.04	.03	.02	.01	.005	.001	Newtons
(RPM)		(m/sec)								
78	O.D.	1.20	93.5	101	111	127	160	202	345	
78	I.D.	.40	31	33.5	37	42.5	53.5	67	115	
45	O.D.	.69	54	58	64	73	92	116	198	
45	I.D.	.24	19	20	22	25.5	32	40	69	
33⅓	O.D.	.51	40	43	47	54	68	86	147	
33⅓	I.D.	.21	16.5	18	19.5	22	28	35	60	

Decreasing the stylus force (maintaining, of course, proper tracking abilities) moves the cutoff frequency *up* in the frequency spectrum and brings the resonant frequency *down* (Table I); it affects the cutoff frequency more than the resonant frequency.

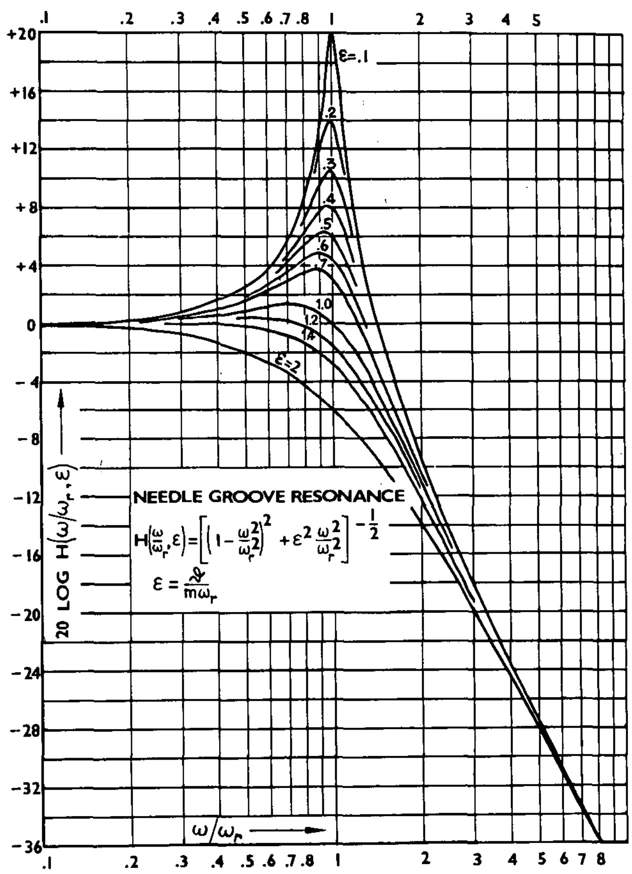

Fig. 4. Graphic presentation of the stylus/groove resonance function $H(\omega/\omega_r, \epsilon)$ for various values of damping parameter ϵ.

Should the recorded wavelength become comparable to the diameter of the contact surface, it is obvious that hardly any information can be "read" by the stylus tip. This phenomenon is termed the scanning loss and compares with the aperture loss encountered in optical and magnetic tape read-out systems. In the phonograph system, it is, however, of a more complex nature and it may partly offset the translation loss discussed in the previous sections. The two phenomena may probably not be simply added, because the scanning loss will tend to impart less amplitude information to the stylus tip and the groove wall forces F_1 and F_2 will differ less.

Miller's expression for the scanning loss function:

$$S\left(\frac{\pi d}{2\lambda}\right) = S\left(\frac{\omega d}{4V}\right) = S(p) = \tag{22}$$

$$= (2/\pi)\int(1-x^2)^{1/2}\cos 2pxdx =$$

$$= \sum_0^\infty \frac{p^{2n}}{(n!)^2}\cdot\frac{(-1)^n}{n+1}.$$

The power series of this equation has its first zero for $\pi d/2\lambda = 1.9$, from which one obtains an indentation width

$$d = 3.8V/\pi f_s, \tag{23}$$

with f_s = scanning loss null frequency.

Introducing d as found in Eqs. (6) and (5), we write

$$3.8V/\pi f_s = 2K^{1/2}2^{-1/6}(F_vR)^{1/3}$$

from which

$$\omega_s = 3.8V\cdot 2^{1/6}/K^{1/2}(F_vR)^{1/3}. \tag{24}$$

Taking the ratio of scanning loss null frequency to the cutoff frequency defined in Eq. (19b), we find that this ratio is a constant = 1.55:

$$\omega_s/\omega_c = 3.8/\sqrt{(6)} = 1.55.$$

Both the function $G(\omega/\omega_c)$ and the function $S(\omega/\omega_s)$ are graphically presented in Figure 5a, the latter curve transposed for an ordinate scale $\omega/\omega_c = 1.55\ \omega/\omega_s$.

