How to calculate the depth of a horn from desired mouth size (OSWG-SE)

Waveguide profile equation:

$$y(x)=OS(x)+SE(x)$$

$$y(x)=(r_0^2+2r_0x\tan(\tau)+x^2\tan^2(\alpha))^{1/2}+LSq^{-1}(1-(1-q^n x^n L^{-n})^{1/n})$$

Design parameters: r_0, theta, alpha, s, q, n, Y.

$$k_1=r_0^2$$

$$k_2=2r_0\tan(\tau)$$

$$k_3=\tan^2(\alpha)$$

$$k_4=\frac{s}{q}(1-(1-q^n)^{1/n})$$

Y = mouth size

Looking for L for which y(L) = Y:

a) if $k_4^2=k_3$, then

$$L=\frac{Y^2-k_1}{2Yk_4+k_2}$$

b) else

$$L=\frac{2Yk_4+k_2-(4Yk_4k_2+k_2^2+4k_4^2k_1+4k_3Y^2-4k_3k_1)^{1/2}}{2(k_4^2-k_3)}$$