For the fundamental of a recorded frequency, the S function may be neglected with little error, provided the recorded wavelength is large compared to the stylus/groove contact surface. As will be shown below, measured pickup response curves coincide with theoretical curves to a remarkably high degree. As has been pointed out earlier, these two functions probably may not be simply added. To illustrate the type of response that might result, however, the two effects are directly combined in one curve which is shown in Fig. 5b.

EXPERIMENTAL RESULTS

In order to check the validity of the theory under discussion, several experiments were carried out in which

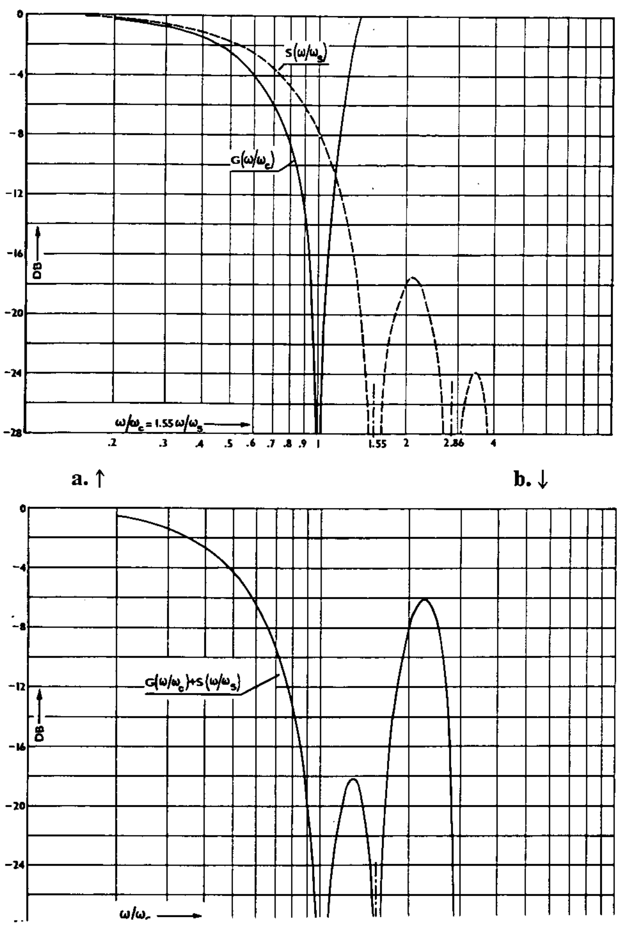

Fig. 5. **a.** Graphic presentation of cutoff function $G(\omega/\omega_c)$ and scanning loss function $S(\omega/\omega_s)$. **b.** Graphic presentation of combined cutoff (G) and scanning loss (S) functions.

measured frequency response curves were obtained and compared with the curves predictable through application of the theory.

Since commercial pickup cartridges are mostly heavily damped and their manufacturers are too often ignorant of the true value of moving mass, we used pickups of our own construction.

The performance of four different models with respect to the H and G function theories was examined. A value of 3.76×10^9 N/m^2 for E_0 is used in all the following examples; all pickups were provided with a tip radius of 17.8 μ (0.7×10^{-3} in.), and a stylus force of 1.5 g was used.

Pickup 1-3

This model had a moving mass (referred to the stylus tip) of 2 mg, and a damping factor $\epsilon = 0.3$.

Special frequency sweep records were cut with a constant stylus velocity of appropriate value, sweeping from 500 to 100,000 Hz. Subsequently, 12-in. Vinylite pressings were made; Young's modulus of the record compound used was not actually measured, but assumed to be 3.3×10^9 N/m^2 (4.8×10^5 psi).

From Eq. (17b) it is found that $f_r = 17{,}800$ Hz and from Eq. (19b) $f_c = 58{,}000$ Hz for $V = 0.495$ m/sec (33⅓ rpm at approximately the outer diameter).

Figure 6a shows the H function plotted for $\epsilon = 0.3$, and also the G function. Furthermore, the two curves are added together to obtain the dashed curve, which is then transferred to Fig. 6b where it is compared with the

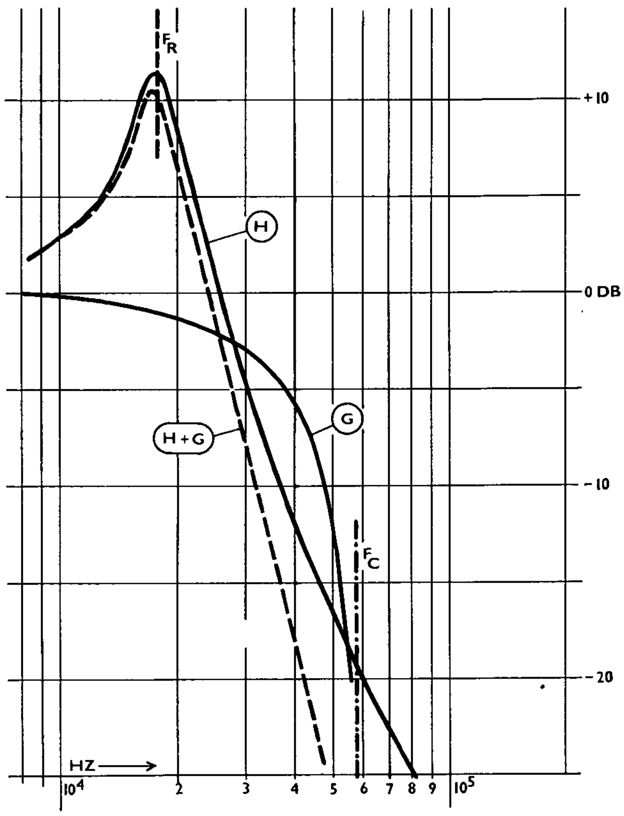

Fig. 6. **a.** Theoretical high-frequency response curve for model 1-3 pickup with moving mass $m = 2$ mg, damping $\epsilon = 0.3$, tip radius $R = 17.8$ μ and stylus force $F_v = 1.5$ g on Vinylite with groove velocity $V = 0.495$ m/sec.

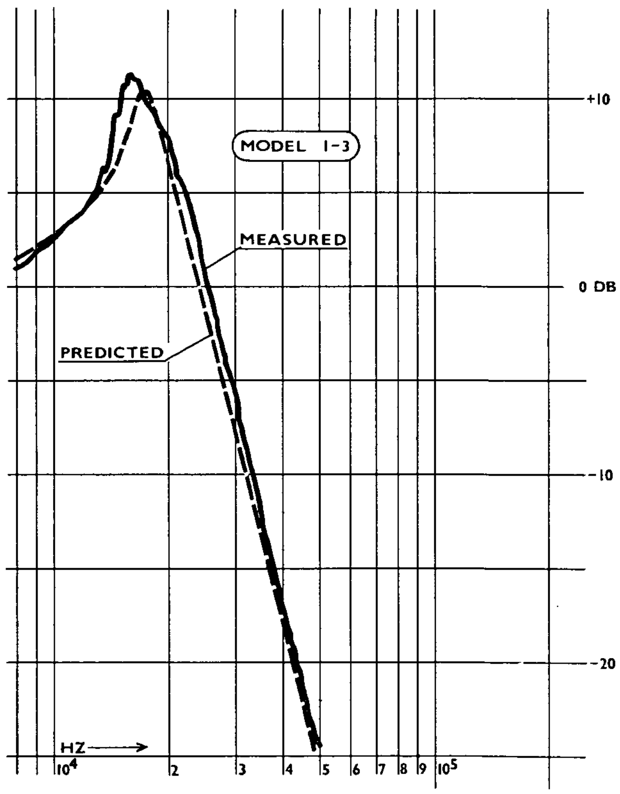

Fig. 6. **b.** Comparison of measured and theoretical response curves for model 1-3 pickup. Parameters as listed for Fig. 6a.

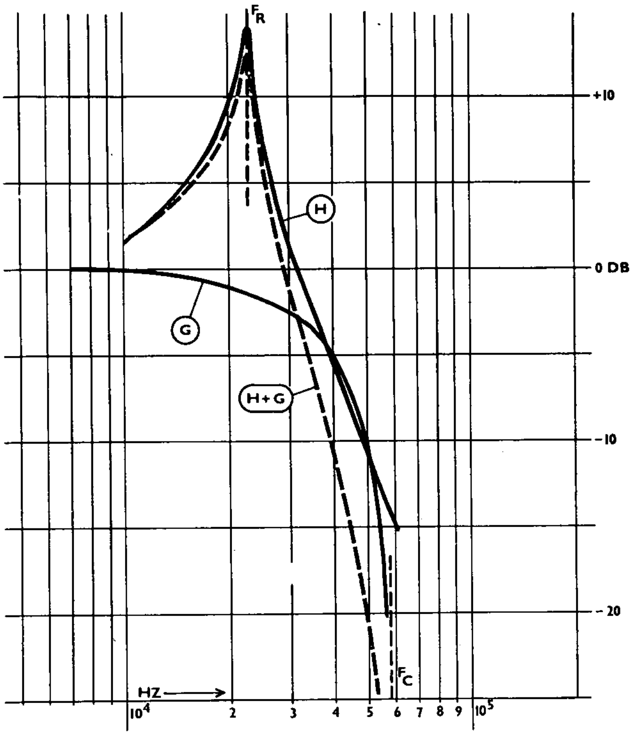

Fig. 7. **a.** Theoretical high frequency response curve for model 2-2 pickup with moving mass $m = 1.2$ mg, damping $\epsilon = 0.2$. Other parameters as in Figure 6a.

measured response of this particular pickup. Note that the two curves match within 2 dB or better.

Pickup 2-2

For this model, the moving mass was 1.2 mg and the damping factor $\epsilon = 0.2$.

Again, we calculate $f_r = 23{,}000$ Hz; since the stylus force, tip radius, groove speed and record material were the same as in the preceding example, we again have $f_c = 58{,}000$ Hz.

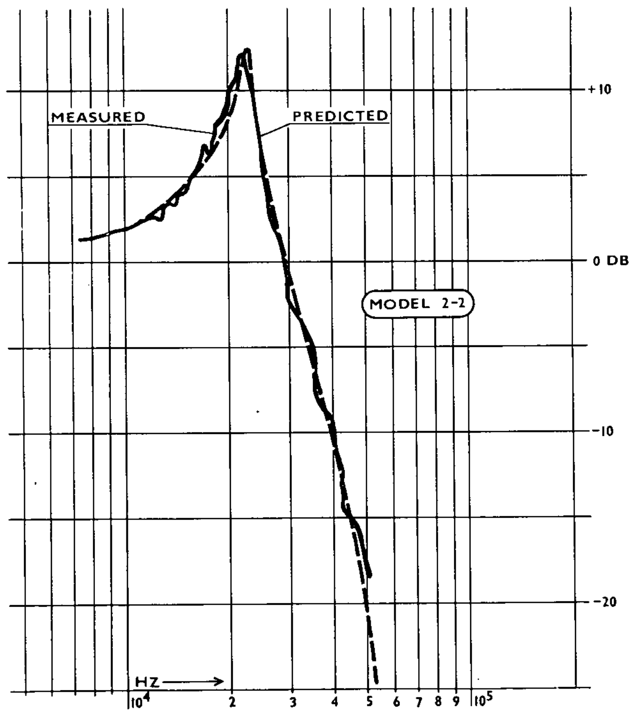

Fig. 7. **b.** Comparison of measured and theoretical response curves for model 2-2 pickup. Parameters as listed for Fig. 7a.

Both functions are plotted and added in Fig. 7a; the resonance peak is somewhat higher with this pickup because of a lower damping value. Again the $(H+G)$ plot is transferred to Fig. 7b and compared with the measured response, which it matches very closely.

Pickup 3E

This pickup has a very much lower moving mass and also more damping. Its moving mass = 0.4 mg and ϵ = 0.7.

With the given characteristics, this pickup should exhibit a stylus/groove resonance $f_r = 40{,}000$ Hz. It was decided to assure a cutoff frequency well beyond this, so a higher groove speed (.79 m/sec) was used for playback. As a result, f_c should occur at 92.5 kHz. Both H and G functions are depicted in Fig. 8a and added to obtain the dashed curve. Comparison of the latter with the measured curve again discloses a very close match, within 2 dB (see Fig. 8b).

Pickup 5B-T

To investigate the pickup response under circumstances where $f_r > f_c$, this pickup was designed to have extremely low moving mass, while the stylus force was increased to 2 grams. Its moving mass was 140 μg, and $\epsilon = 0.2$.

Figure 9a shows the H function for a calculated $f_r = 70{,}000$ Hz and the G function for $f_c = 122.5$ kHz at an

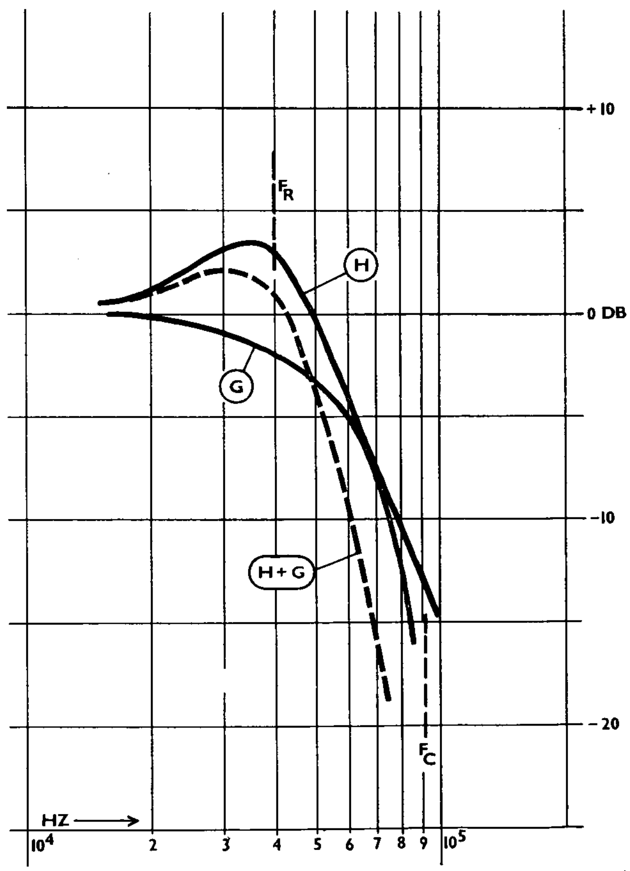

Fig. 8. **a.** Theoretical high frequency response curve for model 3E pickup with moving mass $m = 400$ μg, damping $\epsilon = 0.7$. Other parameters as in Figure 6a, except for groove velocity $V = 0.79$ m/sec.

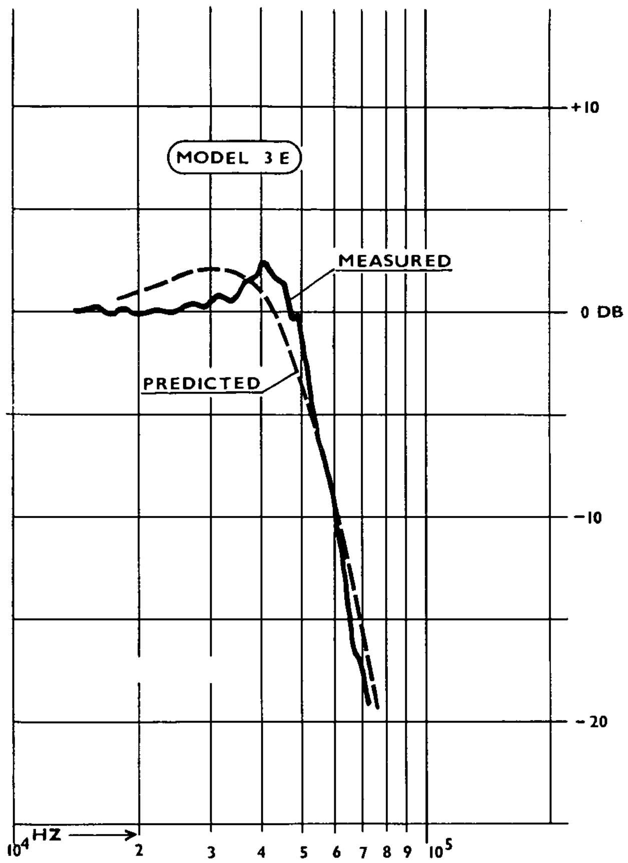

Fig. 8. **b.** Comparison of measured and theoretical response curves for model 3E pickup. Parameters as listed for Fig. 8a.

outer-diameter groove velocity of 1.15 m/sec. The resulting addition (dashed curve) is next transferred to Fig. 9b, where it agrees with the measured response within 2 dB (Curves I and II). Furthermore, the graph depicts the measured response (Curve III) at an inner-diameter groove velocity of 0.6 m/sec (f_c = 64 kHz). The *difference* between this inner diameter response and the *H* function (see Fig. 9a), which should normally "match" the *G* function for f_c = 64 kHz, has also been depicted. To avoid a cluttered graph this Curve IV has been transferred to Fig. 9c. The theoretical *G* function is shown as Curve V and it can be clearly seen that it does not match Curve IV. However, if the scanning loss function is added to the theoretical *G* curve, the response indicated by Curve VI is obtained, which is more of a resemblance to Curve IV. *The conclusion is that the S function should not be ignored at very high frequencies.* This is quite understandable since the recorded wavelengths in this case are indeed comparable to the finite size of the needle/groove contact surfaces! The fact that a very close match between theoretical and measured curves cannot be shown here is probably due to the possibility that the *G* and *S* functions cannot be simply added since these two phenomena are really two aspects of the same physical factors. Moreover, the radius of groove curvature at these frequencies does not satisfy the condition $\rho \geqq 1.5R$.

CONCLUSION

The *H*, *G* and *S* functions prove to be valuable tools in predicting the performance of a pickup design. The vertical stylus force must be kept low enough so as to cause mainly elastic deformation of the groove, at the same time assuring proper tracking. The *G* and *S* functions are the prime factors that limit the passband of a pickup/record playback system. An obvious improvement would be to use a record material that is very much stiffer than the Vinylite compound presently used. Preliminary tests with nickel records have not been successful, possibly due to the fact that almost complete plastic flow prevailed, even at a stylus force of 1 gram. Since the metal is very much harder ($E = 21 \times 10^{10}$ N/m^2 $\approx 30 \times 10^6$ psi), the indentation is indeed very small. The obvious result is that the pressure exerted by the stylus on the groove walls is very high and exceeds the yield point. The sharp knee in the stress/strain curve of nickel means that plastic flow sets in instantaneously, with hardly any elastic components.

These experiments indicate that, at the present state of the art, only a very much lower stylus force offers a solution for increasing the passband of a pickup. This, however, can be possible only if the moving mass is small enough to cope with the extremely high accelerations encountered in the high-frequency region. However, a record material that is very stiff and features a very high yield point, so that deformation of the groove walls is limited to the elastic region with very little indentation, would indeed be a worthwhile move towards a truly wideband phonograph pickup system.

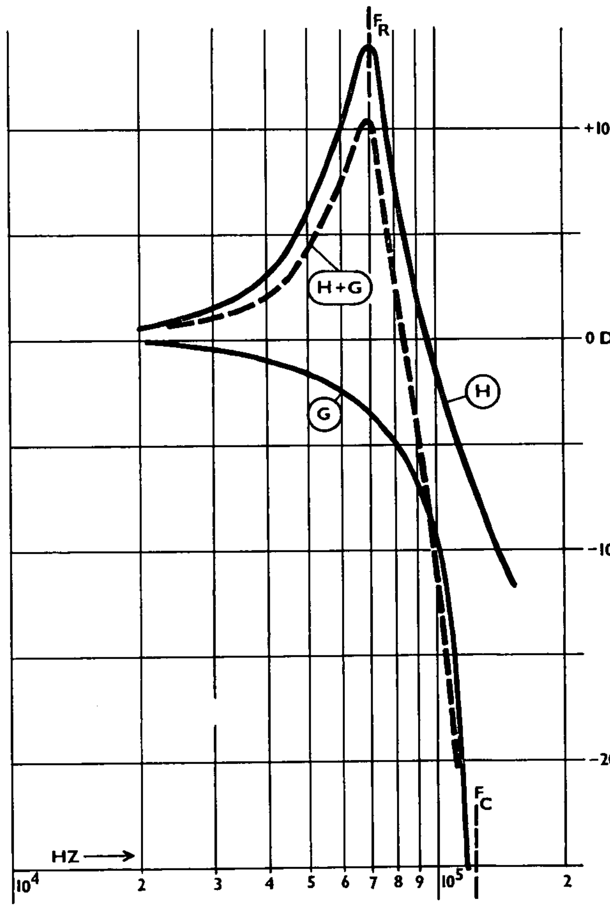

Fig. 9. **a.** Theoretical high frequency response curve for model 5B-T pickup with moving mass $m = 140$ μg, damping $\epsilon = 0.2$, tip radius $R = 17.8$ μ and stylus force $F_v = 2$ g on Vinylite with a groove velocity $V = 1.15$ m/sec.

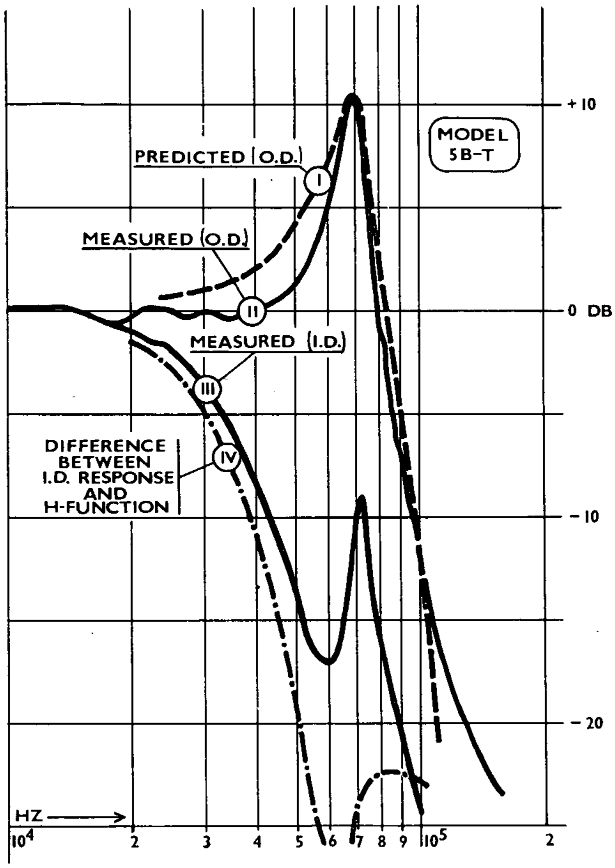

Fig. 9. **b.** Comparison of predicted (Curve I) and measured (Curve II) response curves for model 5B-T pickup. Parameters as in Figure 9a. Curve III is the measured response curve for $V=0.6$ m/sec, and Curve IV depicts the difference between H function and Curve III.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the cooperation of Romolo and Richard Marcucci and Sal Gualtieri of Capps and Company, Inc. who supplied the special stylus tips for the experimental pickups and by H. Bregman of Sonic Recording Products, Inc. who did such an excellent job in the processing of the many test records.

APPENDIX I
Summary of Symbols and Terms Used

(In Alphabetical Order)

Symbol	*Term*	*Dimension (Rationalized MKS)*
A	Recorded amplitude	m, meter
A'	Stylus excursion	m
C	Compliance of record material $= 1/\sigma$	m/N, meter per Newton
d	Indentation width	m
E	Young's modulus of elasticity	N/m^2, Newton per square meter
E_0	Constrained Young's modulus $= E/(1-\nu^2)$	N/m^2
F	Force	N, Newton
F_n	Force, normal to a surface	N
F_v	Vertical stylus force	N
f	Recorded frequency	Hz, Hertz
K	Materials constant	(m^2/N)$^{2/3}$
m	Dynamic moving mass of armature	kg, kilogram
R	Radius of stylus tip	m
S	Stiffness of armature suspension	N/m
V	Tangential groove velocity	m/sec, meters per second
ω	Recorded frequency $= 2\pi f$	rad/sec, radians per second
ω_c	Cutoff frequency	rad/sec
ω_0	Free armature resonance of pick-up	rad/sec
ω_r	Stylus/groove resonance frequency	rad/sec
ω_s	First scanning loss null frequency	rad/sec
β	Half included groove bottom angle	degrees
δ	Indentation depth	m
δ_0	Indentation depth in a silent groove	m
Δ	Difference between A and A'	m
ϵ	Damping factor $= \zeta/(m\omega_r)$	(dimensionless)
ζ	Damping coefficient	kg/sec, kilogram per second
λ	Recorded wavelength	m
ν	Poisson's ratio	(dimensionless)
ρ	Radius of curvature of recorded waveform	m
σ	Stiffness of record material	N/m

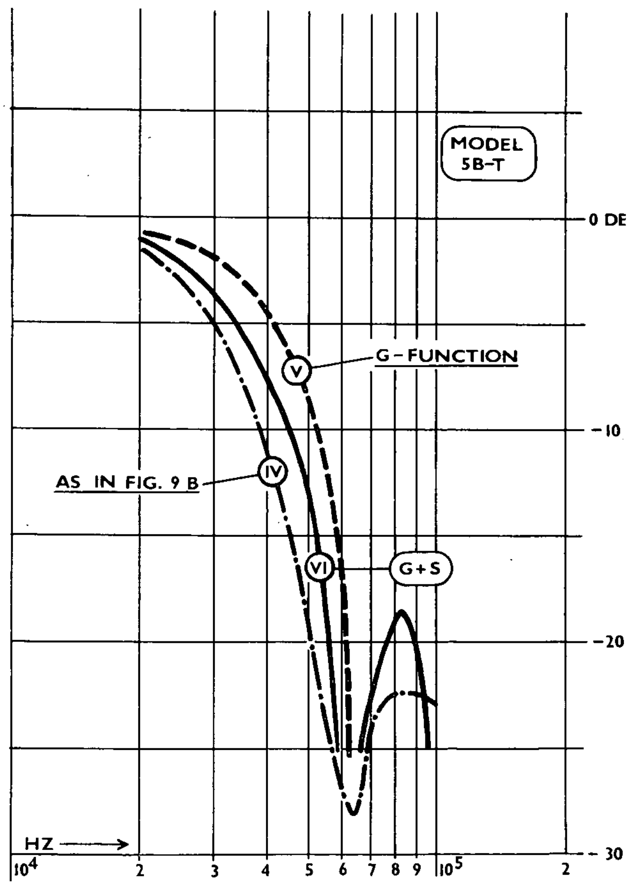

Fig. 9. **c.** Situation at very high frequencies where the scanning loss function cannot be ignored. Curve IV is the same as Curve IV in Figure 9b and Curve V is the theoretical G function, whereas Curve IV is the combined G and S function. Model 5B-T pickup, parameters as listed for Figure 9a, except groove velocity $V=0.6$ m/sec.

APPENDIX II
Calculation of the Radius of Curvature

The radius of curvature at any point of a function $f(y)$ is given[11] by:

$$\rho = (1+y'^2)^{3/2}/y'' \qquad \text{(A1)}$$

where y' stands for the first derivative and y'' is the second derivative of the function $y = f(x)$.

In the case of a sinusoidal groove modulation (see Fig. A1)

$$y = A\sin\omega t = A\sin(2\pi x/\lambda) \qquad \text{(A2)}$$

where λ is the wavelength equal to the tangential groove velocity V divided by the frequency f:

$$\lambda = V/f. \qquad \text{(A3)}$$

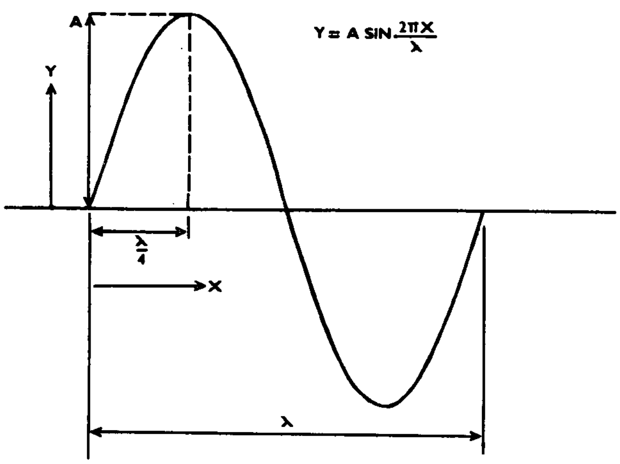

Fig. A1. Sinusoidal groove modulation with amplitude A and wavelength λ.

The first derivative $y' = (2\pi A/\lambda)\cos(2\pi x/\lambda)$, and the second derivative $y'' = -(4\pi^2 A/\lambda^2)\sin(2\pi x/\lambda)$. Equation (A1) then is written as:

$$\rho = \frac{[1+(4\pi^2A^2/\lambda^2)\cos^2(2\pi x/\lambda)]^{3/2}}{(4\pi^2A/\lambda^2)\sin(2\pi x/\lambda)}. \qquad \text{(A4)}$$

It is obvious that a minimum for ρ occurs at $x = \lambda/4$, $3\lambda/4$, etc. Substituting $x = \lambda/4$ in Eq. (A4) gives

$$\rho = \lambda^2/4\pi^2 A. \qquad \text{(A5)}$$

Replacing λ by V/f, Eq. (A5) becomes Eq. (10) in the text:

$$\rho = V^2/4\pi^2 f^2 A = V^2/\omega^2 A. \qquad \text{(10)}$$

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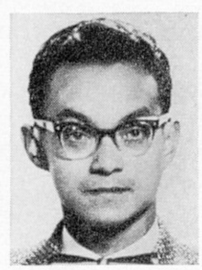
THE AUTHOR



C. R. Bastiaans was born in the former Netherlands East Indies in 1924. He attended school in Holland where he received the diplomas of radio engineer and electrical engineer. In 1950, after five years as an officer in the army, he joined the Philips telecommunication department as a technical-commercial engineer. In 1957, he became head of the electro-mechanical development group of Philips Phonographic Industries. Following this, in 1963, he joined the research staff at Westinghouse in Pittsburgh, Pennsylvania where he is in charge of Special Projects in their Research and Development Center.

Applications have been made for several of Mr. Bastiaans' patents in the audio field. He has been a contributor to the Philips Technical Review and has been published previously in the Audio Engineering Society Journal. Mr. Bastiaans is a member of the Audio Engineering Society